HOMEWORK SET 1
DUE DATE: OCTOBER 11TH 2023

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Problem 1. For $(\alpha, \beta) \in \mathbb{R}^2$, set

$$f_{\alpha,\beta}(z) = z^{\alpha}\bar{z}^{\beta}.$$

Are there any choices of (α, β) (other than $(1, 0)$) for which $f_{\alpha,\beta}$ is a quasiconformal map of the complex plane?

Problem 2. Prove that the definition of module of a quadrilateral is a conformally invariant quantity. In other words, show that if $f : \Omega_1 \to \Omega_2$ is a conformal map and $Q(z_1, z_2, z_3, z_4) \subset \Omega_1$ is a quadrilateral, then $M(Q(z_1, z_2, z_3, z_4)) = M(f(Q)(f(z_1), f(z_2), f(z_3), f(z_4))$.

Problem 3. For $0 < r < R$, let

$$A(r, R) = \{z \in \mathbb{C} : r < |z| < R\} \subset \mathbb{C}$$

be an annulus. Compute the extremal length of the family Γ that consists of smooth arcs that join the circles $\{|z| = r\}$ and $\{|z| = R\}$.

Problem 4. Prove that the standard planar von Koch snowflake based on an equilateral triangle is a quasicircle, in the sense that it satisfies the Ahlfors condition.

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