SYMMETRIES WIDER THAN SUPERSYMMETRY

Towards noncommutative and nonholonomic geometry

> Dictionaries are like watches. The worst is better than none, and the best cannot be absolutely true.
>
> *Samuel Johnson*

In this foreword I will try to give a highly personal overview and sum up in a few words what is done to this day on Supersymmetry and Noncommutative Structures and what is NOT covered in the Concise Encyclopedia, nor in [44]. Following the old slogan "Let all flowers blossom", tested during Mao's culture revolution, the editors did not prevent the authors to dwell on their results (sometimes at the expense of some deeper "rival" results, Berezin's included).

My primary concern is to list main **names** rather than results: with Internet facilities the reader who knows whom to read will be able to do the rest, and it is often the best to read the classics's original papers, however incomprehensible (like Newton's papers in Latin or Lie's papers in German or some of **Berezin**'s papers) and littered with mistakes (if taken too literally) they might be. For example, it does not matter what, say, **Witten** or **Drinfeld** wrote in his latest paper: it is worth reading anyway if one is interested in the latest (and future) developments of physics and mathematics. (Still, some knowledge and books, say, authored by Hj. Tallqvist, an expert in nonholonomic mechanics, are half-sunk in Lethe and can only be dug out of ancient referee journals. Even a masterpiece [27] by F. Klein and Sommerfeld on tops is not translated from German.) For the lack of space, I could not list all the names nor references I'd like to; e.g., on M-**theory** (see a review [45] and other works by **A. S. Schwarz**), D-modules, etc.

Prehistory There lived several giants some of whose ideas and constructions only recently started to contribute in earnest to modern language and Gestalten of today's physics. **Grassmann**, who defined *Grassmann algebra*, not only, see [37]. **Hertz**, known for the unit of frequency, who, in order to eliminate the notion of force, and initiated "geometrodynamics" and introduced the notion 'nonholonomic' for dynamical systems with nonintegrable constraints on velocities, see [25]. More generally, a nonholonomic manifold is a manifold endowed with a nonintegrable distribution. Since any model of *Minkowski superspace* is nonholonomic, this becomes pertinent to us, fans of super. But not only to us: examples of nonholonomic systems are ubiquitous and range from cars and falling cats to pursuing missiles to supergravity to statistical physics to economics to optimal control, etc., so possible applications (not only for grants) are promising, cf. [21]. **Riemann**, whose tensor stands in the left-hand side of Einstein's equations, the generalization of this tensor for any G-structure and further generalization to manifolds and supermanifolds with nonholonomic structures is vital for us. **Lie**, who introduced not only *Lie groups* and their "tangents", **Lie algebras**, but also a form rediscovered by Berezin and **Kirillov** and important in Kirillov's description of classical mechanics. **Weyl**, who introduced **gauge fields**, later generalized to become **Yang–Mills** fields (to the mathematician these are just vector bundles with connections). **Planck** and **Heisenberg** who enforced quantization and prompted development of the "**noncommutative geometry**".

On symmetries and unification: **Maxwell** unified electric and magnetic forces; **Poincaré** whose group is supposed to be the group of symmetries of all fundamental forces and which unites time with space; **Glashow, Salam and Weinberg** unified electromagnetic and weak forces.

Veblen's problem: classification of invariant differential operators (see **Kirillov**'s elucidations [31] and Grozman's solution of particular cases [23]). Importance of this problem dawned after Einstein demonstrated invariance of Einstein equations with respect to the group of diffeomorphisms. In *field theory*, an essential role is played by the invariance of the properties of elementary particles with respect to a transformation group. The invariance of these properties was instrumental in developing a pattern in the zoo of elementary particles. The efficiency of this approach was due to its capacity to predict properties of particles as yet undiscovered; these properties were eventually confirmed by experiments.

Weil's "proche points" = near points is a key word of the working language of modernsupermanifold theory and **Grothendieck**'s revolution in **algebraic geometry**: spectra and *schemes*. For the best and briefest introduction see **Manin**'s lectures [33]. In these lectures it is explicitly stated that one-point spectra may have inner degrees of freedom, like elementary particles.

Introduction of **strings** seems to be the first, after Anaxagoras (ca 500–428 BC), idea that "atoms" are not points. On catch-26 pertaining to *string theory* and how to resolve it by means of **Kaluza–Klein**'s ideas, see [18]. (Regrettably, one seldom if ever acknowledges **Mandel**, perished in 30's, who also developed same ideas at the same time and, moreover, united them with nonholonomicity, cf. [28].)

Penrose's *twistors*: passage from the *Poincaré group* to its complexified deformation, a simple group $SL(4;\mathbb{C})$, for a *superization* see **Manin**'s [36]. Observe that *supertwistor* models naturally lead to nonholonomic structures.

Supersymmetry: first calls: models by Stavraki, Golfand and Likhtman, Volkov and Akulov; superalgebras of Neveu and Schwarz, and Ramond.

History. The first human who realized that he was entering a new field — "*supermathematics*" — was, undoubtedly, **Felix Aleksandrovich Berezin**. Working with questions of second quantization he noticed that it is possible to carry out a parallel description of Bose and Fermi fields, and in mid-sixties came to the conclusion [2] that there exists a nontrivial analogue of Calculus in which the elements of a *Grassmann algebra* play the role of functions. In particular, this meant that our universe is not a space locally equivalent to the *Minkowski space*, but is a superspace. **Wess** and Zumino were the first to consciously demonstrate that **"we live on a supermanifold!"** and some amazing consequences. It was only recently that mathematicians realized that this supermanifold is neither real nor complex and introduced a rigorous notion of complex-real supermanifold, see **J. Bernstein**'s lectures in [11].

- It is universally accepted nowadays that "all" particles fall into two categories: bosons (particles with integer spin satisfying the *Bose–Einstein statistics*) and fermions (particles with half-integer spin satisfying the *Fermi–Dirac statistics*). Recent studies, e.g., of high temperature superconductivity, demonstrate that there are particles more general than Bose and Fermi particles — **anyons** or paraparticles — which obey *parastatistics* rules and whose "spin" is an arbitrary number, (see **Majid**'s elucidations). A fundamental hypothesis of theoretical physics is that all interactions are invariant with respect to the Poincaré group, the group of spacetime transformations in the relativity theory. Under this hypothesis it is impossible to unite bosons and fermions together into one **multiplet** (grouping of particles, indistinguishable in interactions of a given type): one of **no-go theorems**. (Though false as stated, as is clear from, e.g., [11,13,36], these "theorems" inspired further research.) Physicists have long desired to unify particles of different statistics into single **multiplets**, in order to decrease the number of the truly elementary "building blocks" which constitute matter. This can be done by introducing supergroups.
- In topology, *supercommutative superalgebras* have been known for a long time. However, the theory of such algebras has not solicited any special attention. It is also in topology, as well as in various deformation theories, that certain algebras (**Lie superalgebras**), whose structure resembles that of **Lie algebras**, have made their appearance.

  It was also noted that there is a relationship, similar to Lie theory, between **Lie superalgebras** and **Hopf algebras**.
- Classification (ca 1975) of simple finite dimensional complex **Lie superalgebras** due to **Nahm**, Rittenberg and Scheunert, and I. Kaplansky, and skillfully rounded up by **V. Kac** together with

V. Kac's exposition of some elements of their representation theory, both strikingly similar to those of finite dimensional **Lie algebras** (and to some infinite dimensional, namely, $\mathbb{Z}$-graded and of polynomial growth) were extremely impressive and timely.

- Mathematicians also got more interested in **supermanifolds**: mathematics connected with or initiated by E. Witten's papers on **supersymmetry**; **Quillen**'s "**superconnections**" as a natural language for the *index theorem* and elaboration of these works by Alvarez–Gomé, Bismut, Getzler [6], Manin (to mention a few) elucidated even the conventional theorems.
- **Witten** and A. Raina consistently used Weil's near points or the language of families or odd parameters.

Superstructures in the "classical mathematics". The majority of the structures indicated below are not understood. By this I mean that no "practical advantage" is derived from their existence. The lucky exceptions are the index theory ([6]; though its *superization* still is an open problem and the classical invariant theory à la **Roger Howe** (see a review in [13]): in super terms we get a simpler proof of an old result or the hart of the matter becomes clearer.

- Whitehead multiplication of the homotopy groups π_i makes the set $\oplus_i \pi_i$ into a Lie superring. Nobody knows a description of these superrings (what is the semisimple part, what is the radical, etc.). Recently a fantastic description of similar superrings as "positive parts" of certain twisted loops with values in simple **Lie superalgebras** appeared [32].
- Deformations of arbitrary algebraic structures studied by **Gerstenhaber** (as well as the structures themselves) are related with **homological vector fields**, see an ingenious paper by Grusson [24]. Homological fields play also an important role in the formal variational calculus (Gelfand, Dorfman, Daletsky, Tsygan, see [10]).
- The Stinrod algebra in characteristic 2 is identified with the universal enveloping of the "positive part" of the Neveu-Schwarz **Lie superalgebra**, see V. Bukhshtaber's appendix in [8].
- *First order* binary invariant differential operators determine, with a few exceptions, a **Lie superalgebra** structure on their domain [23].
- Quantization of **gauge fields** requires the "odd mechanics" [20].

Selected difficulties and solutions. In [1] **Witten** wrote: Direct experimental confirmation of **supersymmetry** is one of the prime missions of the proposed Superconducting supercollider... More fundamentally, I believe that the main obstacle is that the core geometrical ideas — which must underline *string theory* the way Riemannian geometry underlines *general relativity* — have not yet been unearthed." Actually, by the time Witten wrote this passage a solution — definition of the analog of the curvature tensor for any nonholonomic manifolds or **supermanifolds** — had been unearthed, cf. [21, 22].

Towards noncommutative geometry. To describe physical models, the least one needs is a triple $(X, F(X), L)$, consisting of the "phase space" X, the **sheaf** of functions on it, locally represented by the algebra $F(X)$ of sections of this **sheaf**, and a Lie subalgebra L of the **Lie algebra** of differentiations of $F(X)$ considered as vector fields on X. Here X can be recovered from $F(X)$ as the collection $\mathrm{Spec}(F(X))$, called the spectrum and consisting of maximal, or prime, ideals of $F(X)$. Usually, X is endowed with a suitable topology; for the prime spectra the natural topology (the Zariski one) is very nonHausdorff; since almost any points of such topology is close to any given point this tempts one's imagination to contemplate about possibility of other types of spectra penetrate through walls, etc.

After the discovery of quantum mechanics, the attempts to replace $F(X)$, the algebra of "observables", with a noncommutative ("quantum") algebra A became more and more popular. For a long time, however, nobody, except **Connes**, could develop even elements of differential geometry on spectra of such algebras.

The first successful attempt was *superization* [29, 4] the road to which was prepared in the works of A. Weil, Leray, Grothendiek and Berezin. It turns out that having suitably generalized the notion of the tensor product and differentiation (by inserting certain signs in the conventional formulas) we can reproduce on **supermanifolds** all the **characters** of differential geometry and actually obtain a much reacher and interesting plot than on manifolds. This picture proved to be a great success in theoretical physics since the language of **supermanifolds** and supergroups is a "natural" for a uniform description of Bose and fermi particles. Today there is no doubt that this is the language of the Grand Unified Theories of all known fundamental forces, see [11].

Nevertheless, supergroups are not the largest possible symmetries of **superspaces**; there are transformations that preserve more noncommutativity than just a "mere" supercommutativity. To be able to observe that there are symmetries that unify Bose and fermi particles we had to admit a broader point of view on our Universe and postulate that we live on a **supermanifold**. In [13] we suggest to consider our **supermanifolds** as particular case of metamanifolds.

How noncommutative should $F(X)$ be? To define the space corresponding to an arbitrary algebra is very hard, see Manin's gloomy remarks in [34], where he studies quadratic algebras as functions on "perhaps, nonexisting" noncommutative **projective spaces**.

Manin's idea that there hardly exists one uniform definition suitable for any noncommutative algebra (because there are several quite distinct types of them) was supported by A. Rosenberg's studies. He managed, however, to define several types of spectra in order to interpret ANY algebra as the algebra of functions on a suitable spectrum, see [38]. In particular, there IS a space corresponding to a quadratic (or "quadraticizable") algebra such as the so-called "quantum" deformation $U_q(\mathfrak{g})$ of $U(\mathfrak{g})$, see [12].

Observe that in [34] Manin also introduced and studied symmetries of *supercommutative superalgebras* **wider than supersymmetries**, but he only considered them in the context of quadratic algebras. Regrettably, nobody, as far as I know, investigated consequences of Manin's approach to enlarging supersymmetries.

Unlike numerous previous attempts to devise noncommutative **algebraic geometry**, Rosenberg's theory appears more natural; still, it is algebraic, without any real geometry (no differential equations, integration, etc.). For some noncommutative algebras certain notions of differential geometry can be generalized: such is, now well-known, A. Connes geometry, see [9], and [35]. Arbitrary algebras seem to be too noncommutative to allow to do any physics.

In contrast, the experience with the simplest *noncommutative spaces*, the superspaces, tells us that all constructions expressible in the language of differential geometry (these are particularly often used in physics) can be carried over to the super case. Still, supersymmetry has certain shortcomings which disappear in the theory of metamanifolds [13].

So, it might seem, that the algebras of functions should be either commutative or supercommutative to allow any interesting geometry. **Drinfeld**'s theory of "*quantum groups*" hints that this is not exactly so but to develop the **noncommutative geometry** will not be easy.

Recent results on high temperature superconductivity, presumably carried by **anyons**, particles of fractional spin (whatever that might mean) prepared the audience to these sacreligeous investigations and now, in addition to the beauty of the problems and a natural character of the mathematical formulation I can refer to the authority of physicists, such as **V. Rubakov**.

Selected outstanding results in supermanifold theory and noncommutative geometry: **Serganova**'s description of Kazhdan–Lusztig polynomials and (with **Penkov**) the character formula for a wide class of **irreducible representations** of a wide class of **Lie superalgebras**; **Sergeev**'s invariant theory; **Shander**'s integration theory, nonpolynomial invariant theory and *canonical forms* of various tensors. **Shchepochkina**'s exceptional simple **Lie superalgebras** of vector fields [43] related, perhaps, to the *Standard Model.* On applications of *Kac–Moody superalgebras* or nonpolynomial growth, see **Borcherds** [7] and **Gritsenko** and **Nikulin** [19]. **Egorov**'s *superization* of $\mathfrak{gl}(\infty)$ [15]. Application of noncommutative **algebraic geometry** and theory of $\mathcal{D}$-modules by **Beilinson– Bernstein** and **Brylinsky–Kashivara** to representation theory.

Shortly before his untimely death **Scherk** has demonstrated that supergravity naturally leads to antigravity [42]. Nobody, it seems, investigated this astonishing discovery of a prominet expert.

On notations Good notations help to advance science no less than good theorems. So we should follow best examples (e.g., [11]) and never confuse **Lie superalgebras** with graded **Lie algebras**, *superconformal algebras* with *stringy* superalgebras, never denote the volume element on $4|N$-dimensional supermanifolds "$d^4xd^N\theta$" but write vol(x|θ) or $D(x|\theta)$; be aware of numerous formats of *supermatrices*; stick to most meaningful notations of simple **Lie superalgebras** or invent better.

Summary. Supersymmetry naturally leads to supergravity; the arena for the latter is a nonholonomic supermanifold [21]. Quantization naturally leads to noncommutative algebras of functions while the desire to consider the most broad symmetries of the known entities leads to symmetries wider than supersymmetries [13]. Complexification with odd imaginary numbers also leads us to noncommutative algebra of differential geometry. **M. Vasiliev**'s dissident's study of particles with spin >2 lead to a new class of filtered superalgebras, analogs of **Lie algebra** of matrices of *complex size* [14]. Recent discovery by **Shchepochkina** [43] of exceptional **Lie superalgebras** of vector fields gives new insight to the *Standard Models* [46]. Finally, if Duplij's somewhat incomprehencible conjectures in category theory have a germ of reason they might lead to a revolution.

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