$$ab = q^{-1}ba, \quad cd = q^{-1}dc, \quad ad - da = q^{-1}bc - qcb, \quad (2)$$

which together with (1) gives the structure of the *general matrix quantum supergroup* $\mathrm{GL}_q(2|0)$ [2] (see also **quantum group**).

### Bibliography


[1] Yu. I. Manin, *Quantum groups* and noncommutative geometry, Preprint Montreal Univ. CRM-1561, 1988.

[2] Yu. I. Manin, Topics in noncommutative geometry, Princeton Univ. Press, 1991.

[3] S. Majid, Foundations of quantum group theory, Cambridge Univ. Press, 1995.

[4] P. Bouwknegt, J. McCarthy, P. van Nieuwenhuizen, Phys. Lett. B394 (1997) 82.


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**QUANTUM WEIL ALGEBRA, history** — For any compact *Lie group* $G$, together with an invariant *inner product* on its **Lie algebra** $\mathfrak{g}$, Alekseevand Meinrenken [1] defined quantum Weil algebra $\mathcal{W}_G$ as a tensor product of the universal enveloping algebra $U(\mathfrak{g})$and the **Clifford algebra** $\mathrm{Cl}(\mathfrak{g})$. Just like the usual Weil algebra

$$W_G = S(\mathfrak{g}^*) \otimes \Lambda(\mathfrak{g}^*),$$

the quantum Weil algebra carries the structure of an acyclic, locally *free differential algebra* with an additional structure (*G-differential algebra*) and can be used to define equivariant cohomology for any *G-differential algebra B*. This construction helps to further generalize results of Duflo, Kashiwara–Vergne and Kontsevich on generalization of the *Harish–Chandra isomorphism.* On importance of the classical Weil algebra see [2–3].

### Bibliography


[1] A. Alekseev, E. Meinrenken, Invent. Math. **139** (2000) 135.

[2] B. L. Feigin and E. V. Frenkel, Commun. Math. Phys. **137** (1991) 617; erratum: Commun. Math. Phys. **147** (1992) 647.

[3] E. Poletaeva, C. R. Acad. Sci. Paris Sér. I Math. **326** (1998) 533.


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**QUANTUM WEYL ALGEBRA** — Also the *Wick algebra* with additional relations, is an algebra $W(B, C)$ defined for a vector space $E$ with a basis $\{x^i\}_{i=1}^N$ and its complex conjugate $E^*$ with the dual basis $\{x^{*i}\}_{i=1}^N$ and two *Hermitian linear operators* $B : E \otimes E \to E \otimes E$ and $C : E^* \otimes E \to E \otimes E^*$ as the quotient [1,2]

$$W(B, C) := T(E \oplus E^*)/I_{B,C},$$

where $I_{B,C}$ is an **ideal** in the tensor algebra $T(E \oplus E^*)$, subject the following relations in the algebra

$$x^{*i}\, x^j = \delta^{ij}\, \mathbf{1} + \Sigma_{k,l}^N\, C_{kl}^{ij}\, x^k\, x^{*l},$$
$$x^i\, x^j - \Sigma_{k,l}^N\, B_{kl}^{ij}\, x^k\, x^l = 0,$$
$$x^{*i}\, x^{*j} - \Sigma_{k,l}^N\, \overline{B}_{ij}^{kl}\, x^{*k}\, x^{*l} = 0,$$

and consistency conditions

$$(B \otimes id)(id \otimes B)(B \otimes id) = (id \otimes B)(B \otimes id)(id \otimes B),$$
$$(B \otimes id)(id \otimes C)(C \otimes id) = (id \otimes C)(C \otimes id)(id \otimes B),$$
$$(id_{E\otimes E} + \tilde{C})(id_{E\otimes E} - B) = 0,$$

where $\tilde{C}$ is a matrix with elements $(\tilde{C})_{kl}^{ij} = C_{lj}^{ki}$. The algebra $W(B, C)$ is determined by two operators $B$ and $C$ which satisfy the above consistency conditions. Hence the existence of this algebra is restricted to the existence of these operators. A simple example is provided by diagonal matrices defined by the relations

$$C(x^{*i} \otimes x^j) := c_{ji}\, x^j \otimes x^{*i},$$
$$B(x^i \otimes x^j) := b_{ij}\, x^j \otimes x^i, \quad \text{(no sum)}$$

where $c_{ij}$ are parameters such that $c_{ii} = q_i$ is a complex number, $c_{ij} = b_{ij}$ for $i \neq j$, $b_{ij}b_{ji} = 1$, and $b_{ii} = 1$ for every $i = 1, \ldots, N$, [2]. It is possible to omit the consistency problem by using more general algebras, namely *Wick algebras.* The *Wick algebra* $W(C)$ is defined by one operator $C$ and the corresponding crossing relation

$$x^{*i}\, x^j = \delta^{ij}\, \mathbf{1} + \Sigma_{k,l}^N\, C_{kl}^{ij}\, x^k\, x^{*l},$$

which is related with to Wick ordering of creation and annihilation operators [3].

### Bibliography


[1] E. E. Demidov, Uspekhi Mat. Nauk 48 (1993) 39, in russ; E. E. Mukhin, Commun. Alg. 22 (1994) 451; W. Marcinek, Rep. Math. Phys. 41, 155 (1998).

[2] W. Marcinek, W. Marcinek, On quantum Weyl algebras and generalized quons, in Proceedings of the symposium: *Quantum Groups* and Quantum Spaces, Warsaw, November 20–29, 1995, Poland, in *Quantum Groups* and Quantum Spaces, ed. by R. Budzynski, W. Pusz and S. Zakrzewski, Banach Center Publications, Vol. 40, p. 397, Warszawa 1997.

[3] P. E. T. Jorgensen, L. M. Schmith, and R. F. Werner, J. Funct. Anal. 134 (1995) 33.


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**QUANTUM YANG–BAXTER EQUATION** — The consistency condition for the *scattering matrix* factorization in a quantum mechanical many body problem [1] or *exactly solvable models* [2]. It can be also useful for the *quantum groups* [3]. The constant QYBE has the following form

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12},$$

for a linear operator $R : E \otimes E \to E \otimes E$, where $E$ is a vector space, $R_{ij}$ act on $i$ and $j$ factor of the tensor product $E \otimes E \otimes E$ as $R$, [1]. In the supersymmetric case $E = E_0 \oplus E_1$ is a *$Z_2$–graded vector space*, and $\hat{R}$ is a homogeneous mapping with respect to this gradation, [4]. In the noncommutative braid geometry the QYBE can be given in terms of the matrix $\tilde{R} := PR$, where $P(a \otimes b) := b \otimes a$, as follows

$$(\tilde{R} \otimes id)(id \otimes \tilde{R})(\tilde{R} \otimes id) = (id \otimes \tilde{R})(\tilde{R} \otimes id)(id \otimes \tilde{R}).$$