where ϵ and $\bar{\epsilon}$ are the spinorial parameters of the *supersymmetry transformations*. The *supersymmetry algebra* closes *off-shell* on each component.

The free dynamic of a massless **chiral superfield** is described by the action

$$S_0 = \int d^4x d^2\theta d^2\bar{\theta}\, \Phi\bar{\Phi} \qquad (1)$$

which reduced in components, reads

$$S_0 = \int d^4x \left[\phi\Box\bar{\phi} - \psi^\alpha i\partial_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} + F\bar{F}\right]$$

The **auxiliary field** F does not interact with the dynamical fields and it is governed by an algebraic equation of motion, $F = 0$. Therefore, it can be set to zero without affecting the dynamics of the physical components.

The class of **chiral superfields** defined on a given **superspace** is closed under tensor product: given two **chiral superfields** Φ_1 and Φ_2, the product $\Phi_1\Phi_2$ is still chiral. A consequence of this property is that a mass term can be added to the action (1) as a chiral integral of a chiral quadratic expression

$$S_{mass} = \frac{m}{2}\int d^4x d^2\theta\, \Phi^2 + \frac{m}{2}\int d^4x d^2\bar{\theta}\, \bar{\Phi}^2$$

The action $S_0 + S_{mass}$ describes the free motion of a massive boson and a massive *Dirac fermion.*

Interacting theories can be constructed as theories with potential, by adding a **superpotential** term. The well-known *Wess–Zumino model* given by $S = S_0 + S_{mass} + S_{int}$, where

$$S_{int} = \frac{g}{3!}\int d^4x d^2\theta\, \Phi^3 + \frac{g}{3!}\int d^4x d^2\bar{\theta}\, \bar{\Phi}^3$$

describes the dynamics of a self-interacting massive *scalar superfield.* After elimination of the **auxiliary fields**, the component action contains a massive scalar with cubic and quartic self-interactions, and a massive fermion coupled to the scalar through a *Yukawa potential.*

Another class of highly interacting theories in four dimensions is the class of N = 1 supersymmetric *nonlinear sigma models*, which are described in terms of a *Kähler potential* [3], function of m chiral and m **antichiral superfields**

$$S_\sigma = \int d^4x d^2\theta d^2\bar{\theta}\, K(\Phi^i, \bar{\Phi}_j)$$

Requiring **supersymmetry** implies the $2m$ *scalar superfields* to be the *holomorphic coordinates* of a *Kähler manifold.*

The **chiral superfield** in 4d allows for a consistent coupling to the **supersymmetric Yang–Mills** *vector multiplet.* It is then suitable for describing scalar matter in a given representation, coupled to a **gauge field**. Coupling of **chiral superfields** in the *fundamental representation* to the *vector multiplet* describes supersymmetric matter coupled to a **gauge field**. Coupling of a **chiral superfield**, Lie–algebra valued in the adjoint representation of the *gauge group*, with *Yang–Mills multiplet* gives an *off-shell* realization of pure N=2 super *Yang–Mills theory* [2]. Coupling of three **chiral superfields**, Lie–algebra valued in the adjoint representation of the *gauge group*, with *Yang–Mills multiplet* gives an *on-shell* realization of pure N = 4 super *Yang–Mills theory* [2].

Chiral superfields can be used as building blocks for *reducible representations* of **supersymmetry**. In fact, by the use of *superprojectors* [4], a *generic superfield* realizing a *reducible representation* of **supersymmetry** can be decomposed in terms of chiral *field strengths.*

Bibliography

[1] A. Salam, J. Strathdee, Nucl. Phys. **B76** (1974) 477; S. Ferrara, B. Zumino, J. Wess, Phys. Lett. 51B (1974) 239.

[2] S. J. Gates, Jr., M. T. Grisaru, M. Roček, W. Siegel, Superspace, Benjamin 1983.

[3] B. Zumino, Phys. Lett. **87B** (1979) 203.

[4] W. Siegel, S. J. Gates, Jr., Nucl. Phys. **B189** (1981) 295.

Silvia Penati

CLASSICAL LIE SUPERALGEBRA — The term "classical" is applied to simple Lie (super)groups (and their **Lie superalgebras**) over $\mathbb{C}$ that constitute series ($\mathfrak{sl}$, $\mathfrak{o}$, $\mathfrak{sp}$) and to selected real forms thereof, e.g., unitary (but not other forms, even pseudo-unitary). The experts in finite groups widen the range of applicabillity of the term, leaving out *sporadic groups*, and counting the permutation group S_n as classical.

Though $\mathfrak{gl}$ is not simple, it is always considered as classical over any field.

More tolerant approach is to consider (over complex numbers, reals) "relatives" of *simple Lie superalgebras* — their nontrivial *central extensions* or algebras of differentiations, and all their real forms, like unitary and pseudo-unitary series and extend this definition to any field. The notion of classical object changes with time as some used-to-be new objects become more familiar and classical. For example, Cartan's list of *simple Lie groups* (algebras) is now divided into series and exceptional all of which are now classical and familiar. The former *nonclassical algebras* of series E are the main **characters** in approaches to GUTs and what we consider as *monster algebras* [6] might be the classical algebras of **M-theory**.

For examples of serial and exceptional Lie superalgebras of various types, all of which should be regarded as classical, see [1–5].

Bibliography

[1] M. Scheunert, W. Nahm, V. Rittenberg, J. Mathematical Phys. **17** (1976), no. 9, 1626; P. Freund, I. Kaplansky I., J. Math. Phys.

17 (1976), no. 2, 228; I. Kaplansky, Pacific J. Math. 86 (1980), no. 1, 93; V. Kac, Adv. Math. 26 (1977) 8.
[2] I. Shchepochkina I., Funct. Anal. Appl., 33 (1999), no. 3, 208 (hep-th 9702121); id., Represent. Theory, 3 (1999), 373; I. Shchepochkina, P. Post, Internat. J. Algebra Comput. 8 (1998), no. 4, 479 (physics/9703022).
[3] B. L. Feigin, D. Leites, V. V. Serganova. In: M. Markov et al. (eds.) Harwood Academic Publ., Chur, 1985, Vol. 1–3, 631; J. van de Leur, Comm. Algebra 17 (1989), no. 8, 1815.
[4] P. Grozman, D. Leites, I. Shchepochkina, hep-th 9702120; Acta Mathematica Vietnamica, 26 (2001) 27.
[5] V. Kac, J. van de Leur, In: S. J. Gates, Jr., C. R. Preitschopf and W. Siegel (eds.) Strings '88 Proceedings of the workshop held at the University of Maryland, College Park, Maryland, May 24–28, 1988. World Scientific Publishing Co., Inc., Teaneck, NJ, 1989, 77; V. Kac, Comm. Math. Phys. 186 (1997), no. 1, 233 (Erratum: Comm. Math. Phys. 217 (2001), no. 3, 697); V. Kac, Adv. Math. 139 (1998), no. 1, 1.
[6] P. Henry-Labordere, B. Julia, L. Paulot, hep-th/0203070.

Dimitry Leites

CLASSICAL SUPERALGEBRA — $\mathcal{G}$ of rank r is characterized [1–4] by a **Cartan matrix** (a_{ij}) and a subset $\tau \subset I \equiv \{1,\dots,r\}$ that identifies the odd generators. The **Cartan matrix** can be normalized so that $a_{ii}=2$ if $i \notin \tau$ and $a_{ii}=1$ or 0 if $i \in \tau$. The algebra $\mathcal{G}$ can be constructed from the $3r$ generators $\hat{e}_i$, $\hat{f}_i$ and $\hat{h}_i$, $i \in I$, which satisfy the relations

$$[\hat{e}_i,\hat{f}_j] = \delta_{ij}\,\hat{h}_i, \qquad [\hat{h}_i,\hat{h}_j] = 0,$$
$$[\hat{h}_i,\hat{e}_j] = a_{ij}\,\hat{e}_j, \qquad [\hat{h}_i,\hat{f}_j] = -a_{ij}\,\hat{f}_j,$$

with the following assignments of parity:

$$p(\hat{h}_i) = 0; \quad p(\hat{e}_i) = p(\hat{f}_i) = 0, \quad i \notin \tau$$
$$p(\hat{e}_i) = p(\hat{f}_i) = 1, \quad i \in \tau.$$

In order to obtain a finite-dimensional algebra, the analogs of the *Serre relations* need to be enforced; they read

$$(\mathrm{ad}\,\hat{e}_i)^{1-\tilde{a}_{ij}}\,\hat{e}_j = 0, \quad (\mathrm{ad}\hat{f}_i)^{1-\tilde{a}_{ij}}\,\hat{f}_j = 0, \ \text{for } i \neq j,$$
$$[\hat{e}_i,\hat{e}_i] = 0, \quad [\hat{f}_i,\hat{f}_i] = 0, \ \text{if } a_{ii} = 0,$$

where $(\tilde{a}_{ij})$ is the matrix obtained from the **Cartan matrix** (a_{ij}) by substituting -1 for the strictly positive elements in the rows with 0 on the diagonal entry and setting $\tilde{a}_{ij} = 2\,a_{ij}$ if $a_{ii}=1$ [4]. Further, when an odd root is bordered by two even roots in the **Dynkin diagram**, extra relations need to be imposed. For instance, the **Lie superalgebras** of type $A(m,n)$, $B(m,n)$, $C(n+1)$ and $D(m,n)$ admit a basis (system of simple roots) such that card$(\tau)=1$; any result obtained in this basis can be re-expressed in any other basis with the help of the super **Weyl group**, that acts transitively on the set of all basis. [4] Then, for $\tau=\{m\}$, the extra relations involve the generators $(\hat{e}_{m-1},\hat{e}_m,\hat{e}_{m+1})$ and $(\hat{f}_{m-1},\hat{f}_m,\hat{f}_{m+1})$ associated to the odd root and its two nearest even neighbours and read:

$$\left[(\mathrm{ad}\,\hat{e}_{m-1})\hat{e}_m, (\mathrm{ad}\,\hat{e}_{m+1})\hat{e}_m\right] = 0,$$
$$\left[(\mathrm{ad}\,\hat{f}_{m-1})\hat{f}_m, (\mathrm{ad}\,\hat{f}_{m+1})\hat{f}_m\right] = 0.$$

For a complete set of defining relations see [5].

Bibliography

[1] V. G. Kac, Comm. Math. Phys. 53 (1977) 31.
[2] V. G. Kac, Adv. Math. 26 (1977) 8.
[3] V. G. Kac, in Lecture Notes in Mathematics 676 (1978) 597, (Springer-Verlag, Berlin).
[4] D. Leites, M. V. Saveliev and V. V. Serganova, in Group Theoretical Methods in Physics, V. Markov, V. Man'ko, and V. Dodonov, eds., vol. 1, p. 255 (VNU Science Press, Utrecht, 1986).
[5] P. Grozman, D. Leites, Czech. J. Phys., 51 (1997) 1.

Roberto Floreanini

CLIFFORD ALGEBRA — Cliff (p,q) of signature (p,q) is the real (or complex) associative algebra with a unit generated by elements $a_1, a_2, \dots, a_n$, $n=p+q$, satisfying the defining relations

$$\{a_i,a_j\} := a_ia_j + a_ja_i = 0, \qquad i \neq j,$$
$$a_i^2 = 1, \quad i=1,2,\dots,p, \quad a_j^2 = -1, \quad j = p+1, p+2, \dots, n.$$

Clifford algebras are finite-dimensional and dim Cliff $(p,q) = 2^{p+q}$. The elements

$$a_i,\ i=1,2,\dots,n, \quad a_ia_j,\ i<j,$$
$$a_ia_ja_k,\ i<j<k,\dots, \quad a_1a_2\cdots a_n$$

constitute a basis of Cliff (p,q), $p+q=n$. The **Clifford algebra** Cliff (p,q) is a Z_2-graded associative algebra and Cliff $(p,q) = \mathrm{Cliff}^0(p,q) + \mathrm{Cliff}^1(p,q)$ is its gradation, where $\mathrm{Cliff}^0(p,q)$ and $\mathrm{Cliff}^1(p,q)$ are spanned by products of even and odd numbers of elements a_i, respectively.

The **Clifford algebra** Cliff $(1,3)$ is generated by 4 elements which are denoted by γ_1, γ_2, γ_3, γ_4. The well-known *Dirac matrices* γ_1, γ_2, γ_3, γ_4 realize defining relations for the algebra Cliff $(1,3)$. The *Pauli matrices* σ_1, σ_2, σ_3 realize the defining relations of the algebra Cliff $(3,0)$. We have $\mathrm{Cliff}^0(3,0) \simeq$ Cliff $(2,0)$. The last **Clifford algebra** is isomorphic to the algebra of quaternions.

Anatoli Klimyk

CLIFFORD SEMIGROUP — A **semigroup** that is both an inverse and a **completely regular semigroup.** A simple consequence is that a **Clifford semigroup** is a *semilattice* of groups. This means that the set of *maximal subgroups* $\{S_\alpha : \alpha \in A\}$ can be indexed by the members of a *commutative semigroup* of idempotents A such that $S = \bigcup\{S_\alpha : \alpha \in A\}$ and $S_\alpha S_\beta \subseteq S_{\alpha\beta}$ for each α, $\beta \in A$. The details of the multiplication in S can be readily