

## Arkadii L’vovich Onishchik (on his 70th birthday)

In November 2003 Professor Arkadii L’vovich Onishchik of Yaroslavl State University turned 70.

Onishchik was born in Moscow on 14 November 1933. In 1951 he entered the mechanics and mathematics department of Moscow State University. He was one of the leaders of his class, took an active part in department activities, led mathematical workshops for school students, and organized mathematical olympiads. His scientific advisor was E. B. Dynkin, the author of classical works on Lie group theory. Onishchik obtained his first results when he was still an undergraduate. Some of them were included in a long joint paper with Dynkin on the global structure of compact Lie groups, published in *Uspekhi Matematicheskikh Nauk* in 1955.



In 1956 Onishchik graduated from the university and entered graduate school, specializing in higher algebra. After finishing the graduate school program, he remained in the higher algebra section, at that time chaired by A. G. Kurosh.

In 1960 Onishchik defended his PhD thesis “On transitive Lie transformation groups”, and in 1970 his DSc thesis “Compact homogeneous spaces and decompositions of Lie groups”. In 1975 he became a professor at Yaroslavl University.

His first studies were investigations of the cohomology of loop spaces on topological spaces using the method of spectral sequences. He considered an important special case in which the cohomology algebra of the loop space  $\Omega(X)$  can be determined from the cohomology algebra of the space  $X$  itself. Namely, suppose that  $H^*(X)$  is a free anticommutative algebra of finite type, and let  $y_i$ ,  $i = 1, 2, \dots$ , be its free generators. Then the theorem proved by Onishchik states that the algebra  $H^*(\Omega(X))$  has a free system of generators  $x_i$  of degree  $\deg(x_i) = \deg(y_i) - 1$ . More precisely, the  $x_i$  form a basis of the space of transgressive elements in the cohomological spectral sequence of the fibre bundle of the path space over  $X$ , and  $x_i$  corresponds to  $y_i$  under the transgression. This result was preceded by a joint paper with F. I. Karpelevich, where it was assumed that all the  $y_i$  have odd degree, that is,  $H^*(X)$  is an exterior algebra.

As an application, Onishchik computed the algebra  $H^*(\Omega(M))$ , where  $M = G/U$  is a simply connected homogeneous space of a semisimple compact Lie group  $G$ . In particular, he proved that if the algebra  $H^*(M)$  is a tensor product of algebras with one generator, then the algebra  $H^*(\Omega(M))$  can be computed directly from

$H^*(M)$ . This implies that if the algebra  $H^*(M)$  is given, then either of the two algebras  $H^*(G)$  and  $H^*(U)$  can be found from the other. These results, published in 1958, marked the beginning of many years of research, which was summed up in his book *Topology of transitive transformation groups* (1995). We will return to the topology of homogeneous spaces a little later, but now we present a fundamental result, important for both complex analysis and the theory of algebraic groups of transformations, which Onishchik published in *Doklady Akad. Nauk SSSR* in 1960.

By that time, the methods of sheaf theory had made their way into the theory of functions of several complex variables, leading to an understanding of the role of Stein manifolds in complex analysis. Originally introduced by K. Stein as certain generalizations of domains of holomorphy, these manifolds turned out to form a natural class for which the theorems A and B proved in H. Cartan's seminar of 1951–1952 hold true. Due to the significance of the concept itself, the problem of describing Stein manifolds that are homogeneous with respect to the transitive Lie group of holomorphic transformations becomes especially important. This problem has not yet been fully solved even for complex Lie groups of transformations.

Onishchik's paper talks about Stein manifolds that are homogeneous with respect to reductive complex algebraic groups. Let  $M = G/H$  be a complex homogeneous space of such a group  $G$ . Onishchik proved that (a)  $M$  is a Stein manifold if and only if the subgroup  $H$  is algebraic and reductive; (b) under this condition  $M$  has a structure of an affine complex algebraic manifold. At about the same time this theorem was independently proved by Y. Matsushima. As a byproduct, Onishchik obtained the following important result: if  $K$  is a compact group of linear transformations of a real vector space  $V$ , then the complexification  $G$  of the group  $K$  acts transitively on the complexification of the orbit of any point  $v \in V$ , and moreover, the stationary subgroup of  $v$  in  $G$  coincides with the complexification of its stationary subgroup in the group  $K$ . This result was later re-proved by A. Borel and Harish-Chandra and other authors.

Returning now to the topology of transitive groups of transformations, we consider first the problem of finding all decompositions of the form  $G = G' \cdot G''$ , where  $G$  is a connected compact Lie group and  $G'$  and  $G''$  are connected compact subgroups of it. This problem is equivalent to the problem of finding closed connected subgroups of  $G$  that are transitive on homogeneous spaces of this group. The problem was completely solved by Onishchik in 1962. The following decompositions hold, some of which were known earlier:

$$\begin{aligned} SU_{2n} &= Sp_n \cdot SU_{2n-1} \quad (n \geq 2); & SO_7 &= G_2 \cdot SO_6; & SO_7 &= G_2 \cdot SO_5; \\ SO_{2n} &= SO_{2n-1} \cdot SU_n \quad (n \geq 4); & SO_{4n} &= SO_{4n-1} \cdot Sp_n \quad (n \geq 2); \\ SO_{16} &= SO_{15} \cdot Spin_9; & SO_8 &= SO_7 \cdot Spin_7. \end{aligned}$$

The definitive answer is that, up to a local isomorphism and a rearrangement of factors, any non-trivial decomposition can be obtained from one of the above decompositions by a normal extension of the group  $G''$ . It is remarkable that this algebraic result was proved by investigating the topological properties of the decompositions. No one has yet found a purely algebraic proof.

The theorem on decompositions of compact Lie groups has a variety of applications. In particular, Onishchik has used it to find the automorphism groups of

flag manifolds of simple complex Lie groups. He also applied the theorem to the computation of the isometry group of a Riemannian homogeneous space of a simple compact Lie group.

In the decomposition problem the Lie group is given, but its homogeneous space is not fixed. Conversely, one can ask which Lie groups act transitively on a given homogeneous space  $M = G/H$ . In the case where  $G$  and  $H$  are connected compact Lie groups, much progress has been made in the problem, and the most complete results are due to Onishchik. Its solution for homogeneous spaces requires the introduction of invariants that can be effectively computed from the given pair  $(G, H)$  but in fact depend only on  $M$ . Onishchik introduced an integer-valued invariant which he called the rank of a homogeneous space. Denote by  $P_G$  the space of primitive real cohomology classes of the group  $G$ . Then under the map  $i^*: H^*(G, \mathbb{R}) \rightarrow H^*(H, \mathbb{R})$  induced by the embedding  $i: H \hookrightarrow G$ , we have  $i^*(P_G) \subset P_H$ . By definition, the rank  $r(M)$  of the homogeneous space  $M = G/H$  is equal to the dimension of the kernel of the map  $i^*|_{P_G}$ . We note that  $r(G)$  is the rank of the group  $G$  in the usual sense, and  $r(M) > 0$  if  $M$  has positive dimension. Onishchik obtained the formulae

$$r(M) = \sum_{k=0}^{\infty} \text{rk } \pi_{2k+1}(M), \quad h(M) := r(G) - r(H) = \sum_{k=0}^{\infty} (-1)^k \text{rk } \pi_{k+1}(M),$$

which show that  $h(M) \leq r(M)$  and that both invariants depend only on  $M$ .

Onishchik found all simply connected compact homogeneous spaces of rank 1 and 2. His list of manifolds of rank 1 contains all spheres, complex and quaternion projective spaces, the octave (Cayley) projective plane, and a number of other manifolds. For each manifold he described all transitive actions of connected compact Lie groups. The results by D. Montgomery, H. Samelson, and A. Borel on transitive groups on spheres are contained in this theory as special cases. Another consequence of the theory is that two simply connected compact homogeneous spaces of rank 1 are homotopy equivalent if and only if they are diffeomorphic (this is not true for spaces of rank 2).

Suppose now that for some simply connected compact homogeneous space  $M$  we have a classification of connected compact transitive Lie transformation groups, and consider the question of carrying this classification over to non-compact groups. The problem in this generality was first stated, and largely solved, by Onishchik. To begin with, he proved the following theorem (the real analogue of J. Tits's theorem): if  $G$  is a connected Lie group, and  $H \subset G$  is a closed subgroup such that the manifold  $M = G/H$  is compact, then the normalizer  $N(H^\circ)$  of the connected component  $H^\circ$  of the identity of  $H$  contains a maximal connected triangular subgroup of  $G$ . From this theorem it follows that the commutator subgroup of the radical  $R$  of  $G$  is commutative. Moreover, if the manifold  $M$  is simply connected and  $r(M) = 1$ , then the radical itself is commutative. There are two possible cases: (i)  $h(M) = 1$  and the dimension of  $R$  may be arbitrarily large (say, for an odd-dimensional sphere); (ii)  $h(M) = 0$  and  $R = \{e\}$ , that is,  $G$  is semisimple (say, for an even-dimensional sphere). Using these results, Onishchik found all the connected semisimple Lie groups that act transitively on simply connected compact homogeneous spaces of rank 1, and established which of them can be extended to

non-semisimple transformation groups. We present his list of non-compact simple Lie groups that act transitively on spheres  $S^n$ ,  $n \geq 2$ . Up to a local isomorphism, they are

$$SO_{1,n+1}^\circ, SU_{1,m} (n = 2m - 1), Sp_{1,m} (n = 4m - 1), FII (n = 15), SL_{n+1}(\mathbb{R}), \\ SL_m(\mathbb{C}) (n = 2m - 1), Sp_{2m}(\mathbb{R}) (n = 2m - 1), Sp_{2m}(\mathbb{C}) (n = 4m - 1), \\ SL_m(\mathbb{H}) (n = 4m - 1), \text{ and the universal covering of the group } SL_3(\mathbb{R}) (n = 3).$$

A description of all transitive groups on simply connected compact homogeneous spaces of rank greater than 1 is known only in isolated cases so far. For example, Onishchik made a list of connected transitive Lie transformation groups for real, complex, and quaternion Grassmann and Stiefel manifolds. These results have provided a basis for numerous investigations by other authors in geometry, topology, and algebra.

Onishchik has returned several times to the problem of decompositions of a Lie group into a product of two subgroups, a problem he solved for compact groups. In particular, he described all decompositions of connected reductive Lie groups into a product of two connected reductive subgroups. It turned out that this problem is equivalent to the problem of describing local decompositions. In other words, if  $G$  is a reductive group,  $F$  and  $H$  are reductive subgroups of it, and the orbit of  $F$  in  $G/H$  is open, then this orbit coincides with  $G/H$ . This result, later generalized by other authors, is of principal importance for the theory of algebraic transformation groups.

Onishchik was among the first mathematicians who began to apply the methods of non-Abelian cohomology theory to specific geometric problems. In particular, he generalized de Rham's theorem (for 1-forms) that there exist closed differential forms with given periods and that they are cohomologous. In this generalization the role of forms is played by zero-curvature connections in a given fibre bundle, and the role of periods by their holonomy groups.

Important finiteness theorems in the theory of invariants of compact Lie transformation groups are also due to Onishchik. Namely, let  $G$  be a compact Lie group acting on a compact manifold  $M$ , and  $E$  a vector  $G$ -bundle over  $M$  (all objects are of class  $C^\infty$ ). Denote by  $C^\infty(M)_G$  and  $C^\infty(M)^G$ , respectively, the algebras of smooth spherical and invariant functions on  $M$ . Onishchik proved that the module of smooth invariant sections of  $E$  over the algebra  $C^\infty(M)^G$  has finite type. He also proved that the algebra  $C^\infty(M)_G$  admits a finite system of generators over  $C^\infty(M)^G$ .

A number of fairly recent papers by Onishchik are devoted to the theory of supermanifolds. In particular, he proved a theorem on rigidity of certain super-Grassmannians, proposed a new construction of non-split supermanifolds, and found conditions under which an analytic action of a Lie group lifts to a non-split supermanifold from its retract. In a joint paper with O. V. Platonova, he classified homogeneous and evenly homogeneous supermanifolds whose reduction is a projective space.

Onishchik began his pedagogical activity more than 40 years ago in the mechanics and mathematics department of Moscow University. At that time he and È. B. Vinberg organized a seminar on Lie groups which became a successor of

Dynkin's seminar. Over the subsequent years he has always been one of the leaders of this seminar, even after he moved to Yaroslavl University. Notes from the seminar of 1967/68 formed the basis for a book by Vinberg and Onishchik, *Seminar on Lie groups and algebraic groups* (1988), which became an invaluable source for graduate students and experts in this area.

Onishchik is a founder of the group-theoretic methods laboratory at Yaroslavl University. Since the very beginning of his work there he has regularly edited and published the collections of research papers *Geometricheskie metody v zadachakh analiza i algebrы* (*Geometric methods in problems of analysis and algebra*) and *Voprosy teorii grupp i gomologicheskoi algebrы* (*Problems in group theory and homological algebra*), which have become widely known due to the high level of the papers. These collections have received international recognition: many papers contained in them have been translated into English. Also, Onishchik is a member of the editorial boards of the journals *Annals of Global Analysis and Geometry* and *Transformation Groups*. As an editor he is distinguished by his high standards regarding the mathematical articles as well as by his considerate and tactful attitude towards their authors.

The high level of mathematics in the USSR would hardly have been possible without timely and high-quality translations of international mathematics literature into the Russian language. Onishchik regarded translation of books as a cultural mission, and not just as a secondary part-time job. Without doubt, the readers of the Russian editions of the books *Complex manifolds* by Chern and *Differential geometry and symmetric spaces* by Helgason and other books and papers translated by Onishchik greatly appreciated his mastery as a translator. For several years Onishchik was the librarian of the Moscow Mathematical Society, to the great advantage of readers.

Characteristic of Arkadii L'vovich are his professionalism, high culture, and broad interests, as well as good will, sympathy, and a wonderful sense of humour. These qualities have always attracted young people to him. During his years at Moscow and Yaroslavl Universities he has nurtured several generations of students, many of whom continue active research in mathematics and its applications or teach mathematics. They are ever grateful to their professor, who not only taught them mathematics but also showed them, by his own example, modestly and without many words, how to respect others.

We wish Arkadii L'vovich good health, new interesting mathematical results, and many years of life, as meaningful and fruitful as the part he has left behind.

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