

Reading Course in Topos Theory

The course is intended to give an introduction to the theory of toposes, both Grothendieck toposes (sheaves over generalised spaces) and elementary toposes and their generalisations.

The reading group will meet approximately every second week during January - May 2012. Meetings will be spent discussing texts read and/or presentating selected material. This PhD course will give 7.5 or 10 credit points depending on the amount of work done.

The following is recommended reading before meeting 2.

1. Categories, functors, natural transformations, functor categories. The Yoneda Lemma. Pages 3-11, Lambek and Scott 1986 *or* Ch I.1-4, II.4, III.2 Mac Lane 1997.
2. Adjunctions and equivalences between categories. Pages 12-19, Lambek and Scott 1986 *or* Ch IV.1-2, IV.4, Mac Lane 1997.
3. Limits and colimits, monos and epis in categories. Pages 19-27, Lambek and Scott 1986 *or* Ch I.5, III.3-4, Mac Lane 1997.
4. **Theorem:** Each presheaf is a colimit of representable presheaves. Ch I.1,I.5, Mac Lane and Moerdijk 1992.
5. Subobjects and subobject classifiers. Ch I.2-4, Mac Lane and Moerdijk 1992.
6. Exponentials, cartesian closed categories. **Theorem:** The category of presheaves on a small category is cartesian closed. Definition of elementary topos. Ch I.6, Mac Lane and Moerdijk 1992.

Items (1) – (3) are a review of basic category theory. A standard reference for category theory is (Mac Lane 1997). Awodey (2006) is a gentle introduction to category theory which covers most of the above. Bell (1992) is an introduction to topos theory from a mainly logical point of view.

Exercises

See the problems at the end of each chapter of the respective references.

References

S. Awodey, *Category Theory*. Oxford University Press 2006.

J.L. Bell. *Toposes and Local Set Theories*. Oxford University Press 1992.

J.L. Lambek and P.J. Scott, *Introduction to Higher Order Categorical Logic*. Cambridge University Press 1986.

S. Mac Lane, *Categories for the Working Mathematician. Second edition*. Springer 1997.

S. Mac Lane and I. Moerdijk, *Sheaves in Geometry and Logic: A First Introduction to Topos Theory*. Springer 1992.