relations $(i, j = 1, \dots, n, h \in \mathfrak{h})$:

$$[e_i, f_i] = \delta_{ij}\check{\alpha}_i,$$
$$[h, h'] = 0, \quad h, h' \in \mathfrak{h},$$
$$[h, e_i] = \langle \alpha_i, h \rangle e_i,$$
$$[h, f_i] = -\langle \alpha_i, h \rangle f_i,$$
$$(\text{ad } e_i)^{1-a_{ij}} e_j = 0, \quad i \neq j,$$
$$(\text{ad } f_i)^{1-a_{ij}} f_j = 0, \quad i \neq j.$$

where $\check{\alpha}_1, \dots, \check{\alpha}_n \in \mathfrak{h}$ (resp. $\alpha_1, \dots \alpha_n \in \mathfrak{h}^*$) is a linearly independent subset in the vector space $\mathfrak{h}$ (resp. dual vector space $\mathfrak{h}^*$), and $\langle \check{\alpha}_i, \alpha_j \rangle = a_{ij}$, where $\langle , \rangle$ is the pairing between $\mathfrak{h}$ and $\mathfrak{h}^*$, and $A = (a_{ij})_{ij=1}^n$ is the affine **Cartan matrix** [1]. A (nontwisted) **affine Kac–Moody algebra** may also be realized as the one dimensional *central extension* of a current algebra [1]. The *Frenkel–Kac–Segal construction* [2] in **conformal field theory** allows us to represent **affine Kac–Moody algebra** by **vertex operators**. In *string theory*, **affine Kac–Moody algebras** are related to the algebras of *BPS states* [3].

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AFFINE KAC—MOODY SUPERALGEBRA — A **Lie superalgebra** of functions on the circle with values in a *simple Lie superalgebra* (more precisely called *loop superalgebra*, perhaps, twisted by means of an automorphism of the *target* algebra) with *pointwise* multiplication or a central extention of such, or a subalgebra of the outer derivations of such [1].

Kac-Moody Lie algebras are described by extended **Dynkin diagrams** [4]. Observe that thanks to the existence of a nondegenerate symmetric bilinear form B on the *target* algebra, *loop algebras* always have a *central extension* given by the *cocycle*

$$c(f, g) = \text{res}(B(f, dg)).$$

Some of simple affine Kac–Moody (super)algebras have no *central extension*; some have an odd one; some have infinitely many *central extensions*; some have no **Cartan matrix**.

The conventional definition of *Kac–Moody Lie algebras* is via **Cartan matrix**; moreover, the **Cartan matrix** is always symmetrizable. *Kac–Moody superalgebras* often have no **Cartan matrix**; the matrix can be nonsymmetrizable. The finite dimensional *Kac–Moody superalgebra* with **Cartan matrix** may have several **Cartan matrices** (their presentations, i.e., defining relations, are listed in [3]). The twisted *loop superalgebra* corresponding to an outer automorphism of a finite dimensional **Lie superalgebra** G with **Cartan matrix** may have no **Cartan matrix** or v.v. therefore an intrinsic definition that does not appeal to **Cartan matrices** is desirable, it is as follows, see [2]: let $G = \oplus_{i=-d}^{\infty} G_i$ be a $\mathbb{Z}$-graded **Lie superalgebra** of polynomial growth and $d = \infty$. Then if there is every root vector corresponding to real root acts locally nilpotently in the adjoint representation, then G is Kac–Moody type **Lie superalgebra**, otherwise it is of *stringy* type. For real forms see [5].

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Dimilry Leites

AFFLECK–DINE–SEIBERG SUPERPOTENTIAL — A **superpotential** dynamically generated by nonperturbative quantum effects in certain **supersymmetric gauge theories**. The *nonrenormalization theorem* in supersymmetric *quantum field theory* only prevents its **superpotential** from getting perturbative quantum corrections and has no restriction on the contribution from nonperturbative quantum effects. Based on this consideration and motivated by exploring a possibility of dynamical *supersymmetry breaking* in a four–dimensional massless **supersymmetric gauge theory**, Affleck, Dine and Seiberg [1] first analyzed that the anomaly–free global symmetries in massless *supersymmetric QCD* are compatible with the existence of such a *dynamical superpotential.* Then they found that for $N_f = N_c - 1$ there arises a **superpotential** contributed by **instanton**, and for $N_f < N_c - 1$ this is still generated, but in conjunction with the *gaugino condensation*, while for $N_f > N_c$, no such *dynamical superpotential* exists, N_f and N_c being the numbers of flavour and color, respectively. Thus, this nonperturbative *dynamical superpotential* in case of $N_f \leq N_c$ is called **Affleck–Dine–Seiberg superpotential**. It has provided a new mechanism for