U

UNITARY MINIMAL MODELS — There are two classes of unitary *highest weight* representations of the N= 2 Neveu-Schwarz algebra, the *continuous class* and the *discrete series* [1]. The models corresponding to the *discrete series* are called the N=2 unitary *minimal models*. For an element of the discrete series there exists a number $m \in \mathbb{N}$, $m \geq 2$ such that $t = \frac{2}{m}$ where the *central extension* is written as $c(t) = 3 - 3t$. Besides, there exist two half-integral numbers $j, k \in \mathbb{N}_{\frac{1}{2}}$ and $0 < j, k,\ j + k \leq m - 1$ such that the **conformal weight** is given by $h = \frac{jk - \frac{1}{4}}{m}$ and the $U(1)$-charge by $q = \frac{j-k}{m}$ [1].

The **character of the** *irreducible representation* $\mathcal{Q}_{j,k,m}$ is the formal power series $P_{\mathcal{Q}_{j,k,m}} = x^{-h} y^{-q} \mathrm{tr}_{\mathcal{Q}_{j,k,m}} \times \{x^{L_0} y^{T_0}\}$ [2] which is

$$\begin{aligned} P_{\mathcal{Q}_{j,k,m}} = P_{\mathcal{V}_{j,k,m}} \Bigg\{ 1 + & \\ & \sum_{n=0}^{\infty} x^{n^2 m} (x^{n(j+k)} - x^{-n(j+k)}) \\ & + \sum_{n=0}^{\infty} x^{(n+1)nm - j} y \\ & \times \left(\frac{x^{2(n+1)m - (n+1)j - (n+1)k}}{1 + x^{(n+1)m - j} y} - \frac{x^{(n+1)j + (n+1)k}}{1 + x^{k + nm} y} \right) \\ & + \sum_{n=0}^{\infty} x^{(n+1)nm - k} y^{-1} \\ & \times \left(\frac{x^{2(n+1)m - (n+1)j - (n+1)k}}{1 + x^{(n+1)m - k} y^{-1}} - \frac{x^{(n+1)j + (n+1)k}}{1 + x^{j + nm} y^{-1}} \right) \Bigg\} \end{aligned}$$

with $P_{\mathcal{V}_{j,k,m}}$ being the **characters** of the corresponding **Verma modules** $\mathcal{V}_{j,k,m}$.

$$P_{\mathcal{V}_{j,k,m}} = \prod_{l=1}^{\infty} \frac{(1 + x^{l - \frac{1}{2}} y)(1 + x^{l - \frac{1}{2}} y^{-1})}{(1 - x^l)^2}.$$

The discrete series of unitary representations of the N=2 Ramond $\pm$ sectors are given by integers $m \geq 2$, J and K such that $t = \frac{2}{m}$, $h = \frac{JK}{m} + \frac{c}{24}$ and $q = \pm \frac{J-K}{m}$ with $0 \leq J - 1, K, J + K \leq m - 1$. The spectral flow [3] maps the unitary representations of the Ramond and of the Neveu-Schwarz algebra onto each other such that $J = j + \frac{1}{2}$ and $K = k - \frac{1}{2}$. Furthermore, the topological twists transform the singular vector *embedding* structure from the *Neveu-Schwarz sector* to representations of the topological N=2 algebra [4].

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Matthias Dörrzapf

UNITARY STRUCTURE — A nondegenerate, *sesquilinear* (i.e., linear in one argument and *antilinear* in another argument) and positive definite form H over the field of complex numbers in a space. The group which preserves **unitary structure** in V is denoted by $U(V)$ or $U(n)$ for $n = \dim V$. The **Lie algebra** $\mathfrak{u}(n)$ consists of antihermitean operators. The adjoint representation of $\mathfrak{su}(3)$, the traceless subalgebra, plays a crucial role in justification of quark's conjecture; $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$ is a standard candidate for the role of the **Lie algebra** of inner symmetries of The *Standard Model*, cf. [1]. If the form H is not sign definite, a the group preserving H is called psedounitary; their representations are important, e.g., in quantization of **gauge fields** (Gupta–Bleuer quantization). Both unitary and pseudounitary **Lie algebras** are real forms of complex **Lie algebra** $\mathfrak{gl}(n)$.

In **superspaces**, the analog of **unitary structure** is not so easy to describe because there is no real form of $\mathfrak{gl}(V) = \mathfrak{gl}(m|n)$ which preserves a sign definite form, [2]. But there is only one form which is sign definite on both $V_{\bar{0}}$ and $V_{\bar{1}}$, though the signs are opposite! This nuisance, as far as interpretation in terms of energy and probability are involved, becomes a blessing in other aspects: it is responsible for cancellation of divergences in some of supersymmetric models, cf. [4].

There are several more notions of unitarity [2]: two queer ones and a *periplectic* one; they additionally preserve a complex or *quaternionic structure* given by an *odd operator* and an odd bilinear form, respectively. If the form H is superskewsymmetric, this is precisely the one used in odd mechanics with **antibracket** [3].

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Dimitry Leites

UNIVERSAL ENVELOPING SUPERALGEBRA — An associative superalgebra $U(\mathfrak{g})$ containing the superalgebra $\mathfrak{g}$ such that for any *embedding* $\rho : \mathfrak{g} \longrightarrow A$ there exists an algebra homomorphism $pr : U(\mathfrak{g}) \longrightarrow A$ such that $\rho = i \circ pr$, where $i : \mathfrak{g} \longrightarrow U(\mathfrak{g})$ is the *canonical embedding* and therefore

$$U(\mathfrak{g}) = T(\mathfrak{g})/(x \otimes y - (-1)^{|x||y|} yx - [x, y]),$$

where $T(\mathfrak{g})$ is the tensor product and $|\ |$ is parity. The superalgebra $U(\mathfrak{g})$ is endowed with the natural ascending filtration

$$0 = U_{-1}(\mathfrak{g}) \subset U_0(\mathfrak{g}) \subset U_1(\mathfrak{g}) \subset \cdots \subset U(\mathfrak{g}),$$

where $U_n(\mathfrak{g}) = \oplus_{j \geq n} T^j(\mathfrak{g}) \cap U(\mathfrak{g})$ (see e.g. [1,2]).

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Steven Duplij