Phd-Project: Pseudo-Caratheodory functions

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Functions that map the unit disc holomorphically into a half plane play an important role in many applications (System theory, electrical technology, to name a few). Such functions are very well studied, useful tools are integral representations, or more generally speaking, representations involving resolvents of self adjoint operators in Hilbert spaces.

It appears that these functions do not cover all applications and in 1986 a new class of so-called **Pseudo-Caratheodory functions** was introduced and studied by scientists working at Philips Research Laboratory in Belgium. By definition a function f belongs to this class if it is the ratio of two bounded analytic functions (H^∞ -functions) and if the real part of its radial limit is non-negative almost everywhere. To such a function f one can associate an index I(f).

- If I(f)=0, then f is a "usual" Caratheordory function, i.e. mapping the unit disc into the right half plane. It is well known that such functions have operator representations in terms of a resolvent of a self adjoint operator in a *Hilbert space*.

- If I(f) is finite, then it turned out that the function is a "generalized Caratheodory function", which had been studied already earlier from a different point of view, namely in connection with self-adjoint operators in *Pontryagin spaces* (these are indefinite inner product spaces where the non-positive part is finite dimensional).

- If I(f) is not finite not so much is known about operator representations.

However, every locally analytic function admits (locally) an operator representation with a self-adjoint operator in a *Krein space* (these are indefinite inner product spaces where both the positive and the negative parts can be infinite dimensional). Hence the question appears what are the operator theoretic properties in these representations for Pseudo-Caratheodory functions with infinite index. It should be possible to characterize these functions in terms of properties of the representing operator.

As the project deals with a link between analytic functions and operators you should be familiar with basic complex analysis and some functional analysis. Moreover, some prior experience with Krein spaces can be useful, but is not mandatory.

Deadline for application: May 2nd, 2013

If you are interested in the project, you are welcome to contact me (luger@math.su.se) for more information!