the symmetric *Christoffel symbol*) is determined by two conditions, namely the covariant constancy of the metric ($g_{\mu\nu;\rho} \equiv \partial_\rho g_{\mu\nu} - \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} - \Gamma^\lambda_{\rho\nu} g_{\lambda\mu} = 0$) and the absence of *torsion* ($\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$). The **Levi–Civita connection** appears in Einstein's theory of gravity based on the *Equivalence Principle* [2], but more general connections (with, for example, *torsion* [3]) may appear in certain supergravity theories [4].

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Frans Klinkhamer

LEVI–CIVITA SYMBOL — A pseudotensor defined for d-dimensional orientable manifolds, whose components $\epsilon_{\mu_1\mu_2\cdots\mu_d}$ are completely antisymmetric and normalized to, say, $\epsilon_{12\cdots d} = +1$. In two dimensions, for example, the components are $\epsilon_{12} = -\epsilon_{21} = 1$ and $\epsilon_{11} = \epsilon_{22} = 0$. The **Levi–Civita symbol** is a pseudotensor rather than a tensor, since it picks up a Jacobian factor under general coordinate transformations $x^\mu \to x'^\mu$, cf. [1].

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Frans Klinkhamer

LIE ALGEBRA — A vector space $\mathfrak{g}$ in which an additional operation, called a commutator of elements, is defined. A commutator for elements X and Y of a **Lie algebra** $\mathfrak{g}$ is denoted as $[X, Y]$. A commutator is a bilinear operation on $\mathfrak{g}$ satisfying the following conditions:

(a) $[X, Y] \in \mathfrak{g}$ for all $X, Y \in \mathfrak{g}$;

(b) $[X, Y] = -[Y, X]$ (anticommutativity);

(c) $[X, [Y, Z]] + [Z, [X, Y]] + [Y, [Z, X]] = 0$

(**Jacobi identity**).

Instead of "commutator" also the term "*Lie bracket*" is used. If $X_1, X_2, \ldots, X_n$ is a basis of the vector space $\mathfrak{g}$, then the relation $[X_i, X_j] = \sum_k c^k_{ij} X_k$, where c^k_{ij} are numbers, determines *structure constants* of the **Lie algebra**. *Structure constants* determine their **Lie algebra** uniquely up to an isomorphism. Anticommutativity of the commutator and **Jacobi identity** are equivalent to the following relations for *structure constants*:

$$c^k_{ij} = -c^k_{ji}, \qquad \sum_s (c^p_{is} c^s_{jk} + c^p_{js} c^s_{ki} + c^p_{ks} c^s_{ij}) = 0,$$

respectively. The subspace of even elements of any **Lie superalgebra** is a **Lie algebra**.

Anatoli Klimyk

LIE SUPERALGEBRA — Any $\mathbb{Z}_2$-graded (super)algebra whose product, denoted $[\cdot, \cdot]$, satisfies the generalized **Jacobi identity** and

$$[a, b] + (-1)^{|a||b|}[b, a] = 0,$$

where $|a|, |b|$ are the degrees of $a, b \in A$ (either 0 or 1). It reduces to some ordinary **Lie algebra** when A is ungraded, i.e., if all its elements have degree 0. (see e.g. [1]).

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Gert Roepstorff

LIE SUPERALGEBRA $\mathfrak{gl}(\infty|\infty)$ — An infinite dimensional and super version of $\mathfrak{gl}(n)$. There are several types of infinite dimensional $\mathfrak{gl}$'s that differ a lot in their properties. In order to describe them, let us identify any linear transformation with its matrix having chosen a basis in the space V of the identity representation of the **Lie algebra** $\mathfrak{gl}(V)$. We thus replace the **Lie algebra** of operators $\mathfrak{gl}(V)$ by the **Lie algebra** of matrices $\mathfrak{gl}(\dim V)$ and for an infinite (countably) dimensional $V = \mathbb{C}^\infty$ we get three definitions:

$\mathfrak{gl}(\infty) = \{X \mid X$ is a linear endomorphism of $\mathbb{C}^\infty\}$;

$\mathfrak{gl}_c(\infty) = \{X \in \mathfrak{gl}(\infty) \mid$ nonzero entries of X belong to a stripe of finite width containing the main diagonal and parallel to it$\}$;

$\mathfrak{gl}_f(\infty) = \{X \in \mathfrak{gl}(\infty) \mid X$ has finitely many nonzero entries$\}$.

Any of these three **Lie algebras** of infinite in both directions matrices has a subalgebra of matrices whose rows and columns are only labelled with positive integers. Denote these subalgebras by $\mathfrak{gl}^+_f$, etc. The algebras $\mathfrak{gl}_f(\infty)$ are limits of $\mathfrak{gl}(n)$ as $n \longrightarrow \infty$. These algebras are generated by countably many Chevalley generators subject to the relations governed by the **Cartan matrix** corresponding to one of the following infinite **Dynkin diagrams** (that describe $\mathfrak{sl}$) corresponding to the two ways to tend n to ∞:

○ – · · · – ○
$\longrightarrow$ ○ – · · · (corresponds to $\mathfrak{gl}^+_f(\infty)$)

and

○ – · · · – ○
$\longrightarrow$ · · · – ○ – · · · (corresponds to $\mathfrak{gl}_f(\infty)$)

Penkov proved that the **Lie algebras** $\mathfrak{gl}^+_f(\infty)$ and $\mathfrak{gl}_f(\infty)$ are isomorphic, $\mathfrak{o}^+_f(2\infty)$ and $\mathfrak{o}_f(2\infty + 1)$ are isomorphic, and similar statements hold for **Lie superalgebras**.

For some applications these algebras turn out to be too small, cf. [5–8]. Moreover, they have no nontrivial *central extension*, so it is desirable to enlarge them so that the enlarged **Lie algebra** has a nontrivial *central extension*. This *central extension* is needed in "quantization".

On the other hand, the algebra $\mathfrak{gl}(\infty)$, though a natural enlargement of $\mathfrak{gl}_f(\infty)$, is too large for the modern technique to handle it.

For the above-defined intermediate **Lie algebras** $\mathfrak{gl}_c(\infty)$ and $\mathfrak{gl}_c^+(\infty)$ the *central extension* $\tilde{\mathfrak{gl}}$ with the center z is given by the *cocycle*

$$X, Y \longmapsto \operatorname{tr} J[X, Y] \text{ for any } X, Y \in \mathfrak{gl}_c(\infty),$$

where $J = \operatorname{diag}(\dots, 1, -1, \dots)$ with the 1's occupying the negative positions and the -1's the remaining positions.

A simple finite dimensional **Lie superalgebra** $\mathfrak{g}$ can possess several bases (systems of simple roots) not equivalent with respect to the **Weyl group** of $\mathfrak{g}_{\bar{0}}$. Serganova suggested several *superizations* of the **Weyl group** all of which act by permutations of all the bases of a *simple Lie algebra* $\mathfrak{g}$ (either finite dimensional or a (twisted) loop one) with **Cartan matrix** [3].

As was first noted by M. Saveliev, in applications of **Lie superalgebras** to integrable dynamical systems the base of a *simple Lie superalgebra* that only consists of odd elements plays a distinguished role, [3]. In Table 6 of [3] there are listed **Lie superalgebras** with which there are associated by a *superization* of a method by Drinfeld and Sokolov, also described in [3], super versions of KdVs and (with infinite diagrams) KPs, see a pioneering paper [4] and later works [1].

Note that only a method to recover an equation from a superalgebra is written in [3], the exceptional *super KdVs* themselves corresponding to exceptional diagrams from Table 6 of [3] were never written explicitly.

Not every **Lie superalgebra** possesses such a basis. Serganova's result implies that if the **Lie superalgebra** is finite dimensional or a, perhaps, twisted loop one, or a Kac–Moody one, it does not matter which base we start with as long as the superalgebra possesses a distinguished base since we can reach any of the bases from any given one with the help of any of Serganova's super **Weyl groups**.

Is it true for $\mathfrak{gl}_c(\infty|\infty)$ which was chosen so far [1] to study a superized version of KdV and KP with that there is just one class of bases with respect to an analogue of the **Weyl group**? G. Egorov showed [2] that the answer is **no** and offered several versions of $\mathfrak{gl}(\infty|\infty)$ for which the answer is **yes**.

In what follows $\mathfrak{gl} := \mathfrak{gl}(\infty)$ and an arbitrary map $p\colon \mathbb{Z} \longrightarrow \mathbb{Z}/2$ will be called a parity function. We often encounter the following two parity functions:

$$p_+(x) = \begin{cases} \bar{1}, & \text{if } x \geq 0 \\ \bar{0}, & \text{otherwise} \end{cases}$$

and $p_{st}(x) \equiv x \pmod 2$. On $\mathfrak{gl}_f$, introduce a **Lie superalgebra** structure by defining the *supermatrices* with the help of a fixed parity function p and by setting

$$[a, b]_{ij} = \sum_k (a_{ik} b_{kj} - (-1)^{(p(i)+p(k))(p(j)+p(k))} b_{ik} a_{kj}) \qquad (*)$$

and denote the obtained **Lie superalgebra** by $\mathfrak{gl}_{p,f}$. The **Dynkin diagrams** corresponding to p_+ and p_{st} are (one grey node and all grey nodes), respectively:

$$\cdots - - \bigcirc - - \cdots - - \bigcirc - -$$
$$\otimes - - \bigcirc - - \cdots - - \bigcirc - - \cdots$$

and

$$\cdots - - \otimes - - \cdots - - \otimes - - \cdots$$

Consider the following four enlargements of $\mathfrak{gl}_{p,f}$ with the bracket defined by the above formula $(*)$:

$\mathfrak{gl}_{p,g} = \{a = (a_{ij}) \in \mathfrak{gl} \mid$ for any i_0, j_0 there exists a $c = c(i_0, j_0) > 0$ such that $a_{ij} = a_{ji} = 0$ for all i, j such that $(i - i_0)(j_0 - j) > c\}$.

There are finitely many entries in every row and every column as well as in the first and the fourth quadrants of every matrix from $\mathfrak{gl}_{p,g}$.

Proposition. The *supercommutator* makes the **superspaces** $\mathfrak{gl}_{p,*}$, where $* = g, l, o$, or c, into **Lie superalgebras**, i.e., $[a, b] \in \mathfrak{gl}_{p,*}$ for any $a, b \in \mathfrak{gl}_{p,*}$.

Weyl groups for $\mathfrak{gl}_{p,g}$ and its subalgebras. Let there be two different parity functions p_1 and p_2 on $\mathbb{Z}$. To find out what are the conditions for these two functions to determine isomorphic **Lie superalgebras** of classes $\tilde{\mathfrak{gl}}_{p,c}$, $\tilde{\mathfrak{gl}}_{p,o}$, $\tilde{\mathfrak{gl}}_{p,l}$ and $\tilde{\mathfrak{gl}}_{p,g}$ for $p = p_1$ and $p = p_2$, we need the following infinite permutations groups that will serve as analogues of the **Weyl group** of $\mathfrak{gl}(n)$. In what follows we will only consider isomorphisms of the **Lie superalgebras** of classes $* = g, l, o$, or c which transfer their common Lie subsuperalgebra of finite matrices $\mathfrak{gl}_f$ into itself.

Let $S_\mathbb{Z}$ be the group of all permutations of $\mathbb{Z}$, i.e. the group of all one-to-one maps $\sigma\colon \mathbb{Z} \longrightarrow \mathbb{Z}$; define the groups

$S_g = \{\sigma \in S_\mathbb{Z} \mid$ for some $c_\sigma > 0$ we have $\frac{\sigma(i)}{i} > 0$ for all i such that $|i| > c_\sigma\}$;

$S_l = \{\sigma \in S_g \mid \lim_{|i| \longrightarrow \infty} \frac{|\sigma(i)-i|^\lambda}{|i|} = 0$ for some $\lambda_\sigma = \lambda > 0\}$;

$S_m = \{\sigma \in S_g \mid \limsup_{|i| \longrightarrow \infty} \frac{|\sigma(i)-i|}{|i|} < \infty$, and $\liminf_{|i| \longrightarrow \infty} \frac{|\sigma(i)-i|}{|i|} > 0\}$;

$S_n = \{\sigma \in S_\mathbb{Z} \mid \lim_{|i| \longrightarrow \infty} \frac{|\sigma(i)-i|}{|i|} = 0\}$;

$S_o = \{\sigma \in S_\mathbb{Z} \mid \lim_{|i| \longrightarrow \infty} \frac{|\sigma(i)-i|^\lambda}{|i|} = 0$ for all $\lambda > 0\}$;

$S_c = \{\sigma \in S_\mathbb{Z} \mid |\sigma(i) - i| < c_\sigma$ for some $c_\sigma > 0$ and all $i \in \mathbb{Z}\}$.

The following inclusions

$$S_c \subset S_o \subset S_n \subset S_m \subset S_l \subseteq S_g \subset S_{\mathbb{Z}}$$

hold valid. Define the action of $S_{\mathbb{Z}}$ on parity functions by setting $(\sigma p)(i) = p(\sigma(i))$.

Proposition. Let p and p' be parity functions, $\sigma \in S_g$ and $p' = \sigma\ p$. Then the following formulas determine an isomorphism $\phi_\sigma\colon \tilde{\mathfrak{gl}}_{p,g} \longrightarrow \tilde{\mathfrak{gl}}_{p',g}$:

$$\phi_\sigma(z) = z,\ \ \phi_\sigma(a_{ij}) = (a_{\sigma(i),\sigma(j)}) + \mathrm{str}(J_\phi a)z,$$

where $(J_\phi)_{ij} = J_{\sigma^{-1}i,\,\sigma^{-1}j} - J_{ij}$.

If $\sigma \in S_l$ (or S_o, S_c), then ϕ_σ induces an isomorphism of $\tilde{\mathfrak{gl}}_{p,l}$ with $\tilde{\mathfrak{gl}}_{p',l}$ (respectively of $\tilde{\mathfrak{gl}}_{p,o}$ with $\tilde{\mathfrak{gl}}_{p',o}$, or $\tilde{\mathfrak{gl}}_{p,c}$ with $\tilde{\mathfrak{gl}}_{p',c}$).

The following theorem shows that the isomorphism problem for **Lie superalgebras** $\tilde{\mathfrak{gl}}_{p,g}$, $\tilde{\mathfrak{gl}}_{p,l}$, $\tilde{\mathfrak{gl}}_{p,o}$, $\tilde{\mathfrak{gl}}_{p,c}$ can be reduced to the *equivalence problem* for the parity functions on $\mathbb{Z}$ with respect to permutations groups S_g, S_l, S_o and S_c, respectively.

Theorem Let $\tau \in S_{\mathbb{Z}}$ be given by the formula $\tau(i) = -i - 1$. The **Lie superalgebras** $\tilde{\mathfrak{gl}}_{p_1,g}$ and $\tilde{\mathfrak{gl}}_{p_2,g}$ (respectively, $\tilde{\mathfrak{gl}}_{p_1,l}$ and $\tilde{\mathfrak{gl}}_{p_2,l}$, or $\tilde{\mathfrak{gl}}_{p_1,o}$ and $\tilde{\mathfrak{gl}}_{p_2,o}$, or $\tilde{\mathfrak{gl}}_{p_1,c}$ and $\tilde{\mathfrak{gl}}_{p_2,c}$) are isomorphic if and only if there exists $\sigma \in S_g$ (resp. S_l, or S_o, or S_c) such that $p_2 = \sigma(p_1)$ (respectively $p_2 = \tau\sigma(p_1)$).

Description of equivalence classes of parity functions. For an arbitrary parity function p and $-\infty \le a < b \le +\infty$ set:

$$\mathrm{Odd}(p;\, a,\, b) = \mathrm{card}\{i \in \mathbb{Z} \mid a \le i \le b,\ p(i) = 1\},$$

$$\mathrm{Even}(p;\, a,\, b) = \mathrm{card}\{i \in \mathbb{Z} \mid a \le i \le b,\ p(i) = 0\},$$

$$d(p;\, a,\, b) = d(p,\, b,\, a) = \frac{\mathrm{Odd}(p;\, a,\, b)}{(b - a)}.$$

We will call a parity function p finite if either of the following holds: either

$$\mathrm{Odd}(p; -\infty, +\infty) < \infty$$

or

$$\mathrm{Even}(p; -\infty, +\infty) < \infty.$$

Both $\mathrm{Odd}(p; -\infty, +\infty)$ and $\mathrm{Even}(p; -\infty, +\infty)$ are invariant with respect to any of the permutation groups defined above.

On the other hand, two finite parity functions p, p' are S_c-equivalent if and only if

$$\mathrm{Odd}(p; -\infty, +\infty) = \mathrm{Odd}(p'; -\infty, +\infty)$$
$$\mathrm{Even}(p; -\infty, +\infty) = \mathrm{Even}(p'; -\infty, +\infty).$$

Denote the superalgebras corresponding to a finite parity function p by

$$\mathfrak{gl}_{p,*}(m|\infty), \text{where m} = \mathrm{card(Even(p; -\infty, \infty))}$$

or by

$$\mathfrak{gl}_{p,*}(\infty|n), \text{where n} = \mathrm{card(Odd(p; -\infty, \infty))}$$

Proposition For finite parity functions the cardinalities $\mathrm{Odd}(p; -\infty, +\infty)$ and $\mathrm{Even}(p; -\infty, +\infty)$ give the complete system of invariants of the bases (systems of simple roots) of $\mathfrak{gl}_{p,*}(\infty)$ with respect to S_*.

Non-finite parity functions are subdivided into five S_g-invariant classes according to how many infinite values (0 to 4) are there among

$$\mathrm{Odd}(p; -\infty, 0),\ \mathrm{Even}(p; -\infty, 0),$$
$$\mathrm{Odd}(p; 0, +\infty)\mathrm{Even}(p; 0, +\infty).$$

Denote by Inf the set of parity functions for which all the four values are infinite.

Continuous invariants of certain parity functions. Let $\{a_i\}$, $\{b_i\}$ be two arbitrary series such that

$$\lim\ a_i = \lim\ b_i = \pm\infty,\ \lim\ (a_i - b_i) = \infty. \qquad (*)$$

We will say that α is a left (right) density point for p if there is a $-$ (respectively, $+$) sign in $(*)$ and α is a limit point of $d_i = d(p;\, a_i,\, b_i)$. We will call the set of all left (right) density points for all such series the left (right) density spectrum and denote it by $D_l(p)$ and $D_r(p)$, respectively.

Proposition Both D_l and D_r are subsegments of $[0, 1]$.

Let d be a limit point of D_r, i.e., $\lim d_i = d$ for some $d_i \in D_r$. For each d_i select matrices

$$\{(a_{ij},\, b_{ij})\colon \lim_{j\longrightarrow\infty} d(p,\, a_{ij},\, b_{ij}) = d_i\}$$

as in definition of D_r. Then d is a density point for $\{a_{ii},\, b_{ii}\}$.

Let d, $d' \in D_r$ and $\{(a_i,\, b_i)\}$, $\{(a_i',\, b_i')\}$ be some series converging to d and d', respectively. Then for any $\alpha : 0 < \alpha < 1$ we can select (a'', b'') such that $a_i < a'' < a'$, $b_i < b'' < b'$.

Theorem 1) D_l and D_r are S_c- and S_o-invariant; i.e., if $D_l = [\alpha^l, \beta^l]$, $D_r = [\alpha^r, \beta^r]$ and parity functions p and p' are S_o-equivalent, then $\alpha^l_p = \alpha^l_{p'}$, $\alpha^r_p = \alpha^r_{p'}$, $\beta^l_p = \beta^l_{p'}$, $\beta^r_p = \beta^r_{p'}$.

2) If $0 < \alpha^l = \beta^l < 1$ and $0 < \alpha^r = \beta^r < 1$ for both p and p', then these two functions are S_m-equivalent if and only if they have the same density spectrum.

3) If $0 < \alpha^l \le \beta^l < 1$ and $0 < \alpha^r \le \beta^r < 1$ for both p and p', then these two functions are S_n-equivalent.

4) Any two non-finite parity functions that belong to the same invariance class are S_g-equivalent.

Consider the smallest of the permutation groups, S_c. The main reason to investigate it, is that for $p \equiv 0$ the **Lie algebra** $\tilde{\mathfrak{gl}}_{p,c}$ possesses various nice properties, studied in [6], and our $\tilde{\mathfrak{gl}}_{p,c}$ for p not identically zero is its straightforward generalization.

For $\sigma \in S_c$ we have $|\sigma(a) - a| \leq c_\sigma$ and, therefore, obviously

$$|d(\sigma(p); a, b) - d(p; a, b)| \leq \frac{2c_\sigma}{|a-b|} \text{ for any } p.$$

Thus, for any two series a_i and b_i such that $|a_i - b_i| \longrightarrow \infty$ as $i \longrightarrow \infty$ all density limit points are σ-invariant.

Conjecturally, one can describe parity functions from Inf in terms of the density spectrum only for non-finite functions with the property that for some $c_p > 0$ there is no interval (a, b) with $b - a > c_p$ on which p is constant. In other words, the density spectrum $b - a >$of such p should be separated from 0 and 1. We will call such parity $b - a >$ functions tight.

A larger group, S_n, enables us to progress further:

Proposition If two parity functions have coinciding one-point left and right density spectra and are tight, these functions are S_n-equivalent.

Applications: **super Kadomtsev–Petviashvili** hierarchy, see [1].

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Dimitry Leites

LIE SUPERALGEBRA gl(λ) — The **Lie algebra** of "matrices of *complex size*" λ, as the **Lie algebra** constructed on the space of the associative algebra B_λ, the quotient of $U(\mathfrak{sl}(2))$ modulo the *central character* (the **ideal** generated by C_2-const), where C_2 is the quadratic Casimir element), by means of replacing the *dot product* with the bracket [1]. The associative algebra B_λ was known earlier [2].

The associative version, B_λ, did not draw much attention until recently when identified as a central simple algebra, see [4] for construction of central simple superalgebras important for **Morita equivalence** which, in turn, is important in **M-theory** [8].

The Lie version, $\mathfrak{gl}(\lambda)$, was interpreted as a simplest example of a new class of simple (modulo center) Lie (super)algebras: filtered of polynomial growth, see [3], where interpretations and defining relations are given; for representation see [5].

There can be several **supertraces** on generalizations of $\mathfrak{gl}(\lambda)$ [9]. Apart for such unexpected features, many properties of finite dimensional $\mathfrak{gl}(n)$ have analogs for $\mathfrak{gl}(\lambda)$.

In higher spin gauge theories superanalogs of $\mathfrak{gl}(\lambda)$ (quotient of $U(\mathfrak{osp}(1|2))$ modulo the *central character*) naturally appear [7]. For applications to orthogonal polynomials see [6].

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Dimitry Leites

Likhtman, Evgeny Pinkhasovich — (b. Jan. 12, 1946, Moscow, USSR) Together with Yuri Golfand constructed the first four-dimensional supersymmetric *field theory*, supersymmetric quantum electrodynamics with the mass term of the photon/photino fields, plus two chiral matter supermultiplets [1] (a more detailed version was published in [2]). Likhtman was the first to observe that the vacuum energy vanishes in supersymmetric *field theories.* On page 8 of [3] one can read, in particular: "As is known, in relativistic *quantum field theory*, in transforming the free energy operator to the normal-ordered form there emerges an infinite term which is interpreted as the vacuum energy. It is also known that the sign of this term is different for the particles subject to the *Bose statistics* and *Fermi statistics.* The number of the boson states is always equal to the number of the fermion states. From this it follows that the infinite positive energy of the boson states in any