

# Categorial Dependency Grammars: Analysis and Learning (Invited Talk)

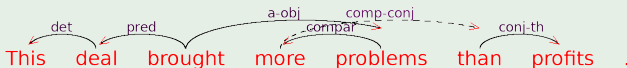
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LACompLing 2021, Montpellier, December 15–17 2021

- 1 Introduction to CDG
- 2 CDG Languages
- 3 CDG Analysis
- 4 Grammatical Inference
- 5 *K*-star CDG
- 6 Conclusion and Open Problems

Surface Dependency Structures (DS) are graphs of surface syntactic relations between the words in a sentence.

## A Dependency Structure



Dependencies are determined by valencies of words

*brought* has +valency **pred** of a **left adjacent** word

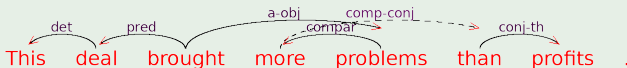
*deal* has -valency **pred** of a **right adjacent** word

Saturation of valency **pred** determines **projective dependency**

*deal* <sup>pred</sup> ← *brought* (**Governor:** *brought*, **Subordinate:** *deal*)

Surface Dependency Structures (DS) are graphs of surface syntactic relations between the words in a sentence.

## A Dependency Structure



Dependencies are determined by valencies of words

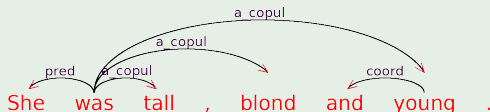
*more* has +valency *comp-conj* of a word *somewhere* on its right

*than* has -valency *comp-conj* of a word *somewhere* on its left

Saturation of *comp-conj* determines *non-projective dependency*

*more* *comp-conj* *than* (**Governor:** *more*, **Subordinate:** *than*)

## Some dependency valencies are MULTIPLE



`pred` is non-repeatable  
`a_copul` is repeatable

## Principle of Repeatable Dependencies [Mel'čuk'88]

- Every dependency  $d$  is either repeatable or non-repeatable
- $d$  is **repeatable** if SOME governor uses  $d$  in SOME DS at least ( $K =$ ) 2 times
- Any word governing through a repeatable dependency  $d$  in SOME DS may have any number of subordinates through  $d$

CDG Types express dependency valencies

## PROJECTIVE DEPENDENCIES (AND ANCHORS)

**Dependency:**  $Gov \xrightarrow{d} Sub$ :

**Governor Type:**  $Gov \mapsto [..\backslash..\backslash..\backslash d/..]^P$

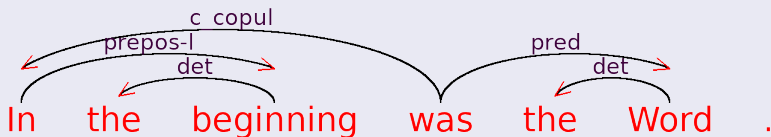
**Subordinate Type:**  $Sub \mapsto [..\backslash d/..]^P$

Anchors are non-important projective dependencies. Used for:

- Anchoring punctuation
- Anchoring the subordinate of non-projective dependencies

# Categorial Dependency Grammars (CDG)

CDG Types express dependency valencies



*in*  $\mapsto$  [*c\_copul* / *prepos-l*]

*the*  $\mapsto$  [*det*]

*beginning*  $\mapsto$  [*det* \ *prepos-l*]

*was*  $\mapsto$  [*c\_copul* \ *S* / *pred*]

*Word*  $\mapsto$  [*det* \ *pred*]

CDG Types express dependency valencies

## NON-PROJECTIVE DEPENDENCIES

Polarized valencies:  $\nearrow d$ ,  $\searrow d$ ,  $\nwarrow d$ ,  $\swarrow d$

Dependency:  $Gov \overset{d}{\dashrightarrow} Sub$ :

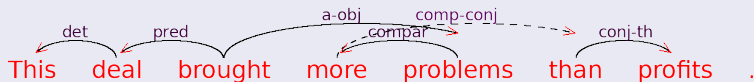
Governor Type Potential:  $Gov \mapsto [..] \nearrow d ..$

Subordinate Type Potential:  $Sub \mapsto [..] \searrow d ..$



# Categorical Dependency Grammars (CDG)

## CDG Types express dependency valencies



*this*  $\mapsto$  [*det*]

*deal*  $\mapsto$  [*det* \ *pred*]

*brought*  $\mapsto$  [*pred* \ *S* / *a-obj*]

*problems*  $\mapsto$  [*compar* \ *a-obj*]

*profits*  $\mapsto$  [*conj-th*]

*more*  $\mapsto$  [*compar*]  $\nearrow$  *comp-conj*

*than*  $\mapsto$  [ / *conj-th*]  $\searrow$  *comp-conj*

CDG Types express dependency valencies

## NON-PROJECTIVE DEPENDENCIES WITH ANCHORS

Polarized valencies:  $\nearrow d$ ,  $\searrow d$ ,  $\nwarrow d$ ,  $\swarrow d$

Anchor valencies:  $\# \searrow d$ ,  $\# \swarrow d$

Dependency and anchor:  $Gov \xrightarrow{d} Sub \xleftarrow{\# \searrow d} Host$ :

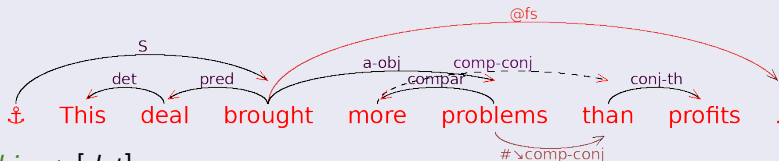
Governor Type:  $Gov \mapsto [..] \nearrow d ..$

Subordinate Type:  $Sub \mapsto [.. \searrow \# \searrow d / ..] \searrow d ..$

Host Type:  $Host \mapsto [.. \searrow \# \searrow d \searrow .. / ..]^P$

# Categorial Dependency Grammars (CDG)

## CDG Types express dependency valencies



*this*  $\mapsto$  [*det*]

*deal*  $\mapsto$  [*det* \ *pred*]

*brought*  $\mapsto$  [*pred* \ *S* / *@fs* / *a-obj*]

*problems*  $\mapsto$  [*compar* \ *a-obj* / *#* \ *comp-conj*]

*profits*  $\mapsto$  [*conj-th*]

*more*  $\mapsto$  [*compar*]  $\nearrow$  *comp-conj*

*than*  $\mapsto$  [*#* \ *comp-conj* / *conj-th*]  $\searrow$  *comp-conj*

*.*  $\mapsto$  [*@fs*]

## Left-oriented rules

$$L!. [C]^P [C \setminus \beta]^Q \vdash [\beta]^{PQ}$$

*Gov*  $\xrightarrow{C}$  *Sub*

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(no new dependency)

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(no new dependency)

$$I^!. [C]^P [C^* \setminus \beta]^Q \vdash [C^* \setminus \beta]^{PQ}$$

*Gov*  $\xrightarrow{C}$  *Sub*

$$\Omega^!. [C^* \setminus \beta]^P \vdash [\beta]^P$$

(no new dependency)

## Left-oriented rules

- $L^!$ .  $[C]^P [C \setminus \beta]^Q \vdash [\beta]^{PQ}$   $Gov \xrightarrow{C} Sub$   
 $L^!_{\varepsilon}$ .  $[ ]^P [\beta]^Q \vdash [\beta]^{PQ}$  (no new dependency)  
 $I^!$ .  $[C]^P [C^* \setminus \beta]^Q \vdash [C^* \setminus \beta]^{PQ}$   $Gov \xrightarrow{C} Sub$   
 $\Omega^!$ .  $[C^* \setminus \beta]^P \vdash [\beta]^P$  (no new dependency)  
 $D^!$ .  $\alpha^{P_1(\swarrow C)P(\nwarrow C)P_2} \vdash \alpha^{P_1PP_2}$   $Gov \dashrightarrow Sub$

## First-Available Rule

**FA:** in  $(\swarrow C)P(\nwarrow C)$ , the valency  $\swarrow C$  is the **first available** for the dual valency  $\nwarrow C$ , i.e.  $P$  has no occurrences of  $\swarrow C, \nwarrow C$

## LEXICON:

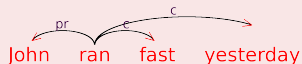
*John*  $\mapsto [pr]$   
*ran*  $\mapsto [pr \setminus S / c^*]$   
*fast, yesterday*  $\mapsto [c]$

### Derivation

$$\begin{array}{c}
 \frac{\frac{\frac{pr \quad ran \quad fast}{[pr \setminus S / c^*]} [c]}{[pr \setminus S / c^*]} I^r \quad yesterday}{[pr \setminus S / c^*]} I^r \\
 \frac{John \quad [pr]}{[pr \setminus S]} \Omega^r \\
 \hline
 S \quad L^!
 \end{array}$$

$$\begin{array}{l}
 L^! \quad [C]^P [C \setminus \beta]^Q \vdash [\beta]^{PQ} \\
 I^! \quad [C]^P [C^* \setminus \beta]^Q \vdash [C^* \setminus \beta]^{PQ} \\
 \Omega^! \quad [C^* \setminus \beta]^P \vdash [\beta]^P \\
 D^! \quad \alpha^{P_1(\swarrow \vee)P(\searrow \vee)P_2} \vdash \alpha^{P_1 P P_2}, \text{ if FA}
 \end{array}$$

### Dependency structures



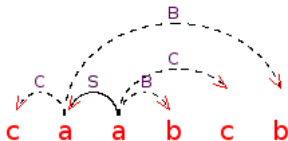
$$\begin{array}{l}
 L^r \quad [\beta / C]^P [C]^Q \vdash [\beta]^{PQ} \\
 I^r \quad [\beta / C^*]^P [C]^Q \vdash [\beta / C^*]^{PQ} \\
 \Omega^r \quad [\beta / C^*]^P \vdash [\beta]^P \\
 D^r \quad \alpha^{P_1(\swarrow \vee)P(\searrow \vee)P_2} \vdash \alpha^{P_1 P P_2}, \text{ if FA}
 \end{array}$$



a [S] <↖B, ↘C>  
 [S\S] <↖B, ↘C>  
 [S] <↗C, ↘B>T  
 [S\S] <↗C, ↘B>  
 [S] <↖B, ↘C>  
 [S\S] <↖B, ↘C>  
 [S] <↖C, ↘B>  
 [S\S] <↖C, ↘B>

b [ ] <↖B>  
 [ ] <↖B>

c [ ] <↖C>  
 [ ] <↖C>

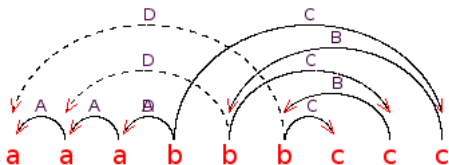


## A CDG for mix with a parse example

In the above grammar, some types have empty heads ; other grammars avoiding empty heads can be provided, but the above one is simpler.

# CDG example: $a^n b^n c^n$

- a [A] <✓D>  
[A\A] <✓D>
- b [B/C] <✓D>  
[A\S/C] <✓D>
- c [C]  
[B\C]



A CDG for  $a^n b^n c^n$  with a parse example

## CdgAnalyst (Dekhtyar-Dikovskiy-Karlov, TCS 2015)

- Dynamic programming parsing algorithm
- Based on CYK parsing algorithm
  - + polarized valency calculus information

## Filtering parsers (Alfared-Béchet-Dikovskiy, Depling 2011)

- Reduction of the search space
- Based on sentence “complexity” of natural languages
  - ⇒ Limit the complexity of potentials

## Greedy parsers (Lacroix-Béchet, Coling 2014)

- Transition-Based Dependency Parser
- 3 steps (local / left non-projective / right non-projective)

## Theorem 8

Algorithm **CdgAnalyst** has time complexity

$$\mathbf{O}(l_G \cdot a_G^2 \cdot (\Delta_G \cdot n)^{2p_G} \cdot n^3).$$

Complexity of CdgAnalyst (Dekhtyar-Dikovskiy-Karlov, TCS 2015)

$n$  : The length of the input string

$l_G$  : The number of assignments in  $G$

$a_G$  : The maximal number of left or right subtypes in  $G$

$\Delta_G$  : The maximal valency deficit in  $G$

$p_G$  : The number of polarized valency names in  $G$

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Grammatical class  $\mathcal{G}$  is learnable if there is an algorithm  $A$  which

- for every target grammar  $G_T \in \mathcal{G}$
- every enumeration  $\sigma = L(G_T)$  and every prefix  $\sigma[n]$ ,

returns a hypothetical grammar  $A(\sigma[n]) \in \mathcal{G}$  and :

- (i) the sequence of languages  $\{L(A(\sigma[n])) \mid n \in \mathbb{N}\}$  converges to the target language  $L(G_T)$
- (ii) this holds for all enumerations  $\sigma$  of  $L(G_T)$

Learning from **strings**:  $\sigma(\mathbb{N}) = L(G_T)$

from **structures**:  $\sigma(\mathbb{N}) = \Delta(G_T)$

# Learnability of $k$ -valued CDG

from strings

(FG'2004)

- $k$ -valued CDG **without** \* iterated types are learnable from structures and from strings
- rigid CDG **with** \* are **not** learnable from strings (a limit point).

Limit point

$$G'_0 = \{a \mapsto [A], b \mapsto [B], c \mapsto [C'_0]\}$$

$$G'_n = \{a \mapsto [A], b \mapsto [B], c \mapsto [C'_n]\}$$

$$G'_* = \{a \mapsto [D], b \mapsto [D], c \mapsto [S / D^*]\}$$

$$L(G'_n) = \{c(b^*a^*)^k \mid k \leq n\} \text{ and } L(G'_*) = c\{b, a\}^*.$$

$$C'_0 = S$$

$$C'_{n+1} = C'_n / A^* / B^*$$

from structures

(FG'2010, ...)

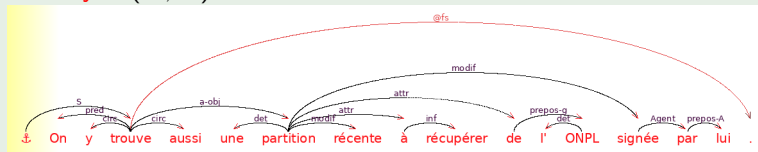
- rigid CDG **with** \* are not learnable from DS

So the CDG are **not** learnable from **dependency treebanks** !

## Type-Generalize-Expand (TGE)

Computes words' types from their VICINITIES in DS

**Vicinity**  $V(w, D)$  of word  $w$  in DS  $D$ :



$V(\textit{partition}, D) = [\textit{det} \setminus \textit{a-obj} / \textit{modif} / \textit{attr} / \textit{attr} / \textit{modif}]$ ,

$V(\textit{de}, D) = [\textit{attr} / \textit{prepos-g}]$

Type-**Generalize**-Expand (TGE) : types with  $d^*$ , repeating principle

Type-**Generalize-Expand** (TGE) : lexicon level, CV for a subclass



**Algorithm** TGE<sup>(K)</sup> (type-generalize-expand):

**Input:**  $\sigma$ , a training sequence of length  $N$ .

**Output:** CDG TGE<sup>(K)</sup>( $\sigma$ ).

let  $G_H = (W_H, C_H, S, \lambda_H)$  where  $W_H := \emptyset$ ;  $C_H := \{S\}$ ;  $\lambda_H := \emptyset$ ;

(loop) for  $i = 1$  to  $N$  // loop on  $\sigma$

let  $D$  such that  $\sigma[i] = \sigma[i-1] \cdot D$ ; // the  $i$ -th DS of  $\sigma$

let  $(X, E) = D$ ;

(loop) for every  $w \in X$  // the order of the loop is not important

$W_H := W_H \cup \{w\}$ ;

let  $t_w = V(w, D)$  // the vicinity of  $w$  in  $D$

(loop) while  $t_w = [\alpha \setminus l \setminus d \setminus \dots \setminus d \setminus r \setminus \beta]$

with at least  $K$  consecutive occurrences of  $d$ ,  $l \neq d$  (or not present) and  $r \neq d$  (or not present)

$t_w := [\alpha \setminus l \setminus d^* \setminus r \setminus \beta]$

(loop) while  $t_w = [\alpha / l / d / \dots / d / r / \beta]$

with at least  $K$  consecutive occurrences of  $d$ ,  $l \neq d$  (or not present) and  $r \neq d$  (or not present)

$t_w := [\alpha / l / d^* / r / \beta]$

$\lambda_H(w) := \lambda_H(w) \cup \{t_w\}$ ; // lexicon expansion

end end

return  $G_H$

**TH** TGE<sup>(K)</sup> learns  **$K$ -star revealing** CDG from DS

(FG 2010)


Importantly, no bound on the number of types is assumed


# Exemple


*John*  $\mapsto [N]$  *to\_the\_station*  $\mapsto [L]$   
*ran*  $\mapsto [N \setminus A^* \setminus S / A^* / L / A^*]$ ,  $[N \setminus A^* \setminus S / A^*]$   
*seemingly, slowly, alone, every\_morning*  $\mapsto [A]$


(Global Simple K-star)

Algorithm TGE<sup>(2)</sup>:

*ran*  $\mapsto [N \setminus S]$  for ( $i = 1$ ):  John ran .

*ran*  $\mapsto [N \setminus S / A]$  for ( $i = 2$ ):  John ran slowly .

*ran*  $\mapsto [N \setminus S / L / A]$  for ( $i = 3$ ):  John ran slowly to\_the\_station .

*ran*  $\mapsto [N \setminus A \setminus S / A^*]$  for ( $i = 4$ ):  seemingly John ran slowly alone .

...

Structured Example	Annotation	Number (k) of Types per word	Repetition number (K) for Indiscernibility
functor-argument (FA, proof-tree)	unlabelled (no dep. name)	bound	no bound
dependency structure (DS)	labelled (dep. names)	no bound	bound

from [Béchet-Foret, Machine Learning, 2021]

## Criteria and readings of the "repetition principle"

consecutive or dispersed in a type ; left-right ; global

- **K-star revealing** (complex equivalence property)
- $\supseteq$  **Simple K-star** (syntactic) : (1) at most  $K - 1$  occurrences of  $d$  and (2) no occurrence of  $d$  if there exists at least one occurrence of  $d^*$  in each  $l_1 \setminus l_2 \setminus \dots \setminus l_p \setminus$  where each  $l_i$  is either  $d$  or some  $x^*$
- $\supseteq$  **Global Simple K-star** : (1) (2) in each  $l_1 \setminus l_2 \setminus \dots \setminus l_p \setminus$   
(both sides)

## UD Corpora

also available for under-resourced languages (breton)

- producing a CDG grammar
- properties of some dependencies, repeating patterns, measures

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<b>Star Scope</b> local count dispersed (global count) sided or both	<b>Star Pattern</b> CDG $d^*$ (FG 2010) + choices $(d_1 d_2)^*$ (LACL 2011) + sequences $(d_1\bullet d_2)^*$ (LACL 2016)	<b>Class constraint</b> (synt — sem) Simple $K$ -star ICFI 2016, MLJ 2021 $K$ -star revealing (on $\Delta(G)$ semantic)
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- beyond  $d^*$  : repeating  $/ d_2 / d_1 / d_2 / d_1$  , etc. as  $/ (d_1\bullet d_2)^*$   
a proposal for extended CDG, with **iterated sequences** of dep.  
and TGE-like algorithm for sequences of length 2 [LACL 2016]

# Some open questions

from [Dekhtyar-Dikovskiy-Karlov, TCS 2015]

(CDG-languages as a class of push-down automata with independent counters)

- An effective tool for showing  $L$  is not a CDG-language ?  
status of the copy language  $\{ww \mid w \in \{a, b\}^*\}$  ?
- Relationships between CDG and other classes of languages ?
- Does the number of non-proj. dep. define a strict hierarchy ?
- Closure under iteration ?

[Kanazawa Wollic (2016) "Abstract Families of Abstract Categorical Languages"]

Abstract Family of Languages (*full* AFL) if closed under

- union ✓, concatenation ✓, Kleene plus / *Kleene star*,
- $\epsilon$ -free homomorphism ✓ / *homomorphism*,  
inverse homomorphism ✓
- and intersection with regular sets ✓

Control over non-projective dependencies ?

THANK YOU !