Generalized Computability and Effective Model Theory in Mathematical Linguistics

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Outline

We consider algorithmic properties of mathematical models, which are used in mathematical linguistics to formalize and represent the semantics of natural language sentences. For example, in the analysis of temporal aspects of verbs the scale of time is usually identified with the ordered set of real numbers or just a dense linear order. There are many results in generalized computability about such structures, and some of them can be applied in this analysis.

As another example, higher order functionals play a crucial role in Montague intensional logic and formal semantics. We discuss some computable models for the spaces of finite-order functionals based on Ershov-Scott theory of domains and approximation spaces.

Outline

- Effective Models of Time
- Effective Models of Types

Time

Time in linguistics is usually represented by the ordered set of real numbers. This model is sufficient to describe formally (and hence analyse effectively) such important features of verbs as *tense* and *aspect*.

There are some properties of dense linear orders (e.g., elimination of quantifiers and decidability), which are well-known for logicians and which could be useful in the analysis of algorithmic properties of interval semantics for verbs in natural languages. However, there are no examples of such analysis in the literature.

Types

R. Montague in "English as a Formal Language" (1970), "Universal Grammar" (1970) and "The Proper Treatment of Quantification in Ordinary English" (1973) proposed a model-theoretic approach to semantics and relations of syntax and semantics of English known as Montague Intensional Logic (IL).

- IL is a typed higher-order logic;
- IL uses finite types and finite-order functionals to formalize grammar categories of natural languages (English).

Neither Montague nor other researchers (to our knowledge) considered complexity issues and algorithmic aspects of objects and constructions of this theory. A natural question is to construct a computable or effective (in some sense) interpretation of Montague IL.

Effective Models

Our approcach is based on

- framework of Σ-definability in admissible sets
  (J. Barwise, Yu. L. Ershov, Y. N. Moschovakis)
- Ershov-Scott theory of approximation spaces and domains

For a set M, consider the set $\mathrm{HF}(M)$ of hereditarily finite sets over M defined as follows: $\mathrm{HF}(M) = \bigcup\limits_{n\in\omega} \mathrm{HF}_n(M)$, where

$\mathrm{HF}_0(M) = \{\varnothing\} \cup M$,
$\mathrm{HF}_{n+1}(M) = \mathrm{HF}_n(M) \cup \{a \mid a \text{ is a finite subset of } \mathrm{HF}_n(M)\}$.

For a structure $\mathfrak{M} = \langle M, \sigma^{\mathfrak{M}}\rangle$ of (finite or computable) signature σ, **hereditarily finite superstructure**

$$\mathbb{HF}(\mathfrak{M}) = \langle \mathrm{HF}(M); \sigma^{\mathfrak{M}}, U, \in, \varnothing\rangle$$

is a structure of signature σ' (with $\mathbb{HF}(\mathfrak{M}) \models U(a) \iff a \in M$).

Fact

$\mathbb{HF}(\mathfrak{M})$ *is the least admissible set over* $\mathfrak{M}$.

Δ_0-formulas and Σ-formulas

Let $\sigma' = \sigma \cup \{U^1, \in^2, \varnothing\}$ where σ is a finite signature.

Definition

*The class of Δ_0-**formulas** of signature σ' is the least one of formulas containing all atomic formulas of signature σ' and closed under $\wedge, \vee, \neg$, $\exists x \in y$ and $\forall x \in y$.*

Definition

*The class of Σ-**formulas** of signature σ' is the least one of formulas containing all Δ_0-formulas of signature σ' and closed under $\wedge, \vee$, $\exists x \in y$, $\forall x \in y$ and $\exists x$.*

Let $\mathfrak{M}$ be a structure of a relational signature $\langle P_0^{n_0}, \ldots, P_k^{n_k}\rangle$ and let $\mathbb{A}$ be an admissible set.

Definition (Yu. L. Ershov 1985)

*$\mathfrak{M}$ is called **Σ-definable in $\mathbb{A}$** if there exist Σ-formulas $\varphi(x_0, y), \psi(x_0, x_1, y), \psi^*(x_0, x_1, y)$, $\varphi_0(x_0, \ldots, x_{n_0-1}, y), \varphi_0^*(x_0, \ldots, x_{n_0-1}, y), \ldots, \varphi_k(x_0, \ldots, x_{n_k-1}, y)$, $\varphi_k^*(x_0, \ldots, x_{n_k-1}, y)$ such that, for some parameter $a \in A$, $M_0 \leftrightharpoons \varphi^{\mathbb{A}}(x_0, a) \neq \varnothing$, $\eta \leftrightharpoons \psi^{\mathbb{A}}(x_0, x_1, a) \cap M_0^2$ is a congruence on $\mathfrak{M}_0 \leftrightharpoons \langle M_0, P_0^{\mathfrak{M}_0}, \ldots, P_k^{\mathfrak{M}_0}\rangle$, where*

$$P_k^{\mathfrak{M}_0} \leftrightharpoons \varphi_k^{\mathbb{A}}(x_0, \ldots, x_{n_k-1}) \cap M_0^{n_k}, \ k \in \omega,$$

$$\psi^{*\mathbb{A}}(x_0, x_1, a) \cap M_0^2 = M_0^2 \setminus \psi^{\mathbb{A}}(x_0, x_1, a),$$

$$\varphi_i^{*\mathbb{A}}(x_0, \ldots, x_{n_i-1}, a) \cap M_0^{n_i} = M_0^{n_i} \setminus \varphi_i^{\mathbb{A}}(x_0, \ldots, x_{n_i-1}) \text{ for all } i \leqslant k,$$

and the structure $\mathfrak{M}$ is isomorphic to the quotient structure $\mathfrak{M}_0 / \eta$.

Σ-definability of a model in an admissible set $\mathbb{A}$ is an extension (on computability in $\mathbb{A}$) of the notion of constructivizability of a model (in classical computability theory CCT).

For a countable structure $\mathfrak{M}$, the following are equivalent:

- $\mathfrak{M}$ is constructivizable (computable);
- $\mathfrak{M}$ is Σ-definable in $\mathbb{HF}(\varnothing)$.

For arbitrary structures $\mathfrak{M}$ and $\mathfrak{N}$, we denote by $\mathfrak{M} \leqslant_\Sigma \mathfrak{N}$ the fact that $\mathfrak{M}$ is Σ-definable in $\mathbb{HF}(\mathfrak{N})$.

For arbitrary cardinal α, let $\mathcal{K}_\alpha$ be the class of all structures (of computable signatures) of cardinality $\leqslant \alpha$. We define on $\mathcal{K}_\alpha$ an equivalence relation $\equiv_\Sigma$ as follows: for $\mathfrak{M}, \mathfrak{N} \in \mathcal{K}_\alpha$,

$$\mathfrak{M} \equiv_\Sigma \mathfrak{N} \text{ if } \mathfrak{M} \leqslant_\Sigma \mathfrak{N} \text{ and } \mathfrak{N} \leqslant_\Sigma \mathfrak{M}.$$

Structure

$$\mathcal{S}_\Sigma(\alpha) = \langle \mathcal{K}_\alpha / \equiv_\Sigma, \leqslant_\Sigma \rangle$$

is an upper semilattice with the least element, and, for any $\mathfrak{M}, \mathfrak{N} \in \mathcal{K}_\alpha$,

$$[\mathfrak{M}]_\Sigma \vee [\mathfrak{N}]_\Sigma = [(\mathfrak{M}, \mathfrak{N})]_\Sigma,$$

where $(\mathfrak{M}, \mathfrak{N})$ denotes the model-theoretic pair of $\mathfrak{M}$ and $\mathfrak{N}$.

It is well-known that

$$\mathbb{C} \leqslant_\Sigma \mathbb{R}.$$

Theorem (Yu. L. Ershov 1985)

$$\mathbb{C} \leqslant_\Sigma \mathbb{L}$$

for any dense linear order of size continuum.

Motivation: find structures $\mathfrak{M}$ such that

1. $\mathfrak{M} \leqslant_\Sigma \mathbb{L}$ with $\mathbb{L}$ used essentially;
2. $\mathfrak{M}$ is "simple" yet natural and useful in applications.

Possible applications appear when $\mathbb{L}$ is treated as the scale of time.

Definition (Yu. L. Ershov)

1. *A first-order theory T is called* **regular** *if it is decidable and model complete.*
2. *A first-order theory T is called c-***simple** *(constructively simple) if it is decidable, model complete, ω-categorical, and has a decidable set of the complete formulas.*

Definition

Structure $\mathfrak{A}$ is called $s\Sigma$-**definable** in $\mathbb{HF}(\mathfrak{B})$ (denoted as $\mathfrak{A} \leqslant_{s\Sigma} \mathfrak{B}$) if $A \subseteq \mathrm{HF}(B)$ is a Σ-subset of $\mathbb{HF}(\mathfrak{B})$, and all the signature relations and functions of $\mathfrak{A}$ are Δ-definable in $\mathbb{HF}(\mathfrak{B})$.

Effective Models of Time

Interval Extensions of Dense Linear Orders

For an arbitrary dense linear order $\mathbb{L} = \langle L, \leqslant \rangle$, define its *interval extension*

$$\mathcal{I}(\mathbb{L}) = \langle I, \leqslant, \subseteq \rangle$$

as follows. A nonempty set $i \subseteq L$ is called an *interval* in $\mathbb{L}$ if, for any $l_1, l_2, l_3 \in L$ such that $l_1, l_3 \in i$ and $l_1 \leqslant l_3$, from $l_1 \leqslant l_2 \leqslant l_3$ it follows that $l_2 \in i$.

Let I be the set of all intervals in $\mathbb{L}$. Elements of L can be considered as intervals of the form $[l, l]$, $l \in \mathbb{L}$.

The relation $\leqslant$ of structure $\mathbb{L}$ induces a partial order relation $\leqslant$ on set I. Namely, for elements $i_1, i_2 \in I$, we set $i_1 \leqslant i_2$ if and only if $l_1 \leqslant l_2$ for any $l_1 \in i_1$ and any $l_2 \in i_2$.

Let $\mathcal{B}(\mathbb{L})$ be the Boolean algebra generated by $\mathcal{I}(\mathbb{L})$.

$\mathbb{L} \models \mathrm{DLO}$ is called *continuous* if for any $A, B \subset L$ such that $A < B$ and $A \cup B = L$, either A has the supremum or B has the infimum.

Theorem (S. 2021)

1. $\mathcal{I}(\mathbb{L})_{Morley} \equiv_{s\Sigma} \mathbb{L}$ iff $\mathbb{L}$ is continuous;
2. $\mathcal{B}(\mathbb{L}) \equiv_{s\Sigma} \mathbb{L}$ iff $\mathbb{L}$ is continuous.

The definition of an approximation space is given below in the most general form. However, in this paper we will consider only very special examples of such spaces, generated by interval extensions.

Definition

An *approximation space* is an ordered triple

$$\mathcal{X} = \langle X, F, \leqslant \rangle,$$

where X is a topological T_0-space, $F \subseteq X$ is a basic subset of *finite elements* and $\leqslant$ is a specialization order on X.

We denote by $a \prec x$ the fact that $a \in F$ and $a \leqslant x$.
Also, we will consider so called *structured* approximation spaces, i.e., we assume F to be the domain of some structure $\mathcal{F}$.

Definition

Let $\mathbb{L}$ be a dense linear order. *The space of temporal processes* over $\mathbb{L}$ is the approximation space

$$\mathcal{T}(\mathbb{L}) = (P(L) \setminus \{\varnothing\}, \mathcal{I}(\mathbb{L}), \subseteq),$$

where $P(L)$ is the set of all subsets of L and $\subseteq$ is the standard set-theoretic inclusion relation on $P(L)$.

Definition

Let $\mathbb{L}$ be a dense linear order. *The atomic space of temporal processes* over $\mathbb{L}$ is the approximation space

$$\mathcal{T}_0(\mathbb{L}) = (P(L) \setminus \{\varnothing\}, \mathbb{L}, \subseteq),$$

where $P(L)$ is the set of all subsets of L and $\subseteq$ is the standard set-theoretic inclusion relation on $P(L)$.

Let σ be a finite predicate signature containing, among other symbols, a binary predicate symbol $\leqslant$. We recall the definition of a formula of dynamic logic DL_σ. Namely, formulas of logic DL_σ have variables of two types — for finite objects and for arbitrary, potentially infinite, objects that can only be accessed with the help of their finite fragments (approximations). We denote these sets by FV and SV, respectively. For the formula θ, the sets of its free variables of these two types are denoted by $FV(\theta)$ and $SV(\theta)$, respectively. If θ is a first-order logic formula of signature σ, then all its variables, including free ones, are considered to be finite. Variables denoted by uppercase letters $(S, P, \ldots)$ are by default considered as variables of type SV.

Definition

The set of Δ_0^{DL}*-formulas of logic* DL_σ is defined as the least set R such that

1) if θ is a first-order logic formula of signature σ, then $\theta \in R$;
2) if $\theta \in R$, $S \in SV$, $a \in FV$, then $[a|S]\theta \in R$, $\langle a|S\rangle\theta \in R$;
3) if $\theta \in R$, $a, s \in FV$, then $[a|s]\theta \in R$, $\langle a|s\rangle\theta \in R$;
4) if $\theta_0, \theta_1 \in R$, then $\neg\theta_0 \in R$, $(\theta_0 \wedge \theta_1) \in R$, $(\theta_0 \vee \theta_1) \in R$ and $(\theta_0 \to \theta_1) \in R$.

Definition

Let $\mathcal{X} = (X, F, \leqslant)$ be a structured approximation space over the structure $\mathcal{F} = (F, \sigma^{\mathcal{F}})$ of signature σ. The *satisfiability relation* on $\mathcal{X}$ for a formula φ of logic DL_σ and an evaluation $\gamma : SV(\varphi) \cup FV(\varphi) \to X$ with $\gamma(x) \in F$ for any $x \in FV(\varphi)$, denoted by $\mathcal{X} \models \varphi \upharpoonright \gamma$, is defined by induction on the complexity of φ:

1) $\mathcal{X} \models [x|S]\theta(x) \upharpoonright \gamma$ if, for all $a \prec \gamma(S)$, $\mathcal{X} \models \theta \upharpoonright \gamma^x_a$;
2) $\mathcal{X} \models \langle x|S\rangle\theta(x) \upharpoonright \gamma$ if there exists $a \prec \gamma(S)$ such that $\mathcal{X} \models \theta \upharpoonright \gamma^x_a$;
3) $\mathcal{X} \models [x|s]\theta(x) \upharpoonright \gamma$ if, for all $a \prec \gamma(s)$, $\mathcal{X} \models \theta \upharpoonright \gamma^x_a$;
4) $\mathcal{X} \models \langle x|s\rangle\theta(x) \upharpoonright \gamma$ if there exists $a \prec \gamma(s)$ such that $\mathcal{X} \models \theta \upharpoonright \gamma^x_a$;
5) $\mathcal{X} \models (\exists S)\theta(S) \upharpoonright \gamma$ if there exists $S_0 \in X$ such that $\mathcal{X} \models \theta \upharpoonright \gamma^S_{S_0}$

and so on.

Definition

An approximation space $\mathcal{X}_1$ is Δ^{DL}-*reducible* to an approximation space $\mathcal{X}_2$ (denoted by $\mathcal{X}_1 \leqslant_{DL} \mathcal{X}_2$), if $\mathcal{X}_1$ as a structure is Δ_0^{DL}-definable in the approximation space $\mathcal{X}_2$, and

1) the structure of finite elements $\mathcal{F}_1$ is Δ_0^{DL}-definable in $\mathcal{X}_2$ inside $\mathcal{F}_2$,
2) there is an effective procedure that associates with every Δ_0^{DL}-formula of space $\mathcal{X}_1$ a Δ_0^{DL}-formula of space $\mathcal{X}_2$, which defines the corresponding predicate in this presentation of space $\mathcal{X}_1$ in space $\mathcal{X}_2$.

Theorem

If $\mathbb{L}$ is continuous, then approximation spaces $\mathcal{T}(\mathbb{L})$ and $\mathcal{T}_0(\mathbb{L})$ are effectively DL-equivalent:

$$\mathcal{T}(\mathbb{L}) \equiv_{DL} \mathcal{T}_0(\mathbb{L}).$$

The basic relations of the temporal logic of J.F. Allen are formalized in dynamic logic as follows: for arbitrary temporal processes $P_1, P_2 \subseteq T$,

P_1 **before** P_2 corresponds to the relation $[i_1|P_1][i_2|P_2](i_1 \leqslant i_2)$;
P_1 **after** P_2 corresponds to the relation $[i_1|P_1][i_2|P_2](i_2 \leqslant i_1)$;
P_1 **while** P_2 corresponds to the relation $[i_1|P_1]\langle i_2|P_2\rangle(i_1 = i_2)$;
P_1 **overlaps** P_2 corresponds to the relation $\langle i_1|P_1\rangle\langle i_2|P_2\rangle(i_1 = i_2)$ (or, in the different interpretation, to the relation $\langle i_1|P_1\rangle\langle i_2|P_2\rangle((i_1 = i_2))\wedge \wedge($ "i_1 is a final subinterval of $P_1) \wedge$ ("i_2 is an initial subinterval of P_2))), etc.

We recall some examples of formalization of semantic meaning of verbs in English proposed by R. Montague. Interval extensions were essentially used by M. Bennett and B. Partee. First, here is the analysis of tense *Present Progressive*.

> The sentence (i.e., state) **John is walking** is true at time p if and only if there is an open interval i such that p is a subinterval of i and for all $t \in i$ state **John walks** is true in moment t.

As another example, consider a formal description of tense *Past Simple*.

The sentence (i.e., state) **John ate the fish** $(= \alpha)$ is true on interval i, if i is a point interval, α refers to the interval i', and there exists an interval $i'' < i'$ such that $i'' < i$ and the state **John eats the fish** is true on i''.

For another example, consider the formal description of tense *Present Perfect*.

The sentence (i.e., state) **John has eaten the fish** $(= \alpha)$ is true on interval i, if i is a point interval, α refers to the interval i', i is a subinterval of i' and there is an interval $i'' < i'$ such that either i is the final point of i'', or $i'' < i$ and the state **John eats the fish** is true on i''.

It is easy to construct Δ_0^{DL}-formulas of signature $\langle \leqslant, \subseteq \rangle$ describing the corresponding relations between these processes (or states) in the space of temporal processes $\mathcal{T}$. Namely,

$$p \subseteq \text{“\textbf{John is walking}”} \iff$$
$$\iff \langle i | \text{“\textbf{John walks}”} \rangle ((p \subseteq i) \wedge (\text{“}i \text{ is an open interval”})),$$

$$p \subseteq \text{“\textbf{John ate the fish}”} \iff [i | \text{“\textbf{John eats the fish}”}](i < p),$$

$$p \subseteq \text{“\textbf{John has eaten the fish}”} \iff [i | \text{“\textbf{John eats the fish}”}](i \leqslant p).$$

In the examples above we consider the states **John walks**, **John is walking**, **John eats the fish**, **John ate the fish** and **John has eaten the fish**, together with the point interval treated as the "present moment". Actually, in these examples it is shown how to define from *Present Simple* more complex tenses. Hence, by the results obtained above, the reasoning about the statements expressed by various combinations of tenses and aspects of English can be carried using some uniform and effective procedure.

The structure of tenses and aspects of verbs in Russian is rather different than that in English. Namely, with three tenses (*Present*, *Past* and *Future*), there are two aspects: *Perfect* and *Imperfect*. The main difficulty for the analysis of Russian verbs is that these two aspects are *independent* in sense there is no basic and no derivable one.

Effective Models of Types

The set D_τ of possible denotations of type $\tau \in Types_{IL}$ is defined by induction on complexity of τ:

- $D_e = A$, $D_t = \{0, 1\}$;
- $D_{(a\to b)} = D_b^{D_a}$ (the set of functions from D_a to D_b);
- $D_{(s\to a)} = D_a^{W\times T}$ (the set of functions from $W \times T$ to D_a).

Function F defines for each constant of type a some element from $D_{(s\to a)}$ which is called its *intension*.

Elements from D_a are called *extensions* of type a.

Grammar categories and types of IL

Category	Grammar equivalent	Corresponding type	Basic expressions
e	no	e	no
t	sentences	t	no
IV	intransitive verbs	$(e \to t)$	walk, talk
CN	common nouns	$(e \to t)$	man, woman
TV	extensional transitive verbs	$(e \to (e \to t))$	love, find
CN/CN	extensional adjectives	$((e \to t) \to (e \to t))$	tall, young
CN/CN	extensional adverbs	$((e \to t) \to (e \to t))$	rapidly, slowly
T	noun phrases and proper names	$((s \to (e \to t)) \to t)$	John, ninety, he
t/t	sentence determinants	$((s \to t) \to t)$	necessarily, possibly
IV/t	connective verbs	$((s \to t) \to (e \to t))$	believe, assert

In general, this approach is intended to represent partial data and operations on this data. The main concepts are:

- *domains*: spaces that are used to define data values and functions (which are also elements of domains);
- *continuity*: consider only continuous functions;
- *approximations*: use simple ("finite") fragments to represent objects which are complex ("infinite");
- *upper bounds*: being able to construct "bigger" approximation for a consistent set of "small" approximations.

Using Σ-predicates in $\mathbb{A}$, concepts that are listed above are realised using three main objects:

- *f- and f^*-bases* on $\mathbb{A}$;
- *Σ-ideals* on f-bases;
- *functional spaces* (or functional products) constructed from two f-bases.

All these objects are defined effectively (in sense of Σ-definability).

f- and f^*-bases are some structured "finite descriptions" of elements of domains.
More precisely, f-base is defined as a quadruple $\mathfrak{B} = \langle B, \leq, Cons, \sqcup \rangle$, where:

- B is a Δ-definable subset of A and $\leq$ is a Δ-definable preorder of B;
- Cons is a family of finite consistent subsets of B;
- $\sqcup : Cons \to B$ is a Σ-definable function which gives the least upper bounds for consistent sets.

f^*-base differs from f-base in that in f^*-base every finite subset of B is consistent and also there exists the least element b_0 (and $\sqcup\emptyset = b_0$). f^*-bases are used to construct functional spaces.

Σ-ideals are constructed from f-bases and are used as elements of domains.
More precisely, Σ-ideal I is a (non-empty) Σ-definable subset of B for which the following holds:

- $b \in I,\ b_1 \in B,\ b_1 \leq b \Rightarrow b_1 \in I$;
- b is a finite subset of $I \Rightarrow b$ is consistent and $\sqcup b \in I$.

Ideals of the form $I_b = \{b_1 : b_1 \in B, b_1 \leq b\}$ for $b \in B$ are called principal ideals.
They can approximate all other ideals, namely $I = \sqcup\{I_b : b \in I\}$ holds for every ideal I.

Informally:

- first property states that an ideal is represented by a set of all elements of f-base which "describe" them;
- second property states that every finite part of an ideal is consistent (hence an ideal is a directed set).

Family of all Σ-ideals with the ordering $\subseteq$ forms a complete poset. We will refer to such poset as a domain (of f-base). It is also a topological T_0-space (with some proper topology which we will not discuss here) and we can define continuous functions in this topology.

Functional spaces are used to define predicates (or functionals) of finite types. Given f-base $\mathfrak{B}_0$ and f^*-base $\mathfrak{B}_1$, we can form functional space $F(\mathfrak{B}_0, \mathfrak{B}_1)$:

- ideals of such functional space can be considered as some approximable (and computable) predicates on $B_0 \times B_1$;
- ordering is a bit technical, but is defined (effectively) so that, having an ideal I of functional space, it is possible to construct corresponding continuous (and computable) mapping f_I on domains;
- for I_0, image $f_I(I_0)$ consists of all approximations of elements related by I to elements of I_0.

To summarise, effecitive interpretations of IL are constructed as follows:

- we define f-bases for our domains of entities, of truth values and of possible worlds;
- having those f-bases, we construct functional spaces over them;
- having functional spaces, we use approximable (and computable) mappings derived from those spaces to model functionals of finite type which are used in IL;
- having those computable functionals, we can argue about semantics of natural language effectively.

Simple model of IL: definition

Further on, we restrict ourselves to the case $\mathbb{A} = \mathbb{HF}(\mathbb{R})$, where $\mathbb{R}$ is the ordered field of real numbers.

In the first (simplest) model:

- truth values are represented by $D_t = \{0, 1\}$ and $0 < 1$;
- singletons from $\mathbb{HF}(\mathbb{R})$ correspond to basic entities from D_e and the order relation is trivial ("$=$");
- real numbers represent possible worlds, each possible world can be considered as a substructure of the whole structure with some partial information about the universe.

Simple model of IL: correspondence

Category	Grammar equivalent	Corresponding type	Object in $\mathbb{HF}(\mathbb{R})$
e	no	e	sets $\{a\}$ for $a \in \mathbb{HF}(\mathbb{R})$
t	sentences	t	no
IV	intransitive verbs	$(\mathrm{e} \to \mathrm{t})$	unary Σ-predicates
CN	common nouns	$(\mathrm{e} \to \mathrm{t})$	unary Σ-predicates
TV	existential transitive verbs	$(\mathrm{e} \to (\mathrm{e} \to \mathrm{t}))$	binary Σ-operators
CN/CN	existential adjectives	$((\mathrm{e} \to \mathrm{t}) \to (\mathrm{e} \to \mathrm{t}))$	Σ-operators
CN/CN	existential adverbs	$((\mathrm{e} \to \mathrm{t}) \to (\mathrm{e} \to \mathrm{t}))$	Σ-predicates
T	noun phrases and proper names	$((\mathrm{s} \to (\mathrm{e} \to \mathrm{t})) \to \mathrm{t})$	Σ-definable families of binary Σ-predicates
t/t	sentence determiners	$((\mathrm{s} \to \mathrm{t}) \to \mathrm{t})$	Σ-definable families of Σ-predicates on $P_1(\mathbb{R})$
IV/t	connective verbs	$((\mathrm{s} \to \mathrm{t}) \to (\mathrm{e} \to \mathrm{t}))$	Σ-operators

(1) **John walks**.

Consider proper name **John** as an object of type e (a set $\{j\}$ for some $j \in \mathbb{HF}(\mathbb{R})$), and (intransitive) verb **walk** as an object of type $(e \to t)$ (unary Σ-predicate **walk'**). The truth value of this sentence is equivalent to the truth value of Σ-formula

$$\{j\} \in \textbf{walk'}$$

in $\mathbb{HF}(\mathbb{R})$.

(2) **John loves Mary**.

As in the previous case, names **John** and **Mary** are considered as objects of type e ($\{j\}$ and $\{m\}$ respectively). Transitive verb **love** is considered as an object of type $(e \to (e \to t))$, hence it is interpreted by some binary Σ-predicate **love'**. Hence the truth value of this sentence is equivalent to the truth value of Σ-formula

$$\langle \{j\}, \{m\} \rangle \in \textbf{love'}$$

in $\mathbb{HF}(\mathbb{R})$.

Ontological model of IL

- $D_t = \{0, 1, \perp, \top\}$ (instead of $D_t = \{0, 1\}$);
- $\perp < 0, < 1, 0 < \top, 1 < \top$, while 0 and 1 are uncomparable (instead of $0 < 1$).

0, 1 and $\perp$ correspond to *no*, *yes*, and *unknown*. The element $\top$ corresponds to inconsistency of data and is necessary for constructing f^*-spaces.

- $D_e = (\mathbb{R} \cup \{\perp, \top\})^{<\omega}$ (instead of singletons);
- ordering on D_e is non-trivial (instead of "$=$").

Ontological model of IL

- Every element from D_e is interpreted as a tuple (sequence) of properties of this element;
- Properties can be discrete (that is, binary) and continuous.

Ontological model of IL

Again, it is possible to interprete by Σ-definable objects in $\mathbb{HF}(\mathbb{R})$ the corresponding objects (types) of IL, e.g.

- common noun "man" (of type $(e \to t)$) could be interpreted by Σ-predicate $(\alpha(\mathit{human}) = 1) \wedge (\alpha(\mathit{gender}) = \text{“}M\text{”})$;
- adjective "tall" (of type $[(e \to t) \to (e \to t)]$) could be interpreted by Σ-operator H such that
  $\alpha \in H(\text{man}) \iff \alpha(\mathit{height}) \geqslant 180$,
  $\alpha \in H(\text{woman}) \iff \alpha(\mathit{height}) \geqslant 175$,
  $\alpha \in H(\text{chair}) \iff \alpha(\mathit{height}) \geqslant 120$.

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Thank You!