for the terms proportional to the *fermionic coordinates* θ [1]. Depending on the amount of **supersymmetry**, **superpotential** may be constrained to obey certain relations. For example, in a four-dimensional N = 1 supersymmetric *field theory* $W(\Phi_i)$ has to be a holomorphic function of **chiral superfields** Φ_i. In other words, it depends on Φ_i but not on $\bar{\Phi}_i$. This property (along with other symmetries) highly restricts perturbative and non-perturbative renormalization of $W(\Phi_i)$ [2]. In the **superspace** action the **superpotential** $W(\Phi_i)$ is integrated over (part of) of the **superspace**, so that the *scalar potential* $V(\varphi_i)$ is typically quadratic in W and its first derivatives [1]. For example, four-dimensional **superspace** action:

$$\int d^4\theta K(\Phi_i, \bar{\Phi}_i) + \int d^2\theta\, W(\Phi_i) + h.c. \qquad (1)$$

is manifestly invariant under $N = 1$ *supersymmetry.* Integrating out the **auxiliary fields**, apart from the kinetic action of φ_i one finds the following *scalar potential*:

$$V(\varphi_i) = \left(\frac{\partial^2 K}{\partial\varphi_i \partial\bar{\varphi}_j}\right)^{-1} \frac{\partial W}{\partial \varphi_i} \frac{\partial \bar{W}}{\partial \bar{\varphi}_j} \qquad (2)$$

Notice that $V(\varphi_i)$ depends on the *Kähler potential* $K(\Phi_i, \bar{\Phi}_i)$ because the **auxiliary fields** also appear in the first term of the action (1). The simplest (and, at the same time, the most general) **superpotential** that leads to unitary, renormalizable four-dimensional N = 1 theory was first written by Wess and Zumino [3]:

$$W_{WZ} = \frac{1}{2} m\Phi^2 + \frac{1}{3} g\Phi^3.$$

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Sergey Gukov

SUPERSCHEME — A **superspace** $(M, \mathcal{O}_M)$, where $\mathcal{O}_M = \mathcal{O}_{M,\bar{0}} \oplus \mathcal{O}_{M,\bar{1}}$ is a **sheaf** of local *supercommutative rings* over space M such that $(M, \mathcal{O}_{M,\bar{0}})$ is a usual *scheme*, and $\mathcal{O}_{M,\bar{1}}$ is a *coherent sheaf* of the $\mathcal{O}_{M,\bar{0}}$-modules [1].

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Steven Duplij

SUPERSCHWARZIAN — *Superization* of the *Schwarz derivative* or *Schwarzian*

$$S_0(g) = \left(\frac{g'''}{g'} - \frac{3}{2}\frac{(g'')^2}{(g')^2}\right) dx^2.$$

and the *Bott cocycle*

$$\mathrm{Bott}_0(g_1, g_2) = \int \log(g_1(g_2(x))' d \log(g_2(x)'),$$

where subscript shows the number of odd parameters. Gelfand and Fuchs established that the only nontrivial 2-*cocycle* c onthe **Lie algebra** V of complex-valued vector fields on the circle S is (up to a *coboundary*) of the form $c(a\frac{d}{dx}, b\frac{d}{dx}) = \int ab''' dx$, where $a, b \in V$ and x the parameteron S. The *central extension* of V given by c is of huge importance in *string theory* and is famous under the name Virasoro algebra, [5].

Clearly, there is an *embedding* i: $H^2(\mathfrak{g}) \longrightarrow H^1(\mathfrak{g}; \mathfrak{g}')$ given by the formula $i(c)(x)(y) = c(x, y)$. (The invariants that describe the deviation of i from the isomorphism are "symmetric *Lie algebra cohomology*" described in [6]; they are related with several new interesting notions, e.g., "*Leibniz algebras*".) The 1-*cocycle* $i(c)$ for V is of the form $i(c)(a\frac{d}{dx}) = a''' dx^2$. These *cocycles* can be integrated to S and Bott, respectively, the *cocycles* on the group of **diffeomorphisms** of S (recall that every **diffeomorphism** $g \in \mathrm{Diff}(S)$ is of the form of a function g: $S \longrightarrow S$).

In [3], 4 series and 5 exceptional simple **Lie superalgebras** pertaining to *string theories* are described (for partial proof of the classification see [7,8]); the ones with nontrivial *central extensions* are called *distinguished stringy superalgebras*; they exist for $N \leq 4$, where N is the odd dimension of the supercircle $S^{1|N}$. The most interesting for applications to **superstrings** are N-extended distinguished superalgebras with the largest N, for them the problem of description of the *Schwarzian* and *Bott cocycle* is still open. The formulas for the super *Bott cocycle* require too muchpreliminaries to express them here; the formulas for *Schwarz derivative* for $N \leq 3$ for the contact cases are as follows. Let $\alpha = dx + \sum_{1\leq i\leq N} \theta_i d\theta_i$ and $\alpha^+ = dx + \sum_{1\leq i\leq N-1} \theta_i d\theta_i + x\theta_N d\theta_N$ be *contact forms* and $K(N)$ and $K^+(N)$ the supergroups of **diffeomorphisms** that preserve Pfaff equations $\alpha = 0$ and $\alpha^+ = 0$, respectively. The multiplier $m(G)$ of $G \in K(N)$ or $K^+(N)$ is a function defined [1] from the formulas $G^*(\alpha) = m(G)\alpha$ and $G^*(\alpha^+) = m(G)\alpha^+$, respectively. Define the contact vector field with generating function f by setting

$$K_f = (2 - E)(f)\frac{\partial}{\partial t} - H_f + \frac{\partial f}{\partial t} E,$$

where $E = \sum \theta_i \frac{\partial}{\partial \theta_i}$, and H_f is the hamiltonian field with Hamiltonian f that preserves $d\alpha$:

$$H_f = -(-1)^{p(f)} \sum_{j\leq N} \frac{\partial f}{\partial \theta_j}\frac{\partial}{\partial \theta_j}.$$

One can verify that $K_f(\alpha) = 2\frac{\partial f}{\partial t}\alpha$. Similarly, set

$$\hat{K}_f = (2 - E)(f)\bar{D} + \bar{D}(f)E + \hat{H}_f,$$

where

$$\bar{D} = \frac{\partial}{\partial t} - \frac{\theta}{2t}\frac{\partial}{\partial \theta}$$

and where

$$\hat{H}_f = (-1)^{p(f)}\left(\sum_{i\leq N-1}\frac{\partial f}{\partial \theta_i}\frac{\partial}{\partial \theta_i} + \frac{1}{t}\frac{\partial f}{\partial \theta}\frac{\partial}{\partial \theta}\right).$$

One can verify that $\hat{K}_f(\alpha^+) = 2\bar{D}(f)\alpha^+$.

Let A_f denote K_f or $\hat{K}_f$ be the vector fields corresponding to contact diffeomorphisms $G \in K(N)$ or $G \in K^+(N)$, respectively. Let $f' = \frac{df}{dt}$. Then for $N = 0$, 1, 2 we have:

$$S_0(G) = \frac{1}{2}\left(\frac{m''}{m} - \frac{3}{2}\frac{m'm'}{m^2}\right)\alpha^2;$$

$$S_1(G) = \frac{1}{2}\left(\frac{A_\theta m'}{m} - \frac{3}{2}\frac{A_\theta mm'}{m^2}\right)\alpha^{3/2};$$

$$S_2(G) = \frac{1}{2}\left(\frac{A_{\theta_1}A_{\theta_2}m}{m} - \frac{3}{2}\frac{A_\theta mm'}{m^2}\right)\alpha$$

The formula for S_3 continues the pattern, to an extent, but is much more complicated, the formulas for S_4 (there are three orfour distinct ones) and for Bott_N for $N = 3, 4$ are unknown yet, cf. [1].

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Dimitry Leites

SUPERSLIVER — A sliver Ξ is a special state of the boundary **conformal field theory** [1] which is an idempotent of the star algebra $\Xi \star \Xi = \Xi$, solves the equation of motion and has the correct quantum numbers (*ghost number* 0 and picture number 0) [2]. The supersliver in the **conformal field theory** given by the *Neveu–Schwarz sector* of the superstring is defined by

$$\langle \Xi | = \langle 0 | U_f,$$

where U_f is the operator associated to the conformal transformation given by $f(z) = \arctan(z)$ and has the form

$$U_f = \mathrm{e}^{\sum_{n=1}^{\infty} a_n L_{-2n}},$$

where the coefficients a_n can be computed explicitly. The Virasoro operators split as $L = L_b + L_f + L_g$, where b, f, g refer respectively to the bosonic matter, fermionic matter and ghost/superghost sectors, and the supersliver will factorize as

$$|\Xi\rangle = |\Xi_b\rangle \otimes |\Xi_f\rangle \otimes |\Xi_g\rangle.$$

The fermionic matter part were calculated in [3] writing $|\Xi_f\rangle$ as a squeezed state (i.e. generalized coherent) and exploiting the technique of **Neumann coefficients** [4]. The ghost/superghost sector was considered in [5]. For slightly different approach [5].

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SUPERSPACE — A locally ringed space $\mathcal{M} = (M, \mathcal{O}_M)$, where $\mathcal{O}_M = \mathcal{O}_{M,\bar{0}} \oplus \mathcal{O}_{M,\bar{1}}$ is a **sheaf** of local *supercommutative rings* over a topological space M such that the fiber $\mathcal{O}_x = \mathcal{O}_{M,x}$ is a local ring for any $x \in M$, and all morphisms of $\mathcal{O}_M$ the are compatible with grading [1–3]. The subsuperspace $\mathcal{U}$ is the restriction $(U, \mathcal{O}_M|_U)$, where $U \subset M$ is open subset. The space M of dimension p is called *underlying space*, and $\mathcal{O}_M$ is a structure **sheaf** of rank q, then the **superspace** $\mathcal{M}$ has dimension $\dim \mathcal{M} = (p|q)$. The *superdimension* $\mathrm{sdim}\,\mathcal{M} = p + \varepsilon q$, where $\varepsilon^2 = 1$, which gives the standard 'multiplication of dimensions' $\mathrm{sdim}\,\mathcal{M} \otimes \mathcal{N} = \mathrm{sdim}\,\mathcal{M} \cdot \mathrm{sdim}\,\mathcal{N}$ [2,4]. A morphism $\Phi\colon (M, \mathcal{O}_M) \to (N, \mathcal{O}_N)$ is a pair $\Phi = (f, g)$ such that $f\colon M \to N$ is a continuous mapping of spaces and $g\colon \mathcal{O}_N \to f^*(\mathcal{O}_M)$ is a morphism of **sheaves** such that for any $x \in M$ the homomorphism $g_x\colon \mathcal{O}_{N,f(x)} \to \mathcal{O}_{M,x}$ is local,