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**ORIENTATION** — For a **superspace** $\mathbb{R}^{n|m}$ or a **supermanifold** $M^{n|m}$, is an **orientation** of the even subspace $\mathbb{R}^n$ or the carrier $M_0^n \subset M^{n|m}$. It is preserved by the transformations that have positive determinant of the even-even block of the *Jacobi matrix*. **Orientation** in this sense naturally appears in *Berezin integration* theory. If a **supermanifold** $M^{n|m}$ is oriented, integrals of *volume forms* $f(x)Dx$ (with compact support) over $M^{n|m}$ are well-defined.

**Orientation** of a vector space can be understood as **coset** of a frame modulo transformations from an *open subgroup* of a general linear group. Unlike the purely even case, in the super case there are two *open subgroups* of index 2: $GL^{+-}(n|m) \subset GL(n|m;\mathbb{R})$ and $GL^{-}(n|m) \subset GL(n|m;\mathbb{R})$ specified by conditions $\det g_{00} > 0$ and $\operatorname{Ber} g > 0$, respectively. Here $\operatorname{Ber} g > 0$ is the **Berezinian** of $g$. **Orientation** defined above corresponds to the first option.

The other option corresponds to the so-called **orientation** of the second kind.

If a **supermanifold** $M^{n|m}$ is endowed with an **orientation** of the second kind, well-defined are integrals of *volume forms* of the type $f(x)D_{01}x$ (with compact support). Suppose on a **supermanifold** $M^{n|m}$ there is an **orientation** of the second kind. Then a usual **orientation** is induced on the total space of the tangent bundle with reversed parity:

$$\Pi TM = (\Pi TM)^{n+m|n+m}.$$

This is used in the integration of **pseudodifferential forms**.

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**ORIENTATION, of supermanifold** — There are five types of orientability depending on the orientability (orientable or not) of the underlying manifold (body) and the total space of the bundle with which the supermanifold is associated, which was described by Shander [1].

Let $\mathcal{U}$ be a superdomain of dimension $n|m$, let $x = (u_1, \dots, u_n, \xi_1, \dots, \xi_m)$ and $y = (v_1, \dots, v_n, \eta_1, \dots, \eta_m)$ be two coordinate systems on $\mathcal{U}$. Let cpr be the natural projection of the Grassmann algebra over a ring onto the ring.

The systems $x$ and $y$ will be called similarly oriented if

$$\operatorname{cpr}\left(\det\left(\frac{\partial u}{\partial v}\right)\right) > 0 \quad \text{and} \quad \operatorname{cpr}\left(\det\left(\frac{\partial \xi}{\partial \eta}\right)\right) > 0,$$

where $\operatorname{cpr} : C^\infty(\mathcal{U}) \longrightarrow C^\infty(U)$ is the canonical projection (hereafter the determinant of the matrix of size $0 \times 0$ is assumed to be equal to 1). An orientation of a superdomain is a class of similarly oriented coordinate systems. Therefore, if $n \cdot m \neq 0$, there are four orientation on $\mathcal{U}$; if either $n$ or $m$ vanishes, there are two orientations. By default we assume in what follows that $n \cdot m \neq 0$.

On the collection of coordinate systems on $\mathcal{M}$, the additive group $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ acts as follows. Denote by $e_1$ (resp. $e_2$) the generator of the first (resp. second) copy of $\mathbb{Z}_2$. Then $e_1$ multiplies by $-1$ the first even coordinate function while $e_2$ multiplies by $-1$ the first odd coordinate function. This $\mathbb{Z}_2 \oplus \mathbb{Z}_2$-action transforms similarly oriented coordinate systems into similarly oriented. The $\mathbb{Z}_2 \oplus \mathbb{Z}_2$-action on orientations is transitive. For $a, b \in \mathbb{Z}_2$, where $(a,b) \neq (0,0)$, denote by $K(a,b)$ the subgroup spanned by $(b,a)$. A connected supermanifold $\mathcal{M}$ is called (completely) orientable if there are 4 connected components of the universal cover, $\widetilde{\mathcal{M}}$, it is called $(a,b)$-semiorientable if there are 2 connected components of $\widetilde{\mathcal{M}}$ and the kernel of the $\mathbb{Z}_2 \oplus \mathbb{Z}_2$-action is $K(a,b)$, and non-orientable if $\widetilde{\mathcal{M}}$ is connected.

Examples: 1) If $\mathcal{M}$ admits an (even) almost complex structure, then $\mathcal{M}$ is orientable.

2) If $\mathcal{M}$ admits an odd almost complex structure, then $\mathcal{M}$ is $(1,1)$-semiorientable.

3) If $\mathcal{M}$ admits a symplectic structure, then $\mathcal{M}$ is $(1,0)$-semiorientable.

4) If $\mathcal{M}$ admits a periplectic (odd symplectic) structure, or an odd Riemannian structure, then $\mathcal{M}$ is $(1,1)$-semiorientable.

5) If $\mathcal{M}$ admits an even Riemannian structure, then $\mathcal{M}$ is $(0,1)$-semiorientable.

The supermanifold associated with the Möbius band (1-dimensional nontrivial vector bundle over the $S^1$) yields an example of a (1, 0)-semiorientable supermanifold. The same Möbius band used as the base of the trivial linear bundle (i.e., the vector bundle with 1-dimensional fiber) gives an example of a (0, 1)-semiorientable supermanifold.

We can also take the Whitney sum of two Möbius bundles and consider it as a linear bundle over the Möbius band. The supermanifold associated with this bundle is an example of a (1, 1)-semiorientable supermanifold. The product of a (1, 0)-semiorientable and a (0, 1)-semiorientable supermanifolds is nonorientable supermanifold.

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**ORIENTIFOLD** — A projection of an oriented closed string theory under the simultaneous action of the worldsheet *parity operator* Ω and a symmetry of the initial background. Per extension any projection of **M-theory**/11D supergravity that reverses the sign of some components of the 3-from potential.

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**OXIDATION** — Reconstructing the higher dimensional theory from the lower dimensional one [1] (see also [2]). An interesting aspect of **oxidation** is that, in contrast to **dimensional reduction**, it is not unique: there can be different branches leading to the same lower dimensional theory. In relevant cases, this nicely coincides with T-duality symmetries from *string theory*. The group theoretical aspects of the **oxidation** procedure are described in [3]. That is a magic triangle can be defined as a table of the U-duality groups appearing in the **oxidation** of $E_n$ cosets from 3 dimensions [3].

The supergravity theory in the highest possible dimension whose toroidal **dimensional reduction** gives back precisely to the pure supergravity in four dimensions with $N$ SUSY **spinors** is called an **oxidation** endpoint [1]. For example, the pure supergravity $\mathrm{N} = 8$ in $d = 4$ can be obtained by **dimensional reduction** of eleven-dimensional supergravity [4].

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