with a sum over i implied, and where $D_\mu\phi^i = (\partial_\mu + igQ_iA_\mu)\phi^i$ is the gauge covariant derivative of the field ϕ^i with a $U(1)$ charge Q_i, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Elimination of the **auxiliary fields** gives

$$\bar{F}^i = -\frac{\partial W}{\partial \phi^i}, \qquad D = -\left[\xi + g\sum_i Q_i\bar{\phi}^i\phi^i\right],$$

where the holomorphic function $W = W(\phi^i)$ is the *superpotential* and the constant ξ comes from a *Fayet–Iliopoulos term*[3] that has been included. The *scalar potential* is

$$V = \sum_i |F^i|^2 + \frac{1}{2}D^2 = \sum_i \left|\frac{\partial W}{\partial \phi^i}\right|^2 + \frac{1}{2}\left[\xi + g\sum_i Q_i\bar{\phi}^i\phi^i\right]^2.$$

If the $U(1)$ symmetry is a global symmetry, rather than a local *local gauge symmetry*, then the *vector multiplet* is not included in L, and $D = 0$. The $U(1)$ invariance of the **superpotential** W requires each term in W to have a net charge of zero. A **superpotential** of the form

$$W = W_0 + a_i\phi^i + b_{ij}\phi^i\phi^j + c_{ijk}\phi^i\phi^j\phi^k$$

allows for renormalizability, and since the constant W_0 is dynamically irrelevant, we can set it equal to zero.

Let us use the derivative notation $X_i = \partial X/\partial\phi^i$, $X_{\bar{j}} = \partial X/\partial\bar{\phi}^j$, $\bar{X}_{\bar{j}} = \partial\bar{X}/\partial\bar{\phi}^j$, $X_{i\bar{j}} = \partial^2 X/\partial\phi^i\partial\bar{\phi}^j$, $X_iX_{\bar{i}} = \sum_i X_iX_{\bar{i}}$, etc. for some function $X(\phi,\bar{\phi})$, with a sum over repeated indices unless otherwise stated. The vacuum expectation value (vev) $\langle\phi^i\rangle = \varphi^i$ is located at the minimum of V where

$$V_i = \bar{W}_{\bar{k}}W_{ki} + DD_i = \bar{F}^kF_i^k + DD_i$$

vanishes. We also note that $V \geq 0$ so that a negative *cosmological constant* does not appear. The vacuum state is supersymmetric if $V(\varphi) = 0$, but **supersymmetry** is spontaneously broken by the vacuum if $V(\varphi) > 0$. From the last equation above, it is seen that a nonzero vacuum expectation value $\varphi^i \neq 0$ can develop from either the *F-term* or the *D-term* in V, resulting in either *F-type strings* or *D-type strings* [4]. Abelian F-type and D-type strings in global SUSY theories are described and discussed in [4] and a global SUSY model of a local **superconducting Witten string** [5], which is an *F-type string*, is described in [6]. See, for example, [7] for a discussion of non-Abelian global **supersymmetry strings** and [8] for a discussion of **supergravity strings**.

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John Morris

SUPERSYMPLECTIC MANIFOLD — A generalization of the notion of **symplectic manifolds** with respect to the definition of supersymplectic spaces. At this point one has two ways: it's known that there are two different kinds of the spaces – odd and even supersymplectic spaces. In the even case we have the following definition [1]: a **supermanifold** equipped with a closed nondegenerated even two – form is called a **supersymplectic manifold**.

Example. Every even "*split*" **supersymplectic manifold** is isomorphic (up to a suitable notion of *equivalence*) to the following data

$$(M, \omega, E, g, \nabla),$$

where (M,ω) is a usual **symplectic manifold**, E is a smooth vector bundle over M, g is a nondegenerated metric on E and ∇ is a connection on E, compatible with the choosen metric [1].

The odd case is much more complicated, so at the moment there are known just a couple of examples, look like the case, related to the cotangent bundle.

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Nikolai Tyurin

SUPERSYMPLECTIC STRUCTURE — The structure that is given by the nondegenerate differential 2-form and can be either even (on **supermanifolds** of dimension $2n|m$) or odd (on **supermanifolds** of dimension $n|n$) with *canonical forms*

$$\omega_0 = \sum dp_i dq_i + \sum \varepsilon_j (d\theta_j)^2,$$

where $\varepsilon_j = \pm 1$ over reals and $\varepsilon_j = 1$ over complex field;

$$\omega_1 = \sum d\theta_i dq_i,$$

respectively.

The odd structure (A. Weil called it *periplectic*) is the one which gives rise to famous **antibracket** [2]. The **antibracket** has deformations parametrized by a singular supervariety; in dimension $2|2$ this deformation miraculously coincides with a deformation of the even *Poisson bracket*. On related peculiar quantizations see [1]. Observe that the "well known" statement that there exists only one quantization of the **Poisson bracket** is only true for polynomials or formal series as functions. For example, for Laurent polynomials (i.e., on tori) and for functions on the orbits of the coadjoint representation of *simple Lie algebra* $\mathfrak{g}$ there are several deformations; the number of parameters in the last example is equal to the rank of $\mathfrak{g}$ and leads to generalizations of the **Lie algebras** of "*matrices of complex size*", cf. [5]. The Posson superalgebra on the vector **superspace** can be realized by vector fields D as

$$\{D \mid L_D(\alpha_1) = 0\},$$
$$\alpha_1 = dt - \sum(p_i dq_i - q_i dp_i) - \sum \varepsilon_j \theta_j d\theta_j.$$

Similarly, the *Buttin superalgebra* (with *Schouten bracket*, i.e., **antibracket**) is

$$\{D \mid L_D(\alpha_0) = 0\}, \quad \alpha = d\tau - \sum(\theta_i dq_i + q_i d\theta_i).$$

The *deformed Buttin superalgebra* is

$$\mathfrak{b}_{a,b}(n) = \{D \in \mathfrak{vect}(n|n+1) \mid L_D({}^{a}_{q,\xi,\tau}\alpha_0^{a-bn}) = 0\}.$$

Instead of a, b, one can consider one parameter

$$\lambda = \frac{2a}{n(a-b)} \in P^1.$$

The *structure functions* (obstructions to flattening the corresponding *G-structure*) are computed in [3]. For infinite dimensional analogs see [4].

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Dimitry Leites

SUPERTIME — A **supermanifold** of values of a dynamical parameter. On manifolds, Time is usually 1-dimensional, a different example is *Kadomtsev–Petviashvili hierarchy* in which an infinite dimensional manifold is interpreted as Time.

On finite dimensional manifolds, Time is always 1-dimensional as follows from the *rectifyability* of vector field theorem studied at early courses of differential equations. Shander generalized the theorem on *rectifyability* of vector fields to nondegenerate fields on **supermanifolds** and gave the following characterization of such fields, in particular, the ones used in SUSY theories: the nondegenerate (at a point) vector field X can locally be reduced to the form $D_0 = \frac{\partial}{\partial x}$, where x is an even coordinate, if X is even, to the form $D_1 = \frac{\partial}{\partial \theta}$, where θ is an odd coordinate, if X is odd and $X^2 = 0$, or to the form $D = \frac{\partial}{\partial \theta} + \theta \frac{\partial}{\partial x}$, if X is odd and $X^2 \neq 0$.

Shander explained that for dynamical systems on **supermanifolds supertime** runs a $(1|1)$-dimensional **supermanifold** with parameters t, τ. Shander gave examples with **Poisson bracket** and **antibracket**, e.g., he showed that the most profound dynamics is given not by $D_0(f) = \{f, H\}$, but by

$$D(f) = \{f, H\},$$

where the parity of the Hamiltonian, H, should be opposite to that of the (anti)bracket $\{\cdot,\cdot\}$, indeed

$$D_0(f) = \frac{1}{2}\{f, \{H, H\}\}.$$

This explanation enables one to pick up odd parameters missed under the conventional crude approach, but no physical paper used this so far.

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Dimitry Leites

SUPERTRACE — A linear functional on a **Lie superalgebra** that vanishes on the superbracket and denoted by str (or just tr). For the definition of the "usual" **supertrace** of the *supermatrix*, not necessarily in the standard format, see [1,2]. There is also *queertrace*, qtr, defined on a "queer" superanalog of the matrix algebra $\mathfrak{q}(n)$ by the same characteristic property but since $\mathfrak{q}(n)$ is a subalgebra in $\mathfrak{gl}(n|n)$ we can compare str and qtr and see that they are totally different functions; in particular, one is even and the other one is odd, [3]. Both **supertrace** and *queertrace* have a contraction into the **Berezin integral** — the **supertrace** on the *Poisson–Lie superalgebra* $\mathfrak{po}(0|n)$ whose parity is equal to that of n.

These **supertraces**, being defined on finite dimensional algebras, can be integrated to groups, so they correspond to **superdeterminants**:

$$\det(\exp(X)) = e^{tr(X)}.$$

There are also superanalogs of trace on infinite dimensional **Lie superalgebras**, they do not necessarily correspond to **superdeterminants**. Examples: *stringy* superalgebras $\mathfrak{k}^L(1|4)$ and $\mathfrak{k}^M(1|5)$, cf. [4], *special Buttin superalgebras* $\mathfrak{sb}(n)$, and divergence free algebras $\mathfrak{svect}(1|n)$. The parity of these **supertraces** is equal to that of the number of odd indeterminates.

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