results in the unique action

$$\int d^4\theta\ \bar{\phi}\phi G^{-1/2} + (\Lambda \int d^2\theta\ \phi^2 + h.c.)$$

(choosing the scale *weight* of the "*superdilaton*" ϕ so that it appears quadratically), where we have left the **conformal supergravity** implicit. As usual, the sign of the kinetic term determines which *scalar multiplet* is physical. In this case the **string gauge** is $G = 1$, while a standard gauge would choose $\phi = 1$ instead. Choosing these gauges does not eliminate the corresponding fields from the theory, but only pushes them into the *supergravity multiplet*, which is no longer conformal after fixing the superscale gauge. Since the two **multiplets** differ in *off-shell* field content, the *supergravity multiplet* will appear to have a different **auxiliary field** content; this is clearly not the case, since the **axion** is physical, not auxiliary as in *new minimal supergravity*. Of course, this can be seen only from the action (or scattering amplitudes), but the physical nature of the **axion** in *string theory* follows from the direct product of the two transverse vectors representing physical states in any two **open strings**. Again, before choosing a gauge **S-duality** SU(1,1) is manifest, but only after **duality** transforming G into a second *chiral multiplet*.

As an example of the **string gauge** for coordinate invariance, we again consider a physical scalar coupled to gravity, starting with the **string gauge** for scale invariance as described above. In *string theory* the actual *dilaton* field that appears through coupling to the ghosts or curvature of the **worldsheet** is the combination $\Phi \equiv (-g)^{1/4}\varphi$, absorbing the measure $\sqrt{-g}$ for purposes of **T-duality** invariance. (In the **superstring** case this definition follows from just **supersymmetry**: The **prepotentials** for **compensators** are necessarily densities, as is clear from the above *supersymmetric cosmological term*.) Now looking at just the part of the action quadratic in the field perturbations h_{ab} and χ, for purposes of producing the simplest propagators, the best gauge-fixing function is simply $\partial^b h_{ab} + \partial_a \chi$ (or its nonlinear version $\partial_m({e_a}^m\Phi)$): Adding its square to the above coordinate invariant action leads to the quadratic part of the gauge-fixed action

$$L_0 \sim -h^{ab}\Box h_{ab} + \frac{1}{D-2}\chi\Box\chi$$

The fact that χ is the *dilaton*, appearing with opposite sign to a physical scalar, was necessary to simplify the h_{ab} kinetic term: Now its trace appears with a physical sign, identifying it as a physical scalar, whereas in pure gravity the trace is the *dilaton*, requiring a separate ${h^a}_a\Box {h^b}_b$ term to give it the opposite sign to the traceless part. Similar remarks apply in the supersymmetric case, where the γ-trace of the *gravitino* becomes the physical **spinor**.

Bibliography

[1] W. Siegel, Phys. Lett. **149B** (1984) 162.
[2] W. Siegel, Phys. Lett. **211B** (1988) 55.

Warren Siegel

STRING THEORY — A theory in which the fundamental constituents are one-dimensional objects, called strings [1]. The dynamics of a relativistic string is naturally governed by a the Nambu–Goto action which is proportional to the area of the "worldsheet", the surface swept by the string during its evolution.

For quantization purposes it turns out to be extremely more convenient to work with the so-called Polyakov action that contains the worldsheet metric as a Lagrange multiplier. For the bosonic string, that contains a tachyon, the absence of BRS anomalies fixes the critical dimension to $D = 26$. For the superstring the critical dimension is $D = 10$ [1].

Bibliography

[1] M. B. Green, J. H. Schwarz and E. Witten, Superstring Theory, Cambridge University Press 1987; M. Kaku, Introduction to Superstrings and M-Theory, 2nd ed., Springer, 1999.

Massimo Bianchi

STRINGY SUPERALGEBRA — A $\mathbb{Z}$*-graded algebra*

$$G = \bigoplus_{i=-d}^{\infty} G_i$$

(not necessarily simple but infinite dimensional and of infinite depth d)such that there exists a root vector corresponding to a real root that does notact locally nilpotently in the adjoint representation (if all root elements actlocally nilpotently, the algebra is said to be of Kac–Moody type [2]). For the list of simple ones and their *central extensions* see [1].

Bibliography

[1] P. Grozman, D. Leites, I. Shchepochkina, hep-th/9702120; Acta Mathematica Vietnamica, v. 26, 2001, no. 1, 27; V. Kac, J. Leur van de, In: S. J. Gates, C. R. Preistschopf, W. Siegel, Strings 1988, Proc. Conf. Univ. Maryland, May 1988, World Sci., Singapore, 77
[2] B. Feigin, D. Leites, V. Serganova, In: M. Markov et. al. (eds.) Group–theoretical methods inphysics, Zvenigorod, 1982, Harwood Academic Publ., Chur, 1985, Vol. 1–3, 631.

Dimitry Leites

SUBGROUP — A subset H of a given group G closed with respect to the group multiplication. In other words: if $\circ\colon G \times G \to G$ is the group multiplication in G then its subset H is a **subgroup** iff H is a group with a multiplication $\bullet\colon H \times H \to H$, where $\bullet$ denotes the restriction $\circ|_{H\times H}$. This is equivalent to the following conditions: (i) for each $a \in H$ its inverse a^{-1} (with respect to the multiplication $\circ$) belongs to H and (ii) for each pair $a, b \in H$ their product $a \circ b$ belongs to H. Sometimes these conditions are written together as $\forall\ a, b \in H\ a \circ b^{-1} \in H$ [1].