

of simple Lie algebra \mathfrak{g} there are several deformations; the number of parameters in the last example is equal to the rank of \mathfrak{g} and leads to generalizations of the **Lie algebras** of “matrices of complex size”, cf. [5]. The Poisson superalgebra on the vector **superspace** can be realized by vector fields D as

$$\{D \mid L_D(\alpha_1) = 0\},$$

$$\alpha_1 = dt - \sum (p_i dq_i - q_i dp_i) - \sum \varepsilon_j \theta_j d\theta_j.$$

Similarly, the *Buttin superalgebra* (with *Schouten bracket*, i.e., **antibracket**) is

$$\{D \mid L_D(\alpha_0) = 0\}, \quad \alpha = d\tau - \sum (\theta_i dq_i + q_i d\theta_i).$$

The *deformed Buttin superalgebra* is

$$\mathfrak{b}_{a,b}(n) = \{D \in \mathfrak{vect}(n|n+1) \mid L_D(\alpha_{q,\xi,\tau}^{a-bn}) = 0\}.$$

Instead of a, b , one can consider one parameter

$$\lambda = \frac{2a}{n(a-b)} \in P^1.$$

The *structure functions* (obstructions to flattening the corresponding G -structure) are computed in [3]. For infinite dimensional analogs see [4].

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SUPERTIME — A **supermanifold** of values of a dynamical parameter. On manifolds, Time is usually 1-dimensional, a different example is *Kadomtsev–Petviashvili hierarchy* in which an infinite dimensional manifold is interpreted as Time.

On finite dimensional manifolds, Time is always 1-dimensional as follows from the *rectifiability* of vector field theorem studied at early courses of differential equations. Shander generalized the theorem on *rectifiability* of vector fields to nondegenerate fields on **supermanifolds** and gave the following characterization of

such fields, in particular, the ones used in SUSY theories: the nondegenerate (at a point) vector field X can locally be reduced to the form $D_0 = \frac{\partial}{\partial x}$, where x is an even coordinate, if X is even, to the form $D_1 = \frac{\partial}{\partial \theta}$, where θ is an odd coordinate, if X is odd and $X^2 = 0$, or to the form $D = \frac{\partial}{\partial \theta} + \theta \frac{\partial}{\partial x}$, if X is odd and $X^2 \neq 0$.

Shander explained that for dynamical systems on **supermanifolds** **supertime** runs a (1|1)-dimensional **supermanifold** with parameters t, τ . Shander gave examples with **Poisson bracket** and **antibracket**, e.g., he showed that the most profound dynamics is given not by $D_0(f) = \{f, H\}$, but by

$$D(f) = \{f, H\},$$

where the parity of the Hamiltonian, H , should be opposite to that of the (anti)bracket $\{\cdot, \cdot\}$, indeed

$$D_0(f) = \frac{1}{2} \{f, \{H, H\}\}.$$

This explanation enables one to pick up odd parameters missed under the conventional crude approach, but no physical paper used this so far.

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SUPERTRACE — A linear functional on a **Lie superalgebra** that vanishes on the superbracket and denoted by str (or just tr). For the definition of the “usual” **supertrace** of the *supermatrix*, not necessarily in the standard format, see [1,2]. There is also *queertrace*, qtr, defined on a “queer” superanalog of the matrix algebra $\mathfrak{q}(n)$ by the same characteristic property but since $\mathfrak{q}(n)$ is a subalgebra in $\mathfrak{gl}(n|n)$ we can compare str and qtr and see that they are totally different functions; in particular, one is even and the other one is odd, [3]. Both **supertrace** and *queertrace* have a contraction into the **Berezin integral** — the **supertrace** on the *Poisson–Lie superalgebra* $\mathfrak{po}(0|n)$ whose parity is equal to that of n .

These **supertraces**, being defined on finite dimensional algebras, can be integrated to groups, so they correspond to **superdeterminants**:

$$\det(\exp(X)) = e^{\text{tr}(X)}.$$

There are also superanalogues of trace on infinite dimensional **Lie superalgebras**, they do not necessarily correspond to **superdeterminants**. Examples: *stringy* superalgebras $\mathfrak{t}^L(1|4)$ and $\mathfrak{t}^M(1|5)$, cf. [4], *special Buttin superalgebras* $\mathfrak{sb}(n)$, and divergence free algebras $\mathfrak{svect}(1|n)$. The parity of these **supertraces** is equal to that of the number of odd indeterminates.

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