

field theories: these **auxiliary fields** vanish if the geometry is restricted by supergauge conditions of Wess-Zumino-type. Actually, it is natural to consider the restriction of the geometry defined by the equation $H_\theta^z = 0$ (and CC) since this condition is invariant under superconformal changes of coordinates. Yet, this constraint implies that the superdiffeomorphism group has to be restricted to the **subgroup** which leaves it stable [2]. At the infinitesimal level, the corresponding stability condition is given by $C^\theta = \frac{1}{2}DC^z$ (and CC) where the superfields

$$C^z \equiv \Xi^z + \Xi^{\bar{z}}H_{\bar{z}}^z + \Xi^\theta H_\theta^z + \Xi^{\bar{\theta}}H_{\bar{\theta}}^z$$

$$C^\theta \equiv \Xi^\theta H_\theta^\theta + \Xi^{\bar{z}}H_{\bar{z}}^\theta + \Xi^{\bar{\theta}}H_{\bar{\theta}}^\theta$$

parametrize infinitesimal superdiffeomorphisms generated by the supervector field $\Xi \cdot \partial \equiv \Xi^z \partial_z + \Xi^{\bar{z}} \partial_{\bar{z}} + \Xi^\theta \partial_\theta + \Xi^{\bar{\theta}} \partial_{\bar{\theta}}$, the latter being a function of the coordinates $(z, \bar{z}, \theta, \bar{\theta})$. For $H_\theta^z = 0$, the differential equation satisfied by the integrating factor Λ takes the simple form

$$\left[\bar{D} - H_\theta^z \partial + \frac{1}{2} (DH_\theta^z) D \right] \Lambda = (\partial H_\theta^z) \Lambda.$$

Transformation laws and holomorphic factorization.

For $H_\theta^z = 0$, the transformation law of H_θ^z under a superconformal change of coordinates $\mathbf{z} \rightarrow \mathbf{z}'(\mathbf{z})$ is given by $(H_\theta^z)' = (\bar{D}\bar{\theta}')^{-1} (D\theta')^2 H_\theta^z$. Moreover, the transformation law of H_θ^z under infinitesimal superdiffeomorphisms reads as [2,3]

$$\delta H_\theta^z = \left[\bar{D} - H_\theta^z \partial + \frac{1}{2} (DH_\theta^z) D + (\partial H_\theta^z) \right] C^z.$$

In the Wess-Zumino supergauge, the induced variations of the ordinary Beltrami differential and of its *fermionic partner* take the form

$$\delta \mu = [\bar{\partial} - \mu \partial + (\partial \mu)] c + \frac{1}{2} \alpha \epsilon$$

$$\delta \alpha = \left[\bar{\partial} - \mu \partial + \frac{1}{2} (\partial \mu) \right] \epsilon + c \partial \alpha - \frac{1}{2} \alpha \partial c,$$

where $c^z \equiv \xi^z + \xi^{\bar{z}} \mu_{\bar{z}}^z$ parametrizes infinitesimal diffeomorphisms and $\epsilon^\theta \equiv \xi^\theta + \xi^{\bar{z}} \alpha_{\bar{z}}^\theta$ local supersymmetry transformations on the underlying Riemann surface. Thus, the chosen parametrization makes the property of holomorphic factorization [4] manifest, both at the level of **superfields** and *component fields*.

Superconformal classes of vielbeins. The Beltrami parametrization of the metric on a Riemann surface can be rewritten in terms of orthonormal frame fields (the so-called *zweibein* forms) and these expressions can be extended to **superspace** [3]. Thus, one writes the super *zweibein* forms in terms of superconformal factors (transforming under **super Weyl transformations**) and Beltrami superfields ' H ' which parametrize superconformal classes of *zweibeins*. This approach to super

Beltrami differentials leads to the same results as the parametrization of supercomplex structures presented above, but it is less economical due to the fact that one has to deal with **super Weyl transformations**.

Bibliography

- [1] D. Friedan, in Unified String Theories, Santa Barbara Workshop, M. B. Green and D. Gross, eds. (World Scientific, 1986); A. M. Baranov, Yu. I. Manin, I. V. Frolov and A. S. Schwarz, Commun. Math. Phys. **111** (1987) 373; L. Crane and J. M. Rabin, Commun. Math. Phys. **113** (1988) 601; M. Batchelor and P. Bryant, Commun. Math. Phys. **114** (1988) 243.
- [2] P. Delduc and F. Gieres, Class. Quantum Grav. **7** (1990) 1907.
- [3] L. Baulieu, M. Bellon and R. Grimm, Phys. Lett. B **198** (1987) 343; S. J. Gates, Jr. and F. Gieres, Nucl. Phys. B **320** (1989) 310; R. Grimm, Nucl. Phys. B (Proc. Suppl.) **5B** (1988) 137; L. Baulieu, M. Bellon and R. Grimm, Nucl. Phys. B **321** (1989) 697; R. Grimm, Ann. Phys. **200** (1990) 49.
- [4] A. A. Belavin and V. G. Knizhnik, Phys. Lett. B **168** (1986) 201; E. D'Hoker and D. H. Phong, Rev. Mod. Phys. **60** (1988) 917; F. Gieres, Lett. Math. Phys. **28** (1993) 43.

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SUPER BRAUER GROUP — The group of classes of Morita equivalent superalgebras. The notion helps to explain, in particular, *Bott periodicity of Clifford algebras*. Deligne [1] further developed *Wall's description* written in pre-super era. For a description over *p-adic fields* see [2].

Bibliography

- [1] P. Deligne et al (eds.) Quantum fields and strings: a course for mathematicians. Vol. 1, 2. Material from the Special Year on Quantum *Field Theory* held at the Institute for Advanced Study, Princeton, NJ, 1996–1997. AMS, Providence, RI; Institute for Advanced Study (IAS), Princeton, NJ, 1999. Vol. 1: xxii+723 pp.; Vol. 2: pp. i–xxiv and 727–1501.
- [2] M. Finkelberg, In: D. Leites (ed.), Seminar on supermanifolds, no. 14/1987-18 Reports of Dept. of Math. of Stockholm Univ., 1986–1990

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SUPER CALOGERO MODEL — A generalization of the *Calogero model* including *anticommuting variables* θ_i and their derivatives $\theta_i^\dagger = \frac{\partial}{\partial \theta_i}$.

The Hamiltonian of SCM

$$\mathcal{H} = \frac{1}{2} \sum_{i=1}^N \left(-\frac{\partial^2}{\partial x_i^2} + \omega^2 x_i^2 \right) + \sum_{i < j} \frac{v^2}{(x_i - x_j)^2}$$

$$+ \frac{\omega_F}{2} \sum_{i=1}^N [\theta_i^\dagger, \theta_i]$$

$$+ \frac{v}{2} \sum_{i < j} \frac{1}{(x_i - x_j)^2} [\theta_i - \theta_j, \theta_i^\dagger - \theta_j^\dagger] + c$$

is supersymmetric when $\omega_F = \omega$ and $c = \frac{v\omega}{2} N(N-1)$ [1]. It is an *integrable system* and has *equidistant spectrum*. Its eigenfunctions have the form $\Phi = \prod_{i < j} (x_i - x_j)^v \Psi$ and the functions Ψ can be obtained