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Jacobi Identity — A relation that expresses an axiom concerning *Lie bracket* (commutator) [.,.] of the elements X, Y, Z of a (complex or real) **Lie algebra** $\mathfrak{g}$ which together with two other axioms – of antisymmetry $[X, Y] = -[Y, X]$ and bilinearity $[aX + bY, Z] = a[X, Z] + b[Y, Z]$ – provide definition of $\mathfrak{g}$. **Jacobi identity** is given in terms of double commutators as

$$[X, [Y, Z]] + [Y, [Z, X]] + [Z, [X, Y]] = 0 \qquad (1)$$

and implies that the product given by *Lie bracket* is in general nonassociative.

Definition of a **Lie superalgebra** involves, in addition to the **Jacobi identity** (1) of its Lie subalgebra (generated by only even elements of the superalgebra), three other Jacobi-like identities involving odd elements so that alltogether the four identities can be written as single super Jacobi identity [1]

$$(-1)^{ac}[A, [B, C]']' + (-1)^{ba}[B, [C, A]']' + (-1)^{cb}[C, [A, B]']' = 0. \qquad (2)$$

In (2), the a, b, c denote grades of elements A, B, C (grade is zero/one for even/odd element respectively) and the prime at the bracket means that it is commutator in all cases except for the bracket of two odd elements which is given by anticommutator $\{.,.\}$.

In the case of a q-deformed analogue of **Lie algebra**, which involves, instead of [.,.], the so-called *q-commutator* of elements A, B defined as $[A, B]_q \equiv AB - qBA$, most general *quantum* **Jacobi identity** takes the form [2]

$$[A, [B, C]_{q_1}]_{q_2} + q_2[B, [C, A]_{q_1}]_{q_2^{-1}} + [C, [A, B]_{q_1 q_2}] = 0. \qquad (3)$$

In the limit of $q_1 = q_2 = 1$ this reduces to the ordinary **Jacobi identity** (1). Direct analogue of **Jacobi identity** (1), in which the *Lie bracket* [.,.] is replaced by the **Poisson bracket**, is a necessary ingredient in the definition of Poisson–Lie algebra of smooth functions on a manifold M.

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Jordan Algebra — An algebra of *Hermitian operators* that is closed under symmetric multiplication (unlike *Lie bracket*) and **Jordan algebras** is an abstract notion which describes this situation. **Jordan algebras** is a commutative algebra J with product $\circ$ satisfying, instead of associativity, the identity

$$(x^2 \circ y) \circ x = x^2 \circ (y \circ x).$$

In [1] McCrimmon gave an account of some applications of **Jordan algebras** from antiquity to nowadays.

Though their applications seem to be numerous [1], their theory did not inspire, perhaps, because, as I. Kantor says "there are no **Jordan algebras**, there are only **Lie algebras**", [2]. For *superization* see [2,3].

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