A study of two explanations in the general theory of relativity

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Abstract

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Keywords
The general theory of relativity · deflection of light rays · Shapiro time delay · delay in the arrival of reflected light · Albert Einstein · Irwin Shapiro · Newton's theory of gravitation · Isaac Newton

Introduction

The purpose here is to present briefly two astronomical phenomena: the deflection of light rays passing close to the sun and the delay in arrival of light sent from the earth and reflected back while passing close to the sun, and to compare explanations proposed to understand these well-known phenomena.

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The comparison concerns the history of science, and raises problems in the philosophy of science.

Early calculations using Newtonian attraction

The first to perform a calculation of the deflection of a light ray passing close to the sun was Henry Cavendish (1731–1810), who did so around 1784; see the article by Clifford M. Will (1988a). This fact is also briefly mentioned by Cervantes-Cota et al. (2020:2). The calculations were not published.

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Erasmus Darwin (1731–1802), grandfather of Charles Darwin (1809–1882), mentioned in a book published in 1791 that a massive object can bend the trajectory of a light ray (Cervantes-Cota et al. 2020:2).

Steven S. Shapiro and Irwin I. Shapiro (2010) write:

“The first calculation of the deflection of light by mass was published by the German astronomer Johann Georg von Soldner in 1801. Soldner showed that rays from a distant star skimming the Sun’s surface would be deflected through an angle of about 0.9 seconds of arc [...]” (Shapiro & Shapiro 2010)

See (von Soldner 1801–1804, 1921).

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See (von Soldner 1801–1804, 1921).

However, as mention above, the first to perform this calculation was Henry Cavendish. Since his work was not published, von Soldner is the one who first published such results.

The general theory of relativity applied to the deflection of light rays

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Einstein's book (1917) was intended to be popular ("*Gemeinverständlich*"), but its translation (1920a) was provided with three more technical and quite useful appendices, the third of which, entitled "The experimental confirmation of the general theory of relativity," is authored by Einstein at the request of the translator Robert W. Lawson, according to the Translator's Note on page ix.

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For reports on solar eclipses, see publications by Will (2006), Lemos (2019), Lemos et al. (2019) and Cervantes-Cota et al. (2020).

Johan von Soldner in 1801–1804 and Albert Einstein in 1911

According to Newton's celestial mechanics, the angle of deflection of a particle with positive mass passing at distance r from the sun's center and with the speed v is

$$\sigma(r, v) = \frac{2GM_\odot}{v^2 r}, \tag{1}$$

where G is Newton's gravitational constant and $M_\odot$ the mass of the sun. For $v = c$, the speed of light, this formula yields an angle of $4 \cdot 10^{-6}$ radians, equal to $0''.83$. He mentions that the calculation leading to this value is a first approximation only (1911:898, 1973a:129).

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Jean-Marc Ginoux (2021) presents von Soldner's calculation and Einstein's two predictions, and discusses in both cases the so-called problem of doubling. A publication by Tilman Sauer (2021) contains a detailed analysis of von Soldners writings, including a study of an original text preserved in the *Bayerische Staatsbibliothek* in Munich, where somebody has made remarks (corrections?) with a pencil.

Einstein in 1916

According to Albert Einstein's general theory of relativity, the sun deflects a light ray: light follows a geodesic in spacetime, and geodesics are influenced by masses. In the words of Clifford M. Will, there are

> "four main phenomena associated with the spacetime geometry that is the central concept of Einstein's theory, namely that it bends light, it warps time, it moves mass, and it makes waves." (Will 2017:19)

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In 1916, Albert Einstein published his general theory of relativity in *Annalen der Physik* (1916a), reprinted in (1916b).

A large part of the 1916 text is devoted to Riemannian geometry and can describe a spacetime with any kind of curvature. To get a description of the universe we are living in, it is necessary to relate the curvature to existing masses. This he does on page 818 in (1916a)—the fiftieth page in his 54-page article—by comparing his gravitational potential with that of Newton. This comparison results in formula (69) on page 818; let me quote:

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“Durch Vergleich ergebt sich

$$\kappa = \frac{8\pi K}{c^2} = 1{,}87 \cdot 10^{-27}. \tag{69}$$

”

(Einstein 1916a:818)

In this equation, κ is Einstein’s gravitational constant and K Newton’s gravitational constant, given the value $6.7 \cdot 10^{-8}$. The unit is cm·g^{-1}.

He determines the deflection, called “die Biegung” and denoted by B, as follows.

“Die Ausrechnung ergibt

$$B = \frac{2\alpha}{\Delta} = \frac{\kappa M}{4\pi\Delta}. \tag{74}$$

Ein an der Sonne vorbeigehender Lichtstrahl erfährt demnach eine Biegung von 1,7″, ein am Planeten Jupiter vorbeigehender eine solche von etwa 0,02″.”

(Einstein 1916a:822; similarly in 1916b:63)

“According to this, a ray of light going past the sun undergoes a deflection of $1{\cdot}7''$ and a ray going past Jupiter a deflection of about $\cdot 02''$.” (Einstein 1973b:171)

The angle of $1''.7$ is that for the smallest possible value of Δ, i.e., $\Delta = R_\odot$, the radius of the sun:

$$\sigma(R_\odot, c) = \frac{4GM_\odot}{c^2 R_\odot}, \tag{2}$$

where, as in (1), G is Newton's gravitational constant, $M_\odot$ the mass of the sun, and c the speed of light. This amount is obtained by combining formula (69) on page 818 and formula (70a) on page 819 in (Einstein 1916a); likewise by combining formula (105a) on page 86 and formula (108) on page 89 in (Einstein 1987), and is stated also in (Weber 1964:233).

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Actually, formula (74) does not result in the claimed deflection of $1''.7$. Carl Runge (1856–1927) discovered this error and asked Einstein about it in 1920 (Sauer 2021: Section 3). To yield the angle of $1''.7$, the denominator $4\pi\Delta$ should be replaced by $2\pi\Delta$. See also the publication by Ginoux (2021).

Concerning the doubling, Einstein writes:

“Es sei hingefügt, daß diese Ablenkung nach der Theorie zur Hälfte durch das (N e w t o n sche) Anziehungsfeld der Sonne, zur Hälfte durch die von der Sonne herrührende geometrische Modifikation („krümmung”) des Raumes erzeugt wird.”

(Einstein 1917:87 = Kox et al., Eds., 1996:511; Einstein 1920b:87)

In translation:

"It may be added that, according to the theory, half of [...] this deflection is produced by the Newtonian field of attraction of the sun, and the other half by the geometrical modification ("curvature") of space caused by the sun."

(Einstein 1920a: Appendix III, (b), page 153)

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(Einstein 1920a: Appendix III, (b), page 153)

These words, *durch* [. . .] *erzeugt wird* and *is produced by*, give me the impression that Einstein considered Newton’s theory of gravitation, even though he now works within his general theory of relativity. I find this simultaneous use of two different theories remarkable.

Other scientists on the doubling

Several scientists have discussed the reasons behind the doubling. These include Julian Schwinger (1968), James J. Callahan (2001), Jean-Marc Ginoux (2021), and Tilman Sauer (2021); Ginoux and Sauer being already mentioned.

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In his book (1986), Julian Schwinger (1918–1994) gives the amount 0.875 arc seconds (1986:140), then mentions the doubling on page 189, and goes on:

“A light beam travels at speed c far from gravitating masses. As it nears the Sun, it takes longer (Einstein, 1911) to go a shorter distance (Einstein, 1916). The two fractional changes are equal. Light slows down by *twice* the fractional change that the consideration of time alone predicted.

[...] we now find that the angular deflection of light in a grazing passage of the Sun is $4gR/c^2$ [to be corrected to $4g'R/c^2$, which in the notation used here is equal to (2)]; the numerical value, an angle measured in radians, is 8.48×10^{-8} [equal to $1''.75$].” (Schwinger 1986:209)

Misner et al. (2002:1103) give the value $\frac{1}{2}(\gamma + 1) \cdot 1''\!.75$, where γ is the parametrized Post-Newtonian (PPN) parameter, close to 1. See also (Will 2017:20) and (Lemos et al. 2019:2).

Joseph Weber (1919–2000) explains:

“It is easy to see why we get an extra factor of 2. The deflection is given by the contribution of two identical terms Γ^x_{oo} and Γ^y_{xx} [...]. In the classical theory, only one term is present (Γ^y_{xx}). At low velocity, $dx/ds \sim v/c$ and $\Gamma^y_{xx} v^2 \ll \Gamma^x_{oo} c^2$; at $v = c$, they contribute equally. We can say that the photon acts as if it has a gravitational mass twice its inertial mass.” (Weber 1964:233)

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Similarly, Callahan (2001:419) writes that the doubling is due to “a fuller account of the gravitational effect.”

Refraction in the sun's atmosphere

Light passing through the sun's atmosphere is subject to refraction: Xu's results (2002) give the value of $26''$ for grazing rays, which has not been observed. However, when the distance to the sun's center is just a little larger than the sun's radius, it is insignificant.

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The refraction decreases very quickly when the distance to the sun's limb increases, much faster than the deflection, which is inversely proportional to the distance to the sun's center—not to the limb.

So it may well be that no observation has been made sufficiently close to the limb as to show such a large refraction.

Delay in the arrival of light

Shapiro time delay

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Irwin Shapiro (1964) predicted that a light signal sent from the earth, passing close to the sun and then reflected back to the earth would be delayed. This phenomenon is now known under the name *Shapiro time delay*.

Importantly, this delay can be measured more easily than the deflection, since it can be done when there is a spacecraft or planet behind the sun instead of a star—a total solar eclipse is not needed.

While the Newtonian and the Einsteinian effects on the deflection of light are said to add to each other, the predictions concerning retardation or acceleration go in opposite directions: By Newtonian gravitation, light moves faster when passing the sun, while relativity predicts that light will be delayed (Eisenhart 1923:516; Wright 2004).

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According to Edward L. Wright's web site, Irwin Shapiro (Shapiro et al. 1977) and Bertotti et al. (2003) have confirmed the predicted delay to a very high degree of accuracy:

“Measurements of the round-trip time of flight of radio signals transmitted from the earth to the Viking spacecraft are being analyzed to test the predictions of Einstein’s theory of general relativity. According to this theory the signals will be delayed by up to $\sim 250\,\mu$s owing to the direct effect of solar gravitation on the propagation. A very preliminary qualitative analysis of the Viking data obtained near the 1976 superior conjunction of Mars indicates agreement with the predictions to within the estimated uncertainty of 0.5%.” (Shapiro et al. 1977: Abstract)

Misner et al. (2002:1108) report that in radar time delay measurements performed in 1971, the value of the Parametrized Post-Newtonian (PPN) parameter γ, was found to be $\gamma = 1.02 \pm 0.08$, thus in agreement with the (even more precise) value given in the following quotation.

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"The bending and delay are proportional to $\gamma + 1$, where the parameter γ is unity in general relativity but zero in the newtonian model of gravitation. The quantity $\gamma - 1$ measures the degree to which gravitation is not a purely geometric effect and is affected by other fields; such fields may have strongly influenced the early Universe, but would have now weakened so as to produce tiny—but still detectable—effects.

Several experiments have confirmed to an accuracy of $\sim 0.1\%$ the predictions for the deflection [...] and delay [...] of photons produced by the Sun. Here we report a measurement of the frequency shift of radio photons to and from the Cassini spacecraft as they passed near the Sun. Our result, $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$, agrees with the predictions of standard general relativity with a sensitivity that approaches the level at which, theoretically, deviations are expected in some cosmological models [...].” (Bertotto et al., 2003: Abstract)

See also the accounts by Will (2006), Lemos (2019), Lemos et al. (2019), and Gilmore & Tausch-Pebody (2020).

Very Long Baseline Interferometry

Using Very Long Baseline Interferometry (VLBI), it has been possible to measure the deflection as well as the time delay with high accuracy; see the quotation from Bertotti et al. above. Among the many publications on VLBI, see also those by Robertson et al. (1991), Lebach et al. (1995), Shapiro et al. (2004), and Titov et al. (2018). They all confirm Einstein's prediction, the results showing that the parameter γ is very close to 1.

Summing up

Newtonian gravitational attraction

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Newtonian gravitational attraction

Gravitation according to Newton's celestial mechanics can act only if the photon is assigned a positive mass—von Soldner and Einstein did so.

As far as reflected light rays are concerned, Newtonian gravitation would accelerate light, whereas this has not been confirmed by measurements. On the contrary, observations have confirmed with high precision that there is a delay, in complete agreement with the general theory of relativity.

Thus, Newton's theory of gravitation

0. *gives zero deflection if photons are assumed to be massless—a light ray is a straight line in four-dimensional spacetime, thus projected onto a straight line in three-dimensional Newtonian space*;

1/2. *gives half of the observed deflection if light is assumed to consist of photons, and that these are treated like corpuscles with positive mass, thus subject to attraction by the sun and travelling like tiny comets along a hyperbola of large excentricty (this is the corpuscular theory of light)*; and

−1. *predicts speeding up of reflected light, in direct opposition to the time delay observed.*

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It must be rejected as an explanation for the two observed phenomena.

Refraction in the sun’s atmoshere

Concerning refraction, it has been mentioned that it is insignificant in the observations made.

Three phenomena and two theories

There are three phenomena that affect light rays passing close to the sun: the deflection of the orbit from a straight line, the Shapiro time delay in the arrival of reflected light, and refraction in the sun's atmosphere.

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There are three phenomena that affect light rays passing close to the sun: the deflection of the orbit from a straight line, the Shapiro time delay in the arrival of reflected light, and refraction in the sun's atmosphere.

There are two theories that have been applied in efforts to explain the first two of these phenomena: Newton's theory of gravitation and Einstein's general theory of relativity. The first gives good results in the absence of massive celestial bodies, but for particles passing close to the sun, it is not accurate.

These two mathematical models of light propagation are totally different, one might say even disjoint, and should not be mixed.

For light rays passing close to the sun, Einstein predicted in 1916 a deflection based, as he wrote, on both Newtonian gravitation and the geometry of spacetime, each contributing half of the deflection. One half is “produced by the Newtonian field of attraction” (Einstein 1920a: Appendix III, (b), page 153), implying that he applies two theories at the same time.

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Why did Einstein mention Newtonian gravitation here? Is it just to try to be understandable? If so, it creates problems in the understanding of the curvature of spacetime. He mentions Newtonian attraction not just as a possibility, but as something which has to be taken into consideration, since half of the deflection "is produced" by Newtonian gravitation.

Question

Newtonian attraction is said to produce an indispensable contribution to the deflection of light rays—for example by Albert Einstein himself in (1917:87), (1920b:87) and (1920a, Appendix III, (b), page 153)—but cannot be allowed in the calculation of the delay in arrival.

Can this contradiction be explained? □

If I may speculate about possible clarifications, my preliminary answer to the question would be that, of the two contributions to the deflection according to the general theory of relativity, each less than one second of arc, the first-mentioned is in its motivation somewhat similar to that of Newtonian attraction (the bending of a straight line), but that this by no means means that Newton's theory should be applied. The basis is said to be the equivalence principle—see, e.g., (Sauer 2021: Section 1)—which was and still is less well known than Newton's theory, and possibly this was a reason for mentioning the latter.

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So, Einstein's explanation should not to be taken literally; it is more like an analogy—but, if so, it is misleading rather than helpful in my opinion.

Conclusion

An intriguing contradiction has been discovered: For predictions of the deflection of light it is stated that Newtonian gravitation is an indispensable component to be combined with the general theory of relativity, but that, on the contrary, consideration of Newtonian attraction is inadmissible, and actually contradicts observations when it comes to predicting the delay in the arrival of reflected light.

Conclusion

An intriguing contradiction has been discovered: For predictions of the deflection of light it is stated that Newtonian gravitation is an indispensable component to be combined with the general theory of relativity, but that, on the contrary, consideration of Newtonian attraction is inadmissible, and actually contradicts observations when it comes to predicting the delay in the arrival of reflected light.

It is suggested that this contradiction has its origin in an attempt to offer a popular explanation.

Acknowledgments

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For references, see the handout or a manuscript that I can send to you.

Thank you!