From Supergravity to Ballbearings

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Abstract. The analog of the Riemann tensor of the phase spaces of the nonholonomic (with constrained velocities) dynamical systems on manifolds and supermanifolds is proposed. These tensors are needed to perform H. Hertz's formulation of mechanics (without the notion of force) and to write supergravity equations on any N-extended Minkowski superspace. Our approach provides one also with a method to select coset superspaces of $\mathcal{SL}(N|4)$ for the role of *the* N-extended Minkowski superspaces. For $N = 1, 2, 4, 8$ certain most symmetric examples are considered. The method is applicable as well to M. Vasiliev's models with $N > 8$.

Introduction

The details of this paper will be given elsewhere. Here we briefly explain how to derive supergravity equations, SUGRA(N) — the analogues of Einstein's equations (EE) on an N-extended Minkowski superspace $\mathcal{M}(N)$ and what are our the criteria for distinguishing suitable Minkowski superspaces among other supermanifolds. As in twistor theory, we consider complex case, the physical reality to be recovered on a suitable real form of $\mathcal{M}(N)$. Our requirements: SUGRA(N) should be (A) a differential equation of order ≤ 2 on the components; (B) the component expantion of SUGRA(N) should contain the ordinary Einstein's equations. For simplicity we assume that the supergroup of motions of $\mathcal{M}(N)$ is $\mathcal{G} = \mathcal{SL}(N|4)$ (though other possibilities can not be eliminated, cf. (Leites e.a. (1998), Manin (1997))), so we wonder: what is the stationary subgroup $\mathcal{P}$ for which $\mathcal{M}(N) = \mathcal{SL}(N|4)/\mathcal{P}$?

As N grows, it becomes clear that to justify the above requirements we have to diminish $\mathcal{P}$, as GIKOS did, cf. (Galperin e.a. (1984)). Then for $N > 3$ we see that the underlying manifold of $\mathcal{M}(N)$ is the direct product of several copies of the Minkowski space M (times, perhaps, an auxiliary space of a yet unclear merit) and SUGRA splits into the usual Einstein equations on each copy of M glued together by odd superfields. In particular, for $N = 4$ there are two copies of M.

For $N = 8$ there are three copies of M, one of them distinguished (say, "our world"), the other two – perfectly interchangeable (in the model considered here; there are other possibililties) – mirrowing, say, "heaven" and "hell"). These extra copies of the Universe (they MUST appear in our approach) embody an idea first, perhaps, voiced in (Sakharov (1986)). Another feature of nonholonomic nature of Minkowski *super*space is the prefered direction of time, an observation we derive by directly looking at the rattleback;

the universality of this observation for nonholonomic systems follows from recent studies by A. Nordmark (1997).

We start with a presentation of Einstein's equations in a form convenient to us — as equations on conformally noninvariant components of the Riemann tensor represented as a section of the bundle whose fiber is certain *Lie algebra cohomology*. This is equivalent to the standard modern treatment of G-structures in differential geometry that uses *Spencer homology* (Sternberg (1985)) but allows a generalization embracing nonholonomic structures, such as SUGRA. *Nonholonomic* manifolds, i.e., manifolds with nonintegrable distributions, see (Hertz (1956)), are encountered quite often. The applications range from the Cat's Problem to electro-mechanical devices. For a moving account of nonholonomic problems and their history see (Vershik and Gershkovich (1994)).

One can apply (Sternberg (1985)) to any supermanifold with a G-structure (such attempts are numerous in the literature) but the tensors obtained do not match the one physicists consider, cf. (Wess and Bagger (1983)). We also offer a general method to derive constraints — analogues of Wess–Zumino constaraints — for any N. We discover that one of the conventional WZ-constraints for $N = 1$ is redundant: its cohomology class is zero. This demonstrates that a computer-aided study (Grozman and Leites (1997)) is a must here: the amount of computations is too vast for a human not to make a slip.

Though the notion of supermanifolds will soon celebrate their 25-th birthday (see (Leites (1974)) for the first definition of supervariety), certain basics are, regrettably, insufficiently known yet. So we will recall them.

1 Structure Functions for Nonholonomic Structures

1.1 Nonholonomic (Super)Manifolds

Let M be a manifold with a distribution D. Let

$$D = D_1 \subset D_2 \subset D_3 \subset \ldots \subset D_d \tag{1}$$

be the sequence of strict inclusions, where $D_i(x) = D_{i-1}(x) + [D_1(x), D_{i-1}(x)]$ for every $x \in M$ and d is the least number for which the sequence (1.0) stabilizes, i.e., such that $D_d(x) \cup [D_1(x), D_d(x)] = D_d(x)$. In case $D_d = TM$ the manifold M is called *completely nonholonomic*. Let $n_i(x) = \dim\ D_i(x)$. The distribution D is called *regular* if all the dimensions n_i are constant functions on M. Each pair: (M, D) with a nonintegrable D will be referred to as a *nonholonomic manifold* if $d \neq 1$. We will only consider completely nonholonomic (super)manifolds with regular distributions.

With the tangent bundle over a nonholonomic manifold (M, D) we can naturally associate a sheaf of nilpotent Lie algebras as follows. At point $x \in M$ set

$$\mathfrak{n}(x) = \bigoplus_{-d\leq i\leq -1} \mathfrak{n}_i(x), \quad \text{where } \mathfrak{n}_{-i}(x) = D_i(x)/D_{i-1}(x), \quad D_0 = 0. \tag{2}$$

Clearly, $\mathfrak{n}(x)$ is a nilpotent Lie algebra.

1.1.1. A flat $(G, \mathfrak{n}(x))$-structure. Let $M = \mathbb{R}^n$ with a nilpotent $\mathbb{Z}$-graded Lie algebra structure, call it $\mathfrak{n}$. Let G be a subgroup of homogeneous (preserving the grading (2)) automorphisms of $\mathfrak{n}$. Let us identify the tangent space at a point $m \in M$ with M by means of a translation from G. The preimages $\mathfrak{n}_{-1}(m)$ of $\mathfrak{n}_{-1}$ under this identification determine a distribution on M. This distribution together with the G-action on the accompanying flag $\mathfrak{n}(m)$ at each $m \in M$ will be called a *flat* $(G, \mathfrak{n})$*-structure*.

1.2 Generalized Cartan's Prolongs

Given a $\mathbb{Z}$-graded nilpotent Lie algebra $\mathfrak{g}_- = \bigoplus_{0>i\geq -d} \mathfrak{g}_i$ and a Lie subalgebra $\mathfrak{g}_0 \subset \mathfrak{der}\, \mathfrak{g}_-$ which preserves the $\mathbb{Z}$-grading of $\mathfrak{g}_-$, define the i-th prolong of $(\mathfrak{g}_-, \mathfrak{g}_0)$ for $i > 0$ to be (here $S^\bullet = \oplus S^k$ and V^* is the dual of V):

$$\mathfrak{g}_i = [(S^\bullet(\mathfrak{g}_-)^* \otimes \mathfrak{g}_0) \cap (S^\bullet(\mathfrak{g}_-)^* \otimes \mathfrak{g}_-)]_i, \tag{3}$$

where the subscript in the rhs singles out the component of degree i and the intersection is well-defined thanks to the fact that $\mathfrak{g}_0 \subset \mathfrak{der}\, \mathfrak{g}_- \subset \mathfrak{g}_-^* \otimes \mathfrak{g}_-$.

Define the *generalized Cartan's prolong*: $(\mathfrak{g}_-, \mathfrak{g}_0)_* = \bigoplus_{i\geq -d} \mathfrak{g}_i$. By the routine arguments, $(\mathfrak{g}_-, \mathfrak{g}_0)_*$ is a Lie algebra. By the same arguments as for the G-structures, cf. (Sternberg (1985), Goncharov (1987)), the space $H^2(\mathfrak{g}_-; (\mathfrak{g}_-, \mathfrak{g}_0)_*)$ is the space of obstructions to flatness of the nonholonomic supermanifold (M, D) and the elements of $H^2(\mathfrak{g}_-; (\mathfrak{g}_-, \mathfrak{g}_0)_*)$ will be called (as for the case $d = 1$) *structure functions*.

The space of structure functions naturally splits into homogeneous components whose degree is induced by the $\mathbb{Z}$-grading of $(\mathfrak{g}_-, \mathfrak{g}_0)_*$. Let $C^s(\mathfrak{g}_-; (\mathfrak{g}_-, \mathfrak{g}_0)_*) = \bigoplus_k C^{k,s}(\mathfrak{g}_-; (\mathfrak{g}_-, \mathfrak{g}_0)_*)$ be this splitting on the cochain level; the corresponding cohomology $H^{k,s}(\mathfrak{g}_-; (\mathfrak{g}_-, \mathfrak{g}_0)_*)$ are precisely the analogues of the Spencer cohomology and coinside with them for $\mathfrak{g}_- = \mathfrak{g}_{-1}$. Sign Rule carries superization.

2 Structure Functions of the N-Extended Minkowski Supermanifold

Recall that the ground field is $\mathbb{C}$. The G-structure of the Minkowski space can be viewed as either (a) (pseudo) Riemannian or, equivalently, (b) twistor structure. "Straightforward" superizations of these structures are distinct. They are considered in (a) (Leites e.a. (1998)) and (b) (Manin (1997)) or (Grozman and Leites (1997)), respectively. Generally, neither of these superizations gives rise to what is *accepted* as supergravity. The reason is that a Minkowski superspace is still another superization of the Minkowski space and is naturally endowed with a nonholonomic structure.

2.1 What is an N-Extended Complexified Minkowski Supermanifold

Recall that the "physical reasons" for the restrictions $N \leq 4$ for the Yang-Mills and $N \leq 8$ for the supergravity theories were put to doubt in (Vasiliev (1995)).

Recapitulations (Leites (1983)). A *supermatrix* is a rectangular table with elements from a supercommutative superalgebra C with given sets of parities P_{row} and P_{col} of its rows and columns. The *size* of a matrix is $P_{row} \times P_{col}$. Usually, the parities are chosen so that the even rows and columns come first followed by the odd ones; such matrices are said to be of the *standard format*. For the square matrices we will only consider the cases when $P_{row} = P_{col}$ and will denote this set of parities by Par. The parity of the matrix unit — the matrix with an element $c \in C$ in the (i, j)-th slot and 0's elsewhere — is defined to be $p(c) + P_{row}(i) + P_{col}(j)$. Hereafter in this paper $C = \mathbb{C}$.

Let $\mathfrak{gl}$ (Par) be the set of square matrices of size $Par \times Par$; let p be the number of 0's and q the number of 1's in Par. It is immediately clear, that for distinct Par's with the same p and q the Lie superalgebras $\mathfrak{gl}$ (Par) have *nonisomorphic* maximal nilpotent (say, upper triangular) subalgebras, though the algebras $\mathfrak{gl}$ (Par) themselves are isomorphic. It often suffices to consider the standard format only, and $\mathfrak{gl}$ (Par) is abbreviated to $\mathfrak{gl}(p|q)$. In supergravity we MUST consider nonstandard formats as well. Generally, we separate collections of even and odd positions in Par, say, $\mathfrak{gl}(a|b|c)$ or $\mathfrak{gl}(a|b|c|d)$, etc.

Since $P_{row} = P_{col}$, the Lie subsuperalgebra of upper triangular matrices in $\mathfrak{gl}$ (Par) is isomorphic to the Lie subsuperalgebra of lower triangular matrices and we will confine ourselves to one of them, denoted by $\mathfrak{n}$. The generators of $\mathfrak{n}$ are the elements just above (below) the main diagonal; we will denote the even generators of $\mathfrak{n}$ by white nodes and the odd generators by "grey" nodes, the nodes corresponding to commuting generators are disconnected, otherwise they are joined by a segment. For instance, for the standard format, i.e., $\mathfrak{gl}(p|q)$, we have:

$$\underbrace{0 - \ldots - 0}_{p-1 \text{ nodes}} - \otimes - \underbrace{0 - \ldots - 0}_{q-1 \text{ nodes}} \tag{4}$$

Consider the Lie supergroup $\mathcal{SL}(N|4)$ and its parabolic subsupergroup $\mathcal{P}$ corresponding to the two marked odd simple roots in the following system of simple roots (this means that $\mathcal{P}$ is generated by all the simple roots except the marked negative ones):

$$0 - - - \overset{+}{\otimes} - - - \underbrace{0 - - \ldots - - 0}_{N-1 \text{ nodes}} - - - \overset{+}{\otimes} - - - 0 \tag{5}$$

The Lie group corresponding to the 0-th term of the $\mathbb{Z}$-grading described by diagram (5) is $G = SL(N) \times SL_L(2) \times SL_R(2) \times \mathbb{C}^*$, i.e., the degree of a marked root is equal to 1 the other simple roots being of degree 0; here indices L and R distinguish the "left" copy of $SL(2)$ from its "right" twin. In this case $\mathfrak{g}_0 = \mathfrak{g} = \mathfrak{o}(4) \oplus \mathfrak{gl}(N) = \mathfrak{sl}_L(2) \oplus \mathfrak{sl}_R(2) \oplus \mathfrak{gl}(N)$, $\mathfrak{g}_- = \bigoplus\limits_{-1 \geq i \geq -2} \mathfrak{g}_i$ with $\mathfrak{g}_{-1} = (id_L \otimes Id) \oplus (id_R \otimes Id^*)$, $\mathfrak{g}_{-2} = id_L \otimes id_R^*$, where id_j is the space of the standard (identity) representation of $\mathfrak{sl}_j(2)$, $j = L, R$; and Id is the space of the identity representation of $\mathfrak{sl}(N)$. The corresponding matrix representaion of $\mathfrak{p} = Lie(\mathcal{P})$ is of format $2|N|2$.

The *N-extended Minkowski superspace* $\mathcal{M}(N)$ is $\mathcal{SL}(N|4)/\mathcal{P}$ endowed with the natural $(G, \mathfrak{g}_-)$-structure. The conventional versions of the Minkowski superspace correspond to a certain real form of the (complex) superspace $\hat{\mathcal{M}}(N)$ with the reduced $(G, \mathfrak{g}_-)$-structure, $(\hat{G}, \mathfrak{g}_-)$-structure for which $\hat{G}$ is semisimple. Clearly,

$$\hat{\mathcal{M}}(N) = \mathcal{P}/\hat{\mathcal{G}}, \text{ where } \hat{G} = SL(N) \times SL_L(2) \times SL_R(2). \tag{6}$$

GIKOS guessed (and we can prove) that these $\mathcal{M}(N)$ never satisfy our requirements (A) and (B) on SUGRA for $N > 1$. GIKOS considered an enlargement $\hat{\mathcal{R}}(N)$ of $\hat{\mathcal{M}}(N)$ defined $\hat{\mathcal{R}}(N) = \mathcal{P}/\hat{\mathcal{G}}'$, where $\hat{G}' = Q \times SL_L(2) \times SL_R(2)$ and Q is a parabolic subgroup of $SL(N)$. In other words, from $\mathcal{P}$ we pass to a smaller parabolic subsupergroup, $\mathcal{P}'$, whose diagram has several middle roots marked as well.

To satisfy (A) and (B), we have to test various $\mathcal{P}'$s. This is impossible without a computer.

We can prove that diagram (4) can not satisfy requirements (A) and (B) regardless of the number of middle roots marked. In particular, it is well known to physicists that even for $N = 1$ the standard format does not satisfy (B). So in order to satisfy requirements (A) and (B) we consider nonstandard formats.

For $N = 8$ the following two possibilities seem to be distinguished:

$$0 - \overset{+}{\otimes} - 0 - \overset{+}{0} - 0 - \overset{+}{0} - 0 - \overset{+}{0} - 0 - \overset{+}{\otimes} - 0 \quad Par = (001111111100)$$

$$0 - \overset{+}{0} - 0 - \overset{+}{\otimes} - 0 - \overset{+}{0} - 0 - \overset{+}{\otimes} - 0 - \overset{+}{0} - 0 \quad Par = (000011110000)$$

Elsewhere we will discuss the assumptions of Haag-Lopuszanski-Sohnius' theorem which lead to the Poincare supergroup and its "twistor enlargment", $\mathcal{SL}(N|4)$. The stationary subgroups we consider here are the simplest ones: the Lie groups, subgroups of $O(4) \times SL(N)$. Notice that even for the same stationary subgroup $O(4) \times SL(N)$ we can consider several realizations.

The experience with the analogues of Einstein's equaitions on symmetric spaces (Leites e.a. (1998)) teaches us to consider the models of Minkowski superspace whose stationary subgroup is smaller: a product of several copies of $SL(2)$: otherwise the equations will be of order > 2.

With all these conventions, we consider the following examples of Minkowski superspaces which we will denote more puristically by $\hat{\mathcal{M}}(Par)$ rather than $M(N)$.

Theorem. *In Table (below) there are listed all the orders and weights of all the structure functions for the indicated* $\hat{\mathcal{M}}(Par)$.

The corresponding cocycles are listed in a detailed version.

Clearly, there are more candidates for the role of $\hat{\mathcal{M}}(Par)$ for the N we have considered; Grozman's package (described in (Grozman and Leites (1997))) allows one to perform corresponding calculations for any model.

To interpret the supergravity in the same way as we have treated the Einstein Equations (Leites e.a. (1998)), define the supergravity equations as follows. On $\hat{\mathcal{M}}(N)$, the stationary subgroup (i.e., $\hat{G}$) of the point preserves $\varepsilon_L \otimes \varepsilon_R \otimes \varepsilon_1 \otimes \ldots \otimes \varepsilon_k$, where ε_i is the volume preserved by $SL_i(2)$, the i-th copy of $SL(2)$, in the 2-dimensional identity representation and $2k = N$.

If there are several, say s, tensors of weight 0 — "scalar curvatures" — we can take for $\mathbf{R}$ their linear combination and the coefficients of this combination determine a parameter which runs over the projective space $\mathbb{C}P^{s-1}$.

Example: $N = 1$. The tensor $\mathbf{R}$ depends on a parameter, the ratio $a : b$ which runs over the projective line $\mathbb{C}P^1$. Physicists call this parameter the *Gates–Sigel* parameter. On $\mathfrak{g}_{-1}$, there is the inner product given by the bracket. Notice that this product is even for $Par = (00100)$ and odd for $Par = (00001)$. (The tacit choice was $Par = (00100)$.) For $Par = (00100)$ the metric g on the Minkowski space M is the product of spinorial metrics ε_L and ε_R on the the maximal isotropic (with respect to the pairing) subspaces of $\mathfrak{g}_{-1}$.

The equation on scalar curvatures takes the form

$$a\mathbf{R}_1(00) + b\mathbf{R}_2(00) = \lambda g. \tag{7}$$

(The numbers $(w_1, \ldots)$ in Table are the components of the highest weight of the irreducible $SL_L(2) \times SL_R(2) \times SL_1 \times \ldots$-modules $R(w_1, \ldots)$. Each SL_i corresponds to a neighboring pair 11 in Par.)

From the explicit form of the cocycles it is clear that the component expansion of the above equation does not contain the usual equation on the scalar curvature for $Par = (00100)$, and only $\mathbf{R}_1(00)$ for $Par = (00001)$ has the right expansion.

Notice immediately, that for (7) to be well-defined, we must demand that all structure functions of orders > 2 vanish. These conditions are called the *Wess-Zumino constraints.* We have fewer of them than, say, in (Wess and Bagger (1983)): one of the constraints is "harmless", its cohomology class is zero (like torsion of the Levi–Civita conneciton).

What shall we take for analogs of Ricci flatness? As for $N = 0$, these should be the vanishing conditions on the part of the Riemann tensor which does not belong to the conformal, i.e., the analog of Weyl, tensor. For $N > 0$ there are several such components and we can equate to zero either or all of them. Different choices correspond to different supergravities (minimal, flexible, etc.). The equations are well-defined provided the constraints vanish.

If all structure functions of order 2 (and of lesser orders) vanish, the higher obstructions are well-defined and we can write an equation on them. For example, for $Par = (00100)$ we can equate to zero one or both of the tensors:

$$\mathbf{R}(3,0) = 0, \quad \mathbf{R}(0,3) = 0. \tag{8}$$

Observe that the "flatness" and the obstructions to flatness we introduced differ drastically from their conventional counterparts. E.g., each contact manifold or supermanifold is flat in our sence, but it is endowed with a connection (whose form is the contact form) with nonzero, moreover, nondegenerate curvature form.

2.2 Table

$\deg(SF)\backslash Par$	(0000)	(00100)	(00001)
$\boxed{0}$	not defined	$(3,1), (1,3)$	—
1	—	$(1,0), (0,1)$	$(1,1)^{\boxed{1}+1}, (0,1)_1, \boxed{(0,3)_1}$
2	$(2,2), (0,0)$ $\boxed{(4,0), (0,4)}$	$(1,1), (0,0)^2$	$(0,0)^2, (2,2),$ $(1,0)_1, \boxed{(3,2)_1}$
3	not defined	$(3,0), (0,3)$	—

$\deg(SF)\backslash Par$	(001100)	(100001)	(000011)
$\boxed{-1}$	not defined	$(0,2), (2,0)$	not defined
$\boxed{0}$	$(222), (123), (321)$	$(0,1)_1, (1,0)_1$	$(222), (123)_1, (301)_1$
1	$(110)_1, (011)_1$	$\boxed{(0,0)}, \boxed{(2,2)^2}$ $(1,1)^2, (1,0)_1^2, (0,1)_1^2$	$\boxed{(110)}, \boxed{(031)_1}$ $(011)_1$
2	$(020)^2, \boxed{\boxed{(200)}}$ $(101), \boxed{\boxed{(002)}}$	$(2,2), (0,0)^3$ $(1,0)_1, (0,1)_1$	$(000), (220), (002)$ $\boxed{(200)}, (020), (101)_1$
≥ 3	—	not defined	—

$\deg(SF)\backslash Par$	(00111100)	(00001111)
$\boxed{-1}$	$(1021), (1023), (1201), (2112),$ $(3201), (1122)_1, (2211)_1$	$(1122), (2211), (1001)_1$ $(1221)_1, (2112)_1, (3001)_1, (1003)_1$
0	—	—
1	$(0110), (0011)_1^{\boxed{2}+1}, (1100)_1^{\boxed{2}+1}$	$(0011)^{\boxed{2}+1}, (1100)^2, (0110)_1^{\boxed{2}+1}$
2	$(0000), (0220), (2000)$ $(1111), (0101)_1, (10010)_1,$ $(0002), (0020), (0200)$	$(0000)^2, (0022), (2200),$ $(0200), (0020)$ $(0101)_1, (1010)_1$
≥ 3	—	—

$\deg(SF)\backslash Par$	(111100001111)
$\boxed{-3}$	$(100001)^2, (100003), (300001), (100201), (102001)$ $(100221), (122001), (101112), (211101), (210012), (111111)$
$\boxed{-2}$	$(100122), (221001), (101211), (112101), (111012)$ $(210111), (030001), (100030)$
-1	—
0	—
1	$(000011)^4, (001100)^4, (110000)^4, (000110)_1^4, (011000)_1^4$
2	$(000000)^3, (000020), (000022)$ $(000200), (002200), (002000), (020000)$ $(220000), (000101)_1, (001010)_1, (010100)_1, (101000)_1$
≥ 3	—

Notations and remarks. Clearly, the cocycles from Table are invariant under the change of parities $1 \longleftrightarrow 0$ in *Par*. The cocycles which also correspond to the "conformal" case — on shell — are $\boxed{\text{boxed}}$; the cocycles of small orders are all conformally invariant, such $\boxed{\text{orders}}$ are boxed; the cocycles *which only exist in the conformal case*, off shell — unheard of in the absence of super — are $\boxed{\boxed{\text{doubleboxed}}}$; the suscript 1 singles out odd cocycles; the exponent denotes the multiplicity of the cocycle; the multiplicity of conformally invariant vectors is boxed.

The entry "not defined" in the Table refers to the "Riemannian" case, i.e., to the semisimple stationary subgroup. It so happened that in these cases there are no conformal cohomologies. The dash — indicates that either there are no structure functions in this order or (if the degree is > 2 and the case is the "Riemannian" one) they are not defined. In the cases considered (but not generally!) "Riemannian" and "on shell" are synonyms.

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