Hybrid Conference

Quantum graphs

in

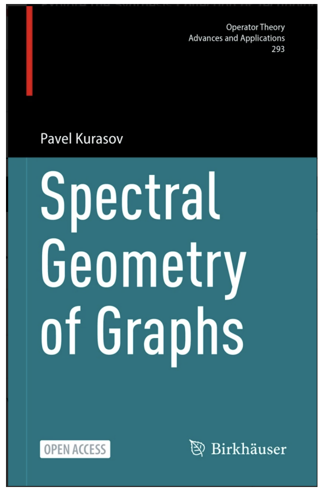
Mathematics, Physics and Applications

QGraph Network Meeting

partially supported by **COST Action CA18232**
"Mathematical models for interacting dynamics on networks"

Stockholm University, 7–8 December 2023

The conference is with the release of the new book entirely devoted to Schrödinger operators on metric graphs



Programme

All ordinary lectures of the meeting will take place at Stockholm University (Cramèr room), and on Zoom via

`https://stockholmuniversity.zoom.us/j/67008814207`

The schedule refers to Stockholm time (CET).

Thursday, 7 December

13.30-14.00	P. Kurasov
14.00-14.45	A. Kostenko
14.50-15.35	K. Naderi
15.40-16.00	**Break**
16.00-16.45	L. Sirko
16.50-17.35	M.-E. Pistol
18.00-20.00	**Dinner**

Friday, 8 December

09.00-11.00	Nobel lectures in physics (L'Huillier, Agostini, Krausz) in Aula Magna (Stockholm Univ.) and via `https://www.kva.se/evenemang/the-nobel-lectures-2023/`
13.30-14.15	R. Band
14.20-15.05	G. Sofer
15.10-15.35	M. Täufer (ZOOM)
15.40-16.00	**Break**
16.00-16.25	J. Kennedy (ZOOM)
16.30-16.55	L. Alon (ZOOM)
17.00-17.25	R. Carlson (ZOOM)
17.30-17.55	D. Mugnolo (ZOOM)

Quantum graphs, quasicrystals and stable polynomials.

L. Alon (MIT Cambridge)

Roughly speaking, Smilansky's trace formula says that the countable sum of $\exp(ikt)$, over all k square-root eigenvalues of a given quantum graph, vanishes whenever t is not the length of some closed path on the graph. Kurasov and Sarnak showed that (up to some technicalities) this means that the spectral measure of a quantum graph is a special type of crystalline measure (a term coined by Meyer) which is called Fourier quasicrystal (a term coined by Lev and Olevskii). Crystalline measures are of interest, partially due to their surprising connection to both physics (quasicrystals) and number theory (the zeros of the Zeta and L functions under GRH). Kurasov and Sarnak showed that the underlying reason for the spectral measure of a quantum graph to have such property is that the secular polynomial is a stable polynomial and that given any stable polynomial and a set of positive frequencies (in analog to the edge lengths) the zeros measure of the resulting trigonometric polynomial is a Fourier quasicrystal. Stable polynomials are multivariate polynomials that are closed under certain operations and were previously used in proving the Lee-Yang circle theorem and the Kadison-Singer conjecture. After providing the needed background, I will discuss a recent work in progress with Cynthia Vinzant on Kurasov-Sarnak quasicrystals. We show that such quasicrystals have well-defined gap distributions, by extending the Barra-Gaspard argument. The implication for quantum graphs is that any quantum graph with scaling invariant vertex conditions has a well-defined gap distribution, for any choice of edge lengths. If time permits, I will also explain how to construct random Kurasov-Sarnak measures with gap distribution as close as we want to the eigenvalues spacing of a random unitary matrix.

Dry Ten Martini Problem for Sturmian Hamiltonians

R. Band (Technion/Potsdam)

"Are all gaps there?", asked Mark Kac in 1981 during a talk at the AMS annual meeting, and offered ten Martinis for the answer. This led Barry Simon to coin the names the Ten Martini Problem (TMP) and the Dry Ten Martini Problem for two related problems concerning the Almost-Mathieu operator. The TMP is to show that the spectrum of the Almost-Mathieu operator is a Cantor set. The Dry TMP concerns the values of the integrated density of states (IDS). The gap labelling theorem predicts the possible set of values which the IDS may attain at the spectral gaps. The Dry TMP is whether or not all these values are attained, or equivalently, "are all gaps there?".

The TMP was fully solved by Artur Avila and Svetlana Jitomirskaya in 2005. The Dry TMP has been recently announced to be solved for the non-critical case (coupling constant different than one). The corresponding preprint by Artur Avila, Jiangong You and Qi Zhou appeared in June 2023.

This talk is about the Dry TMP for Sturmian Hamiltonians. These are one-dimensional Schroedinger operators with aperiodic potentials determined by Sturmian sequences. The potential is determined in terms of two parameters: the frequency and the potential strength (a.k.a coupling constant). As for the Almost-Mathieu operator the Dry TMP is whether all the possible spectral gaps are there for all irrational frequencies and all coupling constants. For large values of the coupling constant, the Sturmian Dry TMP was solved by Raymond in 1997. In 2016, David Damanik, Anton Gorodetski and William Yessen provided a solution if the frequency is the golden mean and for all couplings.

In a current project with Siegfried Beckus and Raphael Loewy we solve the Sturmian Dry TMP for all irrational frequencies and all couplings. In the talk we present the problem and the route to its resolution.

The spectral geometry of biregular graphs: a quantum graph approach

R.Carlson (Colorado Springs)

The spectral theory of the Laplacian on biregular graphs is developed. Generating functions for closed nonbacktracking walks appear when resolvents for trees are related to resolvents for the biregular graphs they cover. This construction leads to a detailed description of the rational extensions of nonbacktracking closed walk generating functions.

Cheeger cuts and Robin spectral minimal partitions of metric graphs

J. Kennedy (Lisbon)

We consider two types of minimisation problems for energy functionals of k-partitions of a fixed compact metric graph. In the first problem, the energy functional is based on the first eigenvalue of a Robin Laplacian on each partition element, where a Robin parameter (or delta condition) $\alpha > 0$ is imposed at all boundary points between partition elements. In the second problem, we look at a purely geometric, Cheeger-type functional based on the isoperimetric ratio of each element, whose minimum corresponds to the k-Cheeger constant of the graph.

We show that, as $\alpha \to 0$, the minimal Robin k-partition energy, suitably normalised, converges to the k-Cheeger constant; and, up to a subsequence, the minimising partitions also converge in a natural Hausdorff sense to a k-Cheeger cut of the graph.

This is based on joint work with João Ribeiro (Lisbon).

Laplacians on infinite metric graphs

A. Kostenko (Ljublajana/Wien)

The main focus in this talk will be on the self-adjointness problem (a.k.a. quantum completeness) for the Kirchhoff Laplacian on an infinite metric graph. More specifically, we will discuss the relationship between one of the classical notions of boundaries for infinite graphs, graph ends, and self-adjoint extensions of the minimal Kirchhoff Laplacian on a metric graph.

Spectral geometry of graphs

P. Kurasov (Stockholm)

The talk is devoted to the newly published book *Spectral Geometry of Graphs*.

Boundary conditions matter II

D. Mugnolo (Hagen)

This is the second part of the same-titled talk. We consider again two distinguished self-adjoint realisations of the Laplacian on infinite metric graphs: In the case of discrete spectrum, under additional metric assumptions, we also extend known upper and lower bounds on Laplacian eigenvalues to metric graphs that are merely locally finite. Most of these bounds are sharp and some of them ï£¡ e.g. in terms of the inradius ï£¡ are new even on compact graphs. This is joint work with Marco Düfel, James. B. Kennedy, Marvin Plümer and Matthias Täufer.

Transmission problems between fractal trees and the Euclidean space

K. Naderi (Oldenburg)

We consider a class of infinitely weighted metric trees obtained as perturbations of self-similar regular trees. For these trees we have found a definition of the trace which maps H^1-functions on the tree to L^2-functions on a compact Riemannian manifold. We succeeded in determining the exact Sobolev regularity of the trace. In the next step we consider the Euclidean space in which we cut out a smooth and compact domain. In this ï£¡holeï£¡ we glue our tree with the help of the trace mapping and thus obtain a new space on which we consider the transmission problem. The aim of our work is to study this problem in terms of well-posedness and to develop methods for the numerical computation of this problem. Based on collaborations with Valentina Franceschi (Padua), Maryna Kachanovska (Palaiseau) and Konstantin Pankrashkin (Oldenburg).

Isospectral but not isomorphic quantum graphs under different boundary conditions

M.-E. Pistol (Lund)

Quantum graphs are defined by having a Laplacian defined on the edges a metric graph with boundary conditions on each vertex such that the resulting operator is self-adjoint. Using computer algebra we have classified all (not too big) isospectral equilateral graphs under different boundary conditions, primarily Neumann boundary conditions. We there find that the loop has at least two isospectral partners. We have also classified isospectral graphs under Dirichlet boundary conditions at pendant edges as well as having ï£¡delta-typeï£¡ boundary conditions at internal vertices. There are pairs of graphs that remain isospectral under a family of boundary conditions with three independent parameters. For most of our graphs we have found all vertices with the same Titchmarsh-Weil m-function, which includes graphs having delta-type boundary conditions. We will present an efficient algorithm which can do such a classification of vertices. If there is time we will demonstrate our open source software. This work is based on https://arxiv.org/abs/2104.12885, but we will also present new results.

Microwave networks with broken and preserved time reversal symmetries. What they can be used for?

L. Sirko (Institute of Physics, Polish Academy of Sciences, Warszawa)

We report experimental study of the distributions of the reflection amplitudes $r_i = |S_{ii}|$ of the two-port scattering matrix S for networks with unitary and symplectic symmetries [1-2] for the intermediate and large absorption strength parameter γ. The experimental results confirm the theoretical predictions obtained within the framework of the Gaussian unitary and symplectic ensembles of the random matrix theory. We also investigate properties of the transmission amplitude of quantum graphs and microwave networks with preserved time reversal symmetries composed of regular polygons. We show that the transmission amplitude of such graphs displays a band of transmission suppression with some narrow peaks of full transmission [3].

This is joint work of Omer Farooq, Afshin Akhshani, Małgorzata Białous, Szymon Bauch, Michał Ławniczak, and Leszek Sirko.

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[1] Oleh Hul, Szymon Bauch, Prot Pakoński, Nazar Savytskyy, Karol Życzkowski, and Leszek Sirko, Phys. Rev. E **69 (5)**, 056209 (2004).

[2] Michał Ławniczak, Afshin Akhshani, Omer Farooq, Małgorzata Białous, Szymon Bauch, Barbara Dietz, and Leszek Sirko, Phys. Rev. E **107**, 024203 (2023).

[3] Afshin Akhshani, Małgorzata Białous, and Leszek Sirko, Phys. Rev. E **108**, 034219 (2023).

Spectral properties of aperiodic metric graphs

G. Sofer (Technion)

Sturmian Hamiltonians appear in mathematical physics as popular models for one-dimensional quasicrystals. This family of discrete one-dimensional Schrödinger operators, including the well known Fibonacci Hamiltonian, is widely studied for its interesting spectral properties.

We study metric analogues of these systems, by considering families of almost periodic metric graphs whose local geometric structure is determined by Sturmian sequences. We show that these graphs share many spectral properties with their discrete Sturmian counterparts. For instance, their spectra can be obtained as the limiting spectra of an appropriate sequence of periodic approximations. We also point at several unique features of these models, which are distinct to the metric setting.

The talk is based on a joint work in progress with Ram Band.

Boundary conditions matter II

M. Täufer (Hagen)

We develop a comprehensive spectral geometric theory for two distinguished self-adjoint realisations of the Laplacian, the so-called Friedrichs and Neumann extensions, on infinite metric graphs. We present a new criterion to characterise compactness of the resolvent of these extensions leading to concrete examples where this depends on the chosen extension. In this context, we present a one-parameter class of graphs that displays a phase transition from discrete to non-discrete spectrum ï£¡ a phenomenon that seems to have no known counterpart for Laplacians on Euclidean domains. This is joint work with Marco Düfel, James. B. Kennedy, Delio Mugnolo and Marvin Plümer