Some (personal) small-group-based experiences teaching Mathematics

A great part of Mathematics research is carried out in research-teams, where the social interaction of the members is fundamental for developing new ideas and tools, as well as learning new tools and techniques. Besides, the peer-review of our work is the fundamental mechanism that we have as a community to ensure scientific progress. Thus, this social peer-learning and interaction shapes our identity as Mathematicians.

So, it seems natural to bring this part of our identity as a community into our teaching, by for instance organising courses around small-group activities and borrowing ideas from the TBL approach (Team Based Learning). In this seminar, I will present some experiences **(and frustrations)** regarding putting this idea into practice. The examples will include MM5021 (campus & online), MM700**5**.

“Those are my principles, and if you don't like them...well I have others.”

— Groucho Marx

Personal transition

Becoming a Mathematician

("similar to produce a fine Cognac")

→ **Mathematical Maturity**

Broad

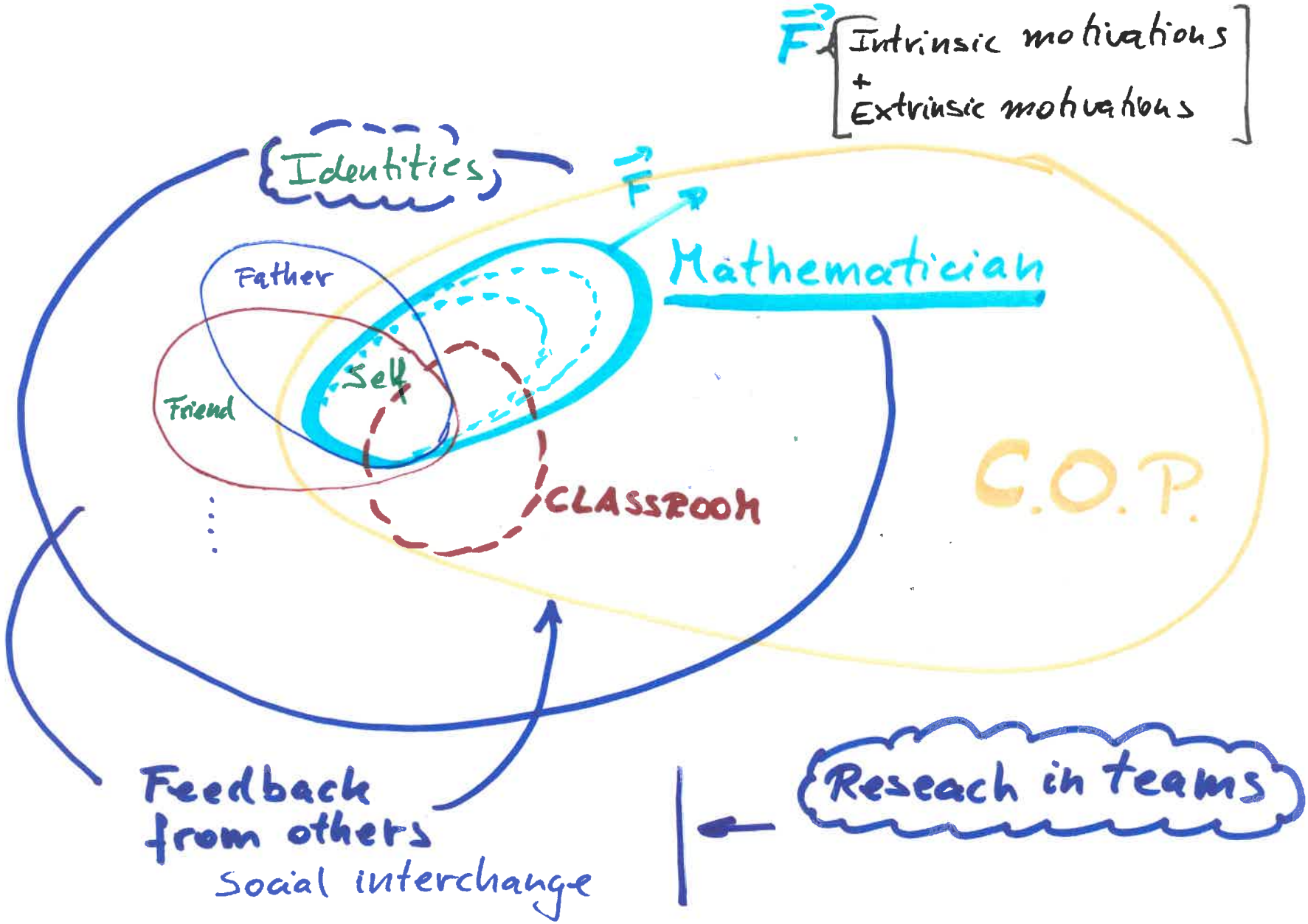
Narrow

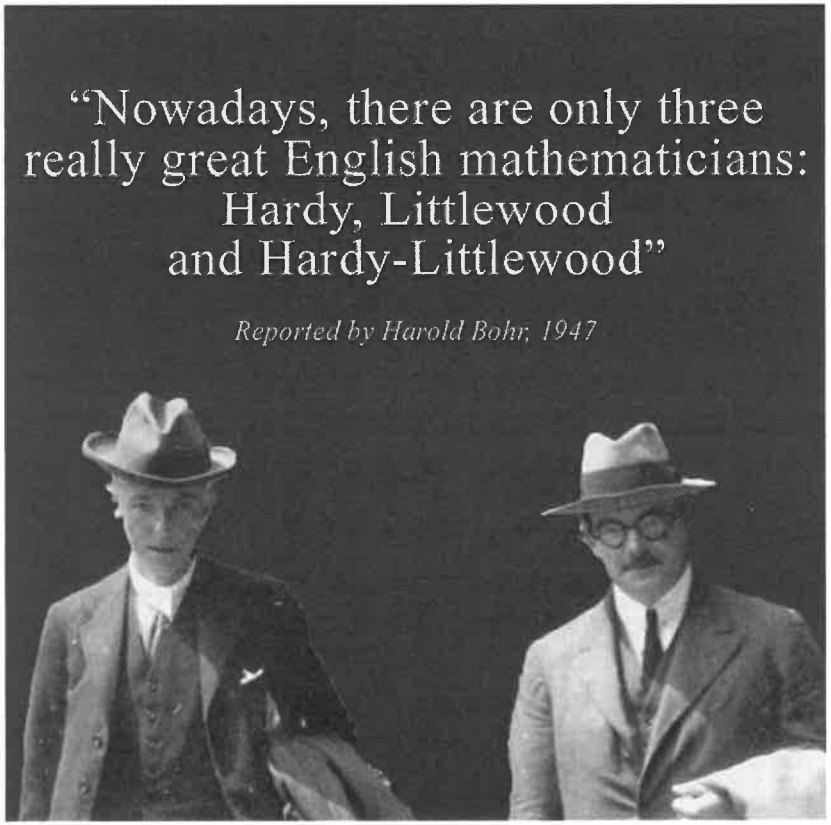
Learning Mathematics is...	Mathematics is...
Developing a way of thinking	An approach to life [...] an integral part of professional & Personal life.
Understanding practical & theoretical aspects	Building and using models of aspects of reality
Acquiring tools & techniques	Individual set of components or techniques (numbers, formulae calculations.

Analogy: How to make chairs: Design & build an item, noone has thought of it before

How to use a hammer & drive a nail

- L.N. Wood, Becoming a Mathematician
- I. Stewart, Letters to a Young mathematician.


$\vec{F}$ [Intrinsic motivations + Extrinsic motivations]
Identities
$\vec{F}$
Mathematician
Father
Friend
Self
CLASSROOM
C.O.P.
Feedback from others
Social interchange
Reseach in teams



Axioms 1 and 2 are particularly interesting. One thing that I enjoy about my own collaborators (and I have had 56 of them) is that we are completely comfortable with each other. When we get together to do mathematics, there is no pretense and no sense of one-upmanship. **Our only goal is to create good mathematics.** We have **mathematical "rap sessions" together,** and these are free and open, with no holds barred. **Anyone can say anything that they like. It does not matter whether it is right or wrong**. The point to is get the juices flowing. To get ideas out in the air so that we can examine them.

*"[John E. Littlewood] is remembered as G. H. Hardy's collaborator—they authored **100 papers together**—but also as a powerful mathematician in his own right.*

Four rules that they adhered to when conducting their seminal collaboration:

(1) **When one man wrote to the other, it was of no matter whether what he wrote was right or wrong. This gave them complete freedom to say whatever they liked.**

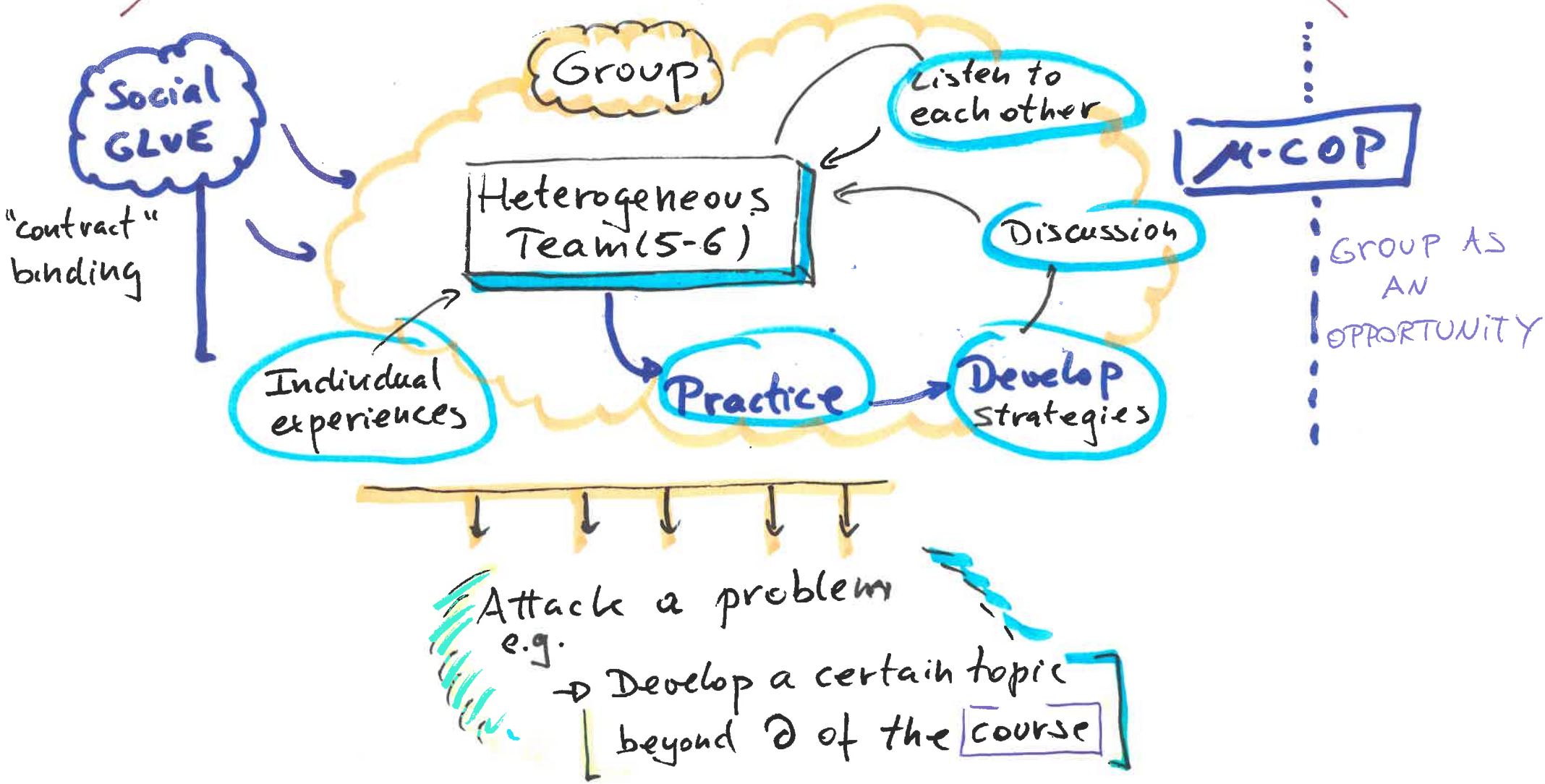
(2) When one received a letter from the other, he was under no obligation to read it.

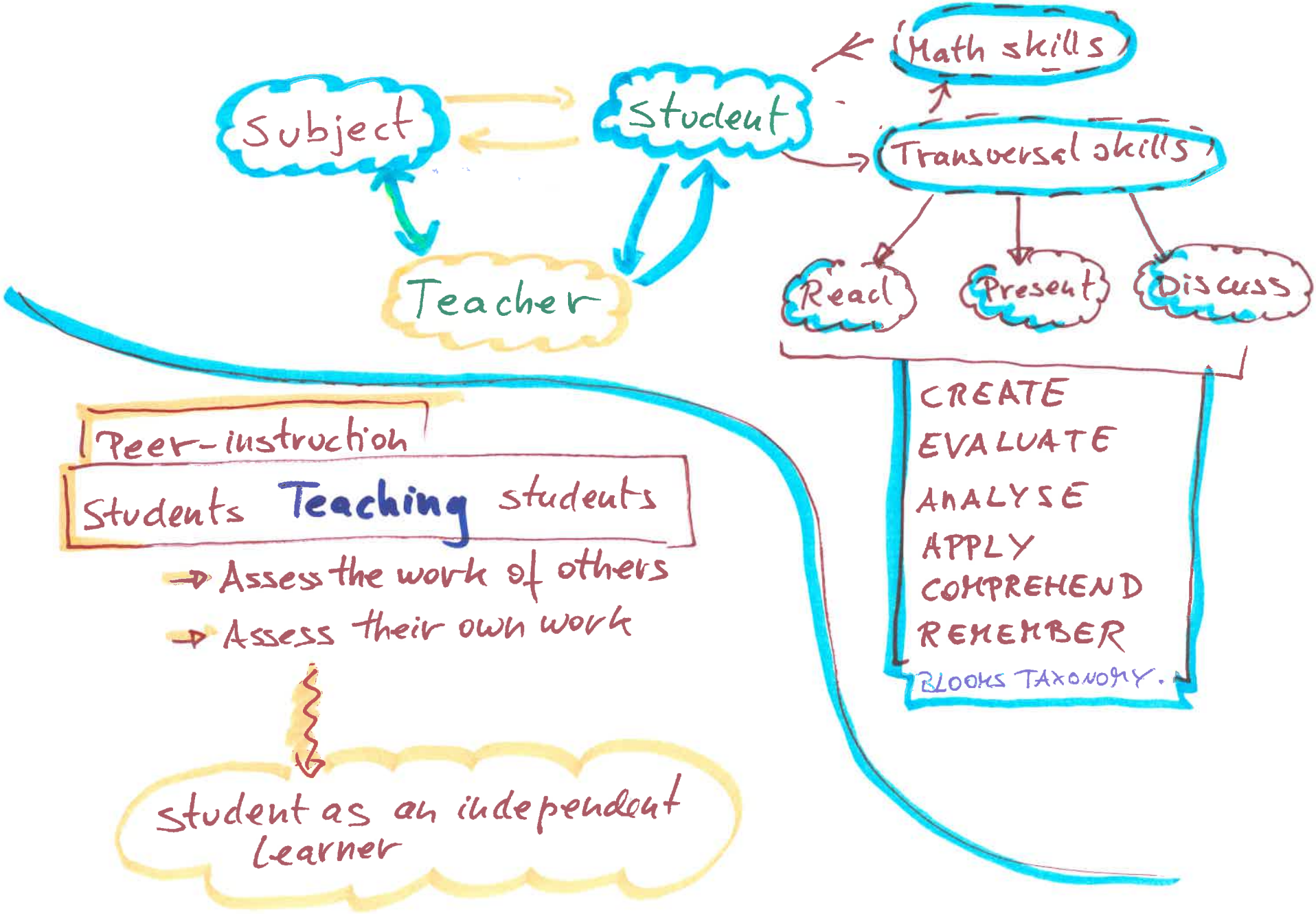
(3) Although it was all right for both of them to think about the same detail, it was better if they did not.

(4) If one of them contributed very little or nothing to a given paper, it did not matter. Both their names would go on the paper, and they would both claim credit for the theorem. "

S. Krantz, *A Mathematician comes of age*, AMS, 2012

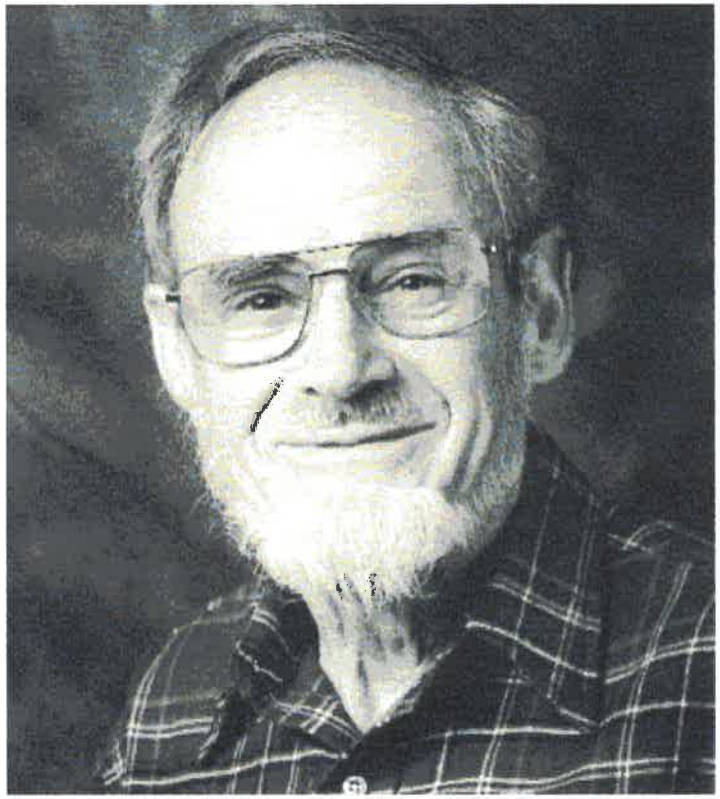
[TEACHER ≡ COACH] Design learning Experiences




Math skills
Subject
Student
Transversal skills
Teacher
Read
Present
Discuss
CREATE
EVALUATE
ANALYSE
APPLY
COMPREHEND
REMEMBER
BLOOMS TAXONOMY.
Peer-instruction
Students Teaching students
Assess the work of others
Assess their own work
student as an independent learner

“I do not know how to present mathematical ideas so effectively that students can take possession of them simply by sitting at my feet and smelling my socks”

G. Piranian (1914-2009),
The Problem of Learning to Teach, *Amer. Math. Monthly* 82, 5, 1975



What is a Mathematician?

What is Mathematics?

What is Mathematics Maturity?

Ian Stewart, *Letters to a young Mathematician*

There is a famous book called *What Is Mathematics?* written by Richard Courant and Herbert Robbins. As with most books whose titles are questions, the question is never quite answered. Yet the authors say some very wise things. Their prologue begins, "Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection." It goes on to tell us that "All mathematical de-

Reuben Hersh. He observed that Courant and Robbins answered their question "by *showing* what mathematics is, not by *telling* what it is. After devouring the book

Hersh's answer is what he calls the humanist philosophy. Mathematics is "A human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context." This is a description, not a definition, since it does not specify the content of that activity. The description may sound a bit

Mathematical maturity is like pornography: I don't know what it is, but I know it when I see it.

John P. D'Angelo (mathematician)

A mathematician is mature up to the point where he becomes interested in mathematics.

Vladimir I. Arnol'd (mathematician)

It seems clear that the role model for a student endeavoring to achieve mathematical maturity is the senior, successful mathematician. Such a person could be an academic professor with a vigorous research program and an international reputation. It could be a successful and innovative worker in the private sector—such as Robert Noyce, inventor of the memory chip and the microprocessor. It could be a leader at one of the many government institutes—such as the National Security Agency (largest employer of math Ph.D.s in the world), or the Institute for Defense Analyses, or Oak Ridge National Laboratory, or Lawrence Berkeley Labs.

S. Krantz, A Mathematician Comes of Age, AMS, 2012

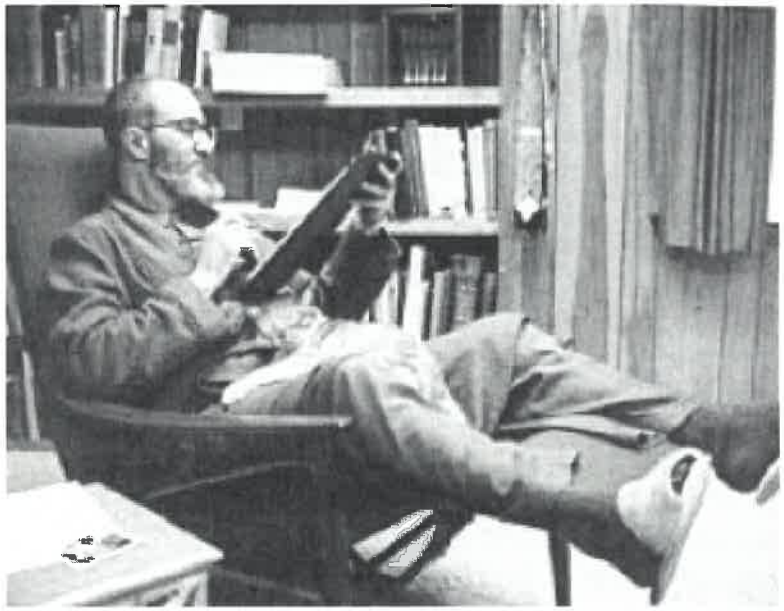
Key Attributes of Mathematical Maturity

- *Learn how to learn mathematics.*
- *Learn to handle abstract ideas.*
- *Recognize mathematical patterns.*
- *Separate ideas from facts.*
- *Learn to draw upon already-learned or already-encountered skills and techniques in order to develop new problems and new techniques.*
- *Learn to read and analyze proofs. Learn to detect incorrect or sloppy proofs. Learn to create your own proofs.*
- *Move comfortably between the analytical (solving equations, analyzing inequalities, manipulating functions) and the visual or graphical.*
- *Learn to develop links between analytical representations of ideas and geometrical representations of ideas.*
- *Learn to communicate mathematical ideas both orally and in writing.*
- *Learn to represent written and verbal problems as mathematical problems. Learn to introduce suitable variables, functions, and notation so as to represent a problem mathematically.*

S. Krantz, A Mathematician Comes of Age, AMS, 2012

I spent most of a lifetime trying to be a mathematician—and what did I learn? What does it take to be one? I think I know the answer: you have to be born right, you must continually strive to become perfect, you must love mathematics more than anything else, you must work at it hard and without stop, and you must never give up.

—*Paul Halmos,* I Want to Be a Mathematician: An Automathography in Three Parts, *Mathematical Association of America, Washington, 1988, p. 400*



My particular attempts...

MM5021 - Foundations of Analysis (Campus VT18+ Online ST18)

MM7005 - Linear Analysis (HT18)

[MM5021 - Campus]

CLASSROOM

Tues	Fri
2h Lectures	Lectures
1h Tutorial	Tutorial

↳ Individual presentations

↳ Affects the group grade

- Hand-in-of homework
- Group discussions
- Pollinisation

Group of 5/6 Individuals (7 groups)

Group-work

Homework

Individual

→ suggested exercises

→ study

34%

8 weeks ≡ 7 Homework

≡ 20 [problems] (3×6+2)

↳ ~3 [Challenging] problems

→ 2 PROOF-BASED

→ 1 TRUE-FALSE Q.

Peer Assessment

(Bonus points)

66% ORAL EXAM.

Course Evaluation MM5021 Mathematics III - Foundations of Analysis, Spring 2018
Respondents: 63Answer Count: 17Answer Frequency: 26.98 %

What was the best aspect of the course?

Working with my group on the homeworks was really fun.

Interesting material, I feel like I learned a lot. Working in groups was quite enjoyable and effective.

Working in groups

What improvements would you suggest in the course?

Personally I would like to remove all group-work, but generally it appear to be quite popular.

The group examination being pass or fail. It's quite stressful having your grade depend on others.

I did not like that if a SINGLE member of the group went up and presented something at the blackboard and performed poorly then EVERYONE's grade in the group got punished. Does not seem fair. The performance of an individual should only affect that individuals grade.

What advice would you like to give to future course participants?

Don't believe the "sarskild behorighet"... The course focuses VERY much on developing mathematical intuition and maturity. The content of the book corresponds well to the "sarskild behorighet", but the construction of the course by the lecturer corresponds to a more advanced level.

Less groupwork and clear instructions on what counts towards the final grade.

Also, I think that the groups should be better monitored. My group didn't function that well but the teachers were not aware of this until the very end of the course.

Other comments

It feels very much as if the lecturer really dreams of giving a doctoral course where students with a great level of mathematical experience can discuss different topics. Well, this is grundniva.

I get that the thought of working in groups seams good and maybe it works for many groups, but when skill and ambition is to diverse in one group I don't think it is good for any party in the group.
Grupparbetena mycket svårare än kursmaterialet i övrigt

MM5021 - Distance ST18

Coursebook + Online material (11+1 weeks)

- Video (Focus on threshold concepts/theorems)
- Further written material

Evaluation

- ~58% 35 → Final written exam
  - 1 TRUE-FALSE
  - 3 QUESTIONS
- ~17% 10 → Individual HW I
  - → 3 Quizzes
    - TRUE/FALSE questions
    - one chance / limited time
    - Answer within a time period
- ~25% 15 → GHW GROUP HW
  - → Telematic discussions
  - → Overleaf.

- 1 grade for the whole group (N=3 homework)
- Multiplying factor affecting the individual grade determined by Peer assessment (Fink method)

$$Hw = \sum_{j=1}^{N} (\underbrace{GHw_j \cdot m_j}_{\text{Ass. 5 students}} + I_j) \qquad m_j \in [3/4, 5/4]$$

Approaches to Peer Evaluation

Peer Evaluation is an essential component of the Team-Based Learning Model. Ensuring that you develop a peer evaluation method that is both fair and "defensible" is one of the keys to a successful TBL course. Some students may express concern when they are to be evaluated by their peers. It is important to stress the value of peer feedback, and how common peer feedback is in the modern workplace.

In the context of TBL, peer evaluation has teammates assessing teammates in order to "measure" the "helping behaviors" and to develop a peer evaluation scores. Peer evaluation does not measure how much a student knows, but how his or her contribution to team cohesion and productivity is perceived by their teammates.

Peer evaluation can have numerous benefits:

1. By motivating students to strive for excellent performance

2. By giving comfort to "stronger" students who fear that they will be doing most of the work

3. By providing the instructor with a more accurate way of assigning individual grades at the end of the course (students know their teammates' contributions better than the instructor does).

4. By helping students develop their feedback and evaluation skills

Fink Method:

This is another anonymous method where the evaluation score is used as a multiplier with the team portion of the course grade. This can be very useful in courses that have a larger team component grade (i.e. project courses). In this method it is possible for a team member to get higher or lower mark then the team product grade - depending on the peer evaluation. For example, one hard working student may get 90% on the report (that was originally marked at 80%) and another student that has contributed less may get 70%. This method can use simple (divide the money) evaluations or rubric-based evaluations. Rubric-based evaluations are often perceived to be fairer by students, as they seem to be more comfortable assigning lower scores, when they can justify them (Ostafichuk, personal communication).

A number of TBL practitioners are now recommending that students develop their own performance criteria, by which everyone will be judged. "Student value that which they create" (Lane, 2007). An activity is added to the start of the semester, where student work in their teams to develop the criteria (the whole class must agree on the same criteria). This can increase student buy-in to peer evaluation.

Pros and cons: Unlike the Michaelsen Method, this method allows students to give the same score to each teammate, which may lead to more student satisfaction. A potential problem is that sometimes students underestimate the degree to which they can affect a teammate's final grade with relatively small changes in the peer evaluation scores.

Rick Goedde, St. Olaf College, goedde@stolaf.edu

Jim Sibley, University of British Columbia, jim.sibley@ubc.ca

A Problematic with online courses: The drop-out is.

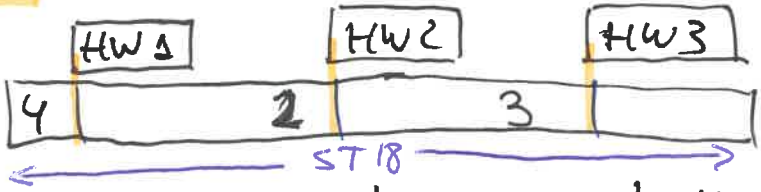
HUGE

In this case

77 in the list from Jennifer (→ Password)

40 logged-in at least once

25 Responded a initial call (Active choice)



9 Drop-out

16 went from beginning to the end

13 took the exam.

5 passed (1B 1C 2D 1E)

↔ Groups binded & regulated by a

5-6 individuals

Group contract

- 1st week of the course
- → Ice-breaking
- → Set: · Aims
  · Expectations
  · Framework.

Research

University of Waterloo (Canada)

Course Evaluation MM5021 Mathematics III - Foundations of Analysis, Summer 2018

Respondents: 37
Answer Count: 7
Answer Frequency: 18.92 %

What improvements would you suggest in the course?

No group contract, or even better, no group homework. It was mostly an obstacle to handle the homework in a group setting. Students can discuss the course content without having 2-4 other students writing solutions to the exercise that was allocated to yourself...

Comment on the overall impression

Too much controlling bullshit surrounding everything, even though I liked the math content.

What was the best aspect of the course?

It has all the course material online. We are not stressed to buy any books.
The course material itself was interesting.
The mandatory group work. I wish all math courses had similar, mandatory forum-based group homeworks, since you get so much out of the discussions.
That it was a distance course and that it had a written exam instead of an oral.
The group work was better functioning than expected.
Baby Rudin.
Getting a better theoretical grounding in real analysis

MM7005 (Flipped system)

[Classroom]

Tues
- 3 h "Lecture"

Fri
- 1 h "Lecture" ⇝ RAT
- 2 h Tutorial ⇝ Slovenian style → Oral presentation

3 Groups ~5 students

(☆) In classroom group discussions
- → Basic definitions
- → Proof of Theorems

9 lists of "Exercises"

Home
- → Material (Slides) available online
- → Supporting material
- → Videos "on demand"

Evaluation
- Written exam 80%
- Homework (Individual) 20%
- Oral presentation → Bonus [≤10]

(☆) I had to remind the students to have a brake!!

MM7005 - Linear Analysis - Fall 18

Respondents: 23 Answer Count: 6 Answer Frequency: 26.09 %

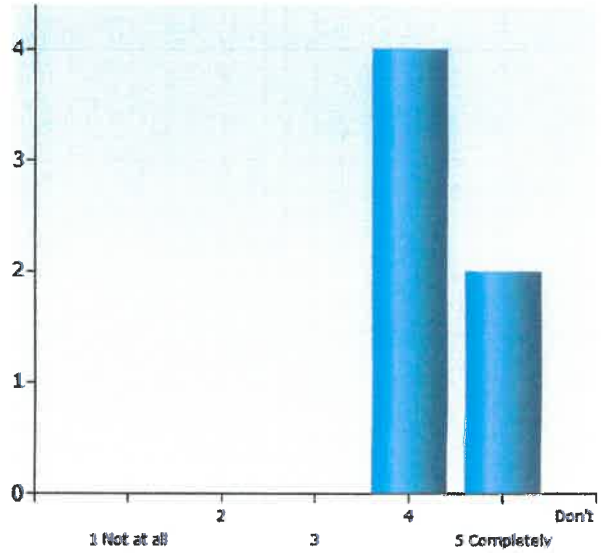
What was the best aspect of the course?

The course has many interesting applications. A good thing was that the course was full of exercises and group discussions about the problems.
The topic of this analysis course was very very interesting
innovative, discussion of problems with little groups and then comparison of the answers with the entire class
Good literature. Encouragement of student participation on exercise sessions.

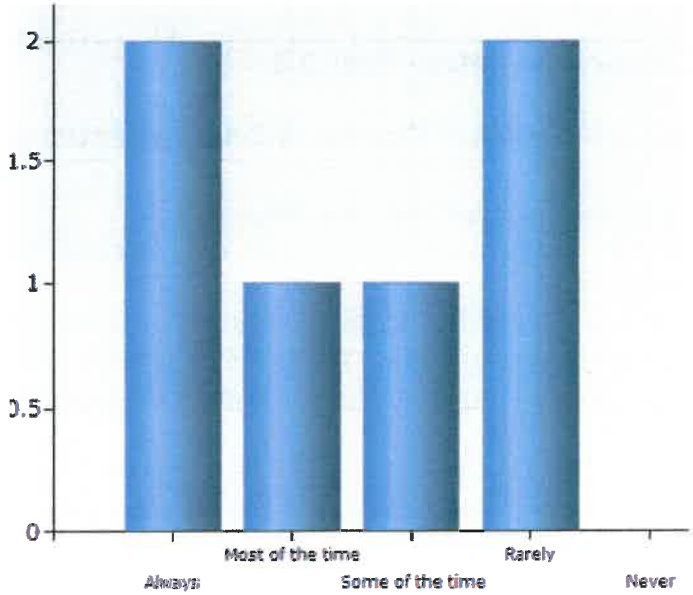
How has been your experience on the flipped system approach implemented on the course? What did you like most?

It was okay but it could have benefited from being mixed with conventional lectures. These sessions were good because they made one extra aware of what one needed to study more.
system that demands more work from the student but it allows to learn more deaply the course
I thought it went well. I liked that the focus during the classroom time was on how to solve problems and apply the results in the slides, since I think one usually only gets that perspective by doing exercises at home.
As it was unfamiliar it was difficult to gauge how well prepared I was for the exam, but other than that it was interesting to try this different system. TH ebest part was when we ourselves were proving important statements (like the uncertainty principle) with just a bit of guidance on what steps to take

Overall I am satisfied with this course



Did you read the material on time?



Did you find working in groups on the suggested problems being useful?