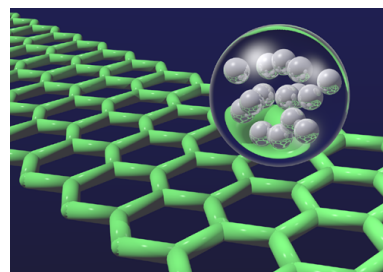# Discrete and continuous quantum graphs

PhD student project

Announced on
www.math.su.se


Credit: Erlend Davidson (Thomas Young Centre)

Quantum graphs - differential operators on metric graphs - is a rapidly growing branch of mathematical physics lying on the border between differential equations, spectral geometry and operator theory. The goal of the project is to compare dynamics given by discrete equations associated with (discrete) graphs with the evolution governed by quantum graphs. Discrete models can be successfully used to describe complex systems where the geometry of the connections between the nodes can be neglected. It is more realistic to use instead metric graphs with edges having lengths. The corresponding (continuous) dynamics is described by differential equations coupled at the vertices. Such models are used for example in modern physics of nano-structures and microwave cavities.

Understanding the relation between discrete and continuous quantum graphs is a challenging task leaving a lot of freedom, since this area has not been studied systematically yet. In special cases such relations are straightforward, sometimes methods originally developed for discrete graphs can be generalized, but often studies lead to new unexpected results. To find explicit connections between the geometry and topology of such graphs on one side and spectral properties of corresponding differential equations on the other is one of the most exciting directions in this research area. As an example one may mention an explicit formula connecting the asymptotics of eigenvalues to the number of cycles in the graph, or the estimate for the spectral gap (the difference between the two lowest eigenvalues) proved using a classical Euler theorem dated to 1736!

Possible directions of the research project are

- Study spectral properties of continuous quantum graphs in relation to their connectivity and complexity (this question is well-understood for discrete graphs);
- Investigate relations between quantum graphs and quasicrystals;
- Develop new models combining features of discrete and continuous graphs and study their properties;
- Transport properties of networks and their complexity.

It is expected that the candidate will take an active part in the work of the Analysis group at Stockholm University, Cooperation group "Continuous Models in the Theory of Networks" (ZIF, Bielefeld, http://www.uni-bielefeld.de/ZIF/KG/2012Models/) and Research and Training Network "QGRAPH" (http://www2.math.su.se/ pak/QGRAPH/) joining 15 research teams from all over the world.
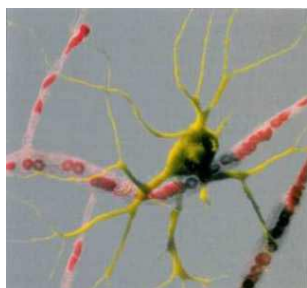
Supervisor: Pavel Kurasov, Stockholm university.
If you are **interested**, please send an e-mail
to Pavel Kurasov at pak@math.su.se
Deadline for application: May 2, 2013