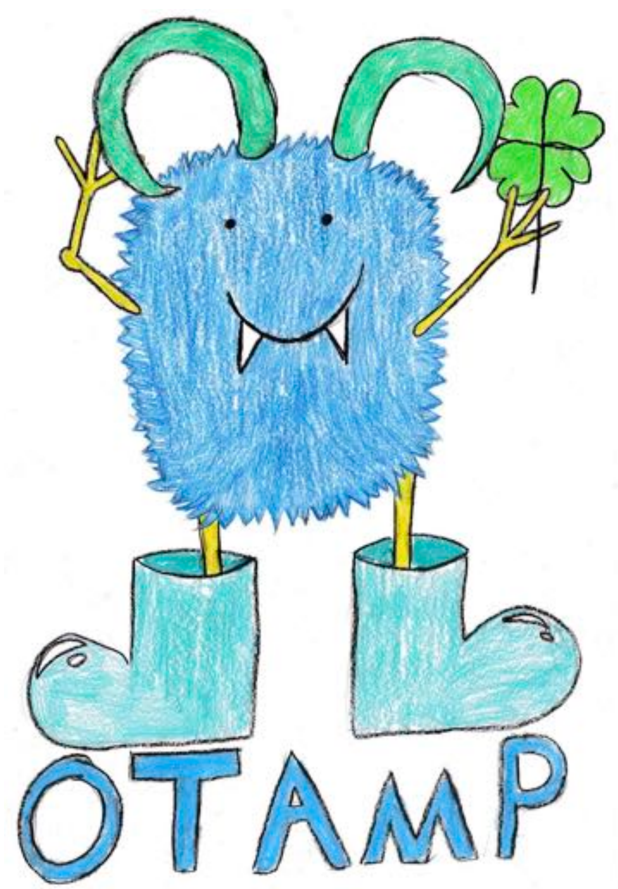
Operator Theory, Analysis and Mathematical Physics
OTAMP2026



June 22 – 25, 2026, **Pärlan**

Stockholms universitet

European Mathematical Society

Wenner-Gren Stiftelserna
Wenner-Gren Foundations

Location:

Conference room **Pärlan**

floor 6, house 1, Albano complex of Stockholm University

Organizers:
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Alice Brolin
Mina Farag
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with assistance from Elena-Sofia Kurasov

Monday

June 22, 2026

Pärlan, house 1, Albano

10:00–10:10	Opening
10:15–11:00	Pavel Exner *Effects of time-reversal symmetry violation in quantum graphs*
11:05–11:50	Jonathan Breuer *The point spectrum of periodic quantum trees*
	Lunch break
13:15–14:00	Christoph Fischbacher *Sharp Polynomial Decay Bounds for Multidimensional Periodic Schrödinger Operators*
14:05–14:35	Jonathan Rohleder *Optimization of the lowest Neumann eigenvalues for parallelograms*
14:40–15:10	Jakob Reiffenstein *Eigenvalues of self-adjoint extensions for defect larger than one*
	Coffee break
15:40–16:10	Alberto Richtsfeld *Boundary Value Problems for Dirac Operators on Graphs*
16:15–16:45	Noah Körner *Eigenvalue asymptotics for strong δ-interactions supported on curves with corners*

Tuesday

June 23, 2026

Pärlan, house 1, Albano

09:00–09:45	Michael Aizenman *Resonant Delocalization in Random Schrödinger Operators on Tree Graph*
09:50 –10:35	Anders Karlsson *Special values of spectral zeta functions of graphs and spaces*
	Coffee break
11:00–11:30	Mark Malamud *To the Birman-Krein-Visic Theory*
11:35–12:05	Rostyslav Kozhan *Zeros of Multiple Orthogonal Polynomials*
	Lunch break
13:15–14:00	Jan Derezinski *One-dimensional Schrödinger operators solvable in terms of the confluent and hypergeometric functions*
14:05–14:35	Timotheus Schmatzler *Inequalities between the eigenvalues of elliptic operators*
14:40–15:10	Pavel Kurasov *From quasicrystals to crystallines*
	Coffee break
15:40–16:10	Pawel Pietrzycki *Characterization of spectral measures and hyperrigidity*
16:15–16:45	Damian Kołaczek *Numerical ranges of real linear operators*
17:00–20:00	**Reception** at the Dept. of Mathematics, Stockholm Univ.

Wednesday

June 24, 2026

Pärlan, house 1, Albano

09:00–09:45	Delio Mugnolo *Landscape Functions And Agmon-Type Estimates for Magnetic Schrödinger Operators on Infinite Graphs*
09:50–10:35	Jacob Shapiro *The index of a pair of pure states and the quantum Hall effect*
	Coffee break
11:00–11:30	Michal Jex *Asymptotic Properties of Atomic Hamiltonians*
11:35–12:05	Alice Brolin *Partial Isospectrality of Rank Two Singular Pertubations*
	Lunch break
13:00–13:45	Julie Rowlett *Extrema of determinants of Laplacians on tori*
15:00–16:15	**Boat trip from Strömkajen** line 11
16:30–17:45	**Walk in Vaxholm**
18:00–20:30	**Dinner at Vaxholm hotel**
	Boat back to Strömkajen at: 19:30, 19:50, 20:30, 21:00, **21:35**. Buss 670 direct to Albano at: 20:01, 20:16, 20:30, 21:00, 21:30, 22:00,...

Thursday

June 25, 2026

Pärlan, house 1, Albano

9:00–9:45 Uzy Smilansky
Combining graph and ergodic theories to introduce and compute the Lyapunov exponent of a quantum Hamiltonian

9:50–10:20 Penelope Gehring
Nonlocal boundary conditions for symmetric hyperbolic systems

Coffee break

10:50–11:20 Mina Farag
Standard Laplacian on Metric Cayley Graphs

11:25–11:55 Jussi Behrndt
Approximation of Dirac operators with delta-shell potentials in the norm resolvent sense

Closing

Resonant Delocalization in Random Schrödinger Operators on Tree Graph

MichaelAizenman

Princeton University

The talk will explain the Aiz-Warzel (2011) criterion for resonant delocalization on tree graphs that was recently analyzed further by Aggarwal-Lopatto (2025) and Drogin-Smart (2025), and its extension to a many body model which is currently of interest in physics.

Approximation of Dirac operators with delta-shell potentials in the norm resolvent sense

Jussi Behrndt

TU Graz

In this talk the approximation of two and three-dimensional Dirac operators with delta-shell potentials supported on curves and surfaces, which may be bounded or unbounded, is discussed. It is shown under suitable conditions on the weight of the delta-interaction that a family of Dirac operators with regular, squeezed potentials converges in the norm resolvent sense to the Dirac operator with the delta-shell interaction. Special attention is paid on singular perturbations that are combinations of electrostatic and Lorentz scalar delta-shell interactions, where an explicit smallness condition on the coupling parameters is derived. Via counterexamples it is shown that this condition is sharp.

This talk is based on joint work with M. Holzmann and C. Stelzer-Landauer.

The point spectrum of periodic quantum trees

Jonathan Breuer

The Hebrew University of Jerusalem

Periodic Jacobi matrices on trees have been the subject of considerable interest in the past several years. Their quantum analogs, namely periodic quantum trees, have received much less attention and are less well understood. We consider the point spectrum of such operators, prove some analogous statements to those known in the discrete case and discuss some differences. This is joint work with Netanel Y. Levi.

Partial Isospectrality of Rank Two Singular Pertubations

Alice Brolin
Stockholm University

We consider a self adjoint operator A and the perturbed operator $A + \alpha_1\langle\varphi_1, \cdot\rangle\varphi_1 + \alpha_2\langle\varphi_2, \cdot\rangle\varphi_2$ where φ_1, φ_2 are from the form domain of A. We investigate under which condition the perturbed operator shares the first two eigenvalues with the unperturbed operator. We show that generally there is a unique non zero choice of α_1, α_2 such that the first two eigenvalues are preserved. As an example of this behaviour we look at metric graphs with delta couplings at two vertices. This is joint work with Mina Farag, Pavel Kurasov and Serge Nicaise.

One-dimensional Schrödinger operators solvable in terms of the confluent and hypergeometric functions

Jan Dereziński
University of Warsaw

I will describe several holomorphic families of one-dimensional Schrödinger operators whose Green functions (the integral kernel of their resolvents) have simple expressions in terms of Bessel, or more generally confluent functions, and Gegenbauer, or more generally hypergeometric functions. Most of them were discovered by physicists in the early years of Quantum Mechanics. They have various surprising properties, including unexpected singularities and "transmutation identities", which link Green functions of distinct families. Some of these operators have nice geometric interpretations: e.g. Gegenbauer Hamiltonians arise when we separate the (pseudo-)Laplacian on the sphere, hyperbolic space and deSitter space.

The talk is based on joint work with Jinyeop Lee and Pedram Karimi.

Effects of time-reversal symmetry violation in quantum graphs

Pavel Exner

Doppler Institute for Mathematical Physics
and Applied Mathematics, Prague

The talk concerns the ways in which the violation of time-reversal invariance in quantum graphs coming from the choice of the vertex coupling conditions can be manifested in spectral properties. We focus on two cases. One is of spectral geometry type; we ask about the topology of a finite graph with a particular vertex coupling of the indicated type which optimizes the eigenvalues. The other is related to quantum chaos: we demonstrate an example of a graph the properties of which, both in the eigenvalue spacing distribution and the form factor, differ from what is expected in this situation. The results come from a common work with Ram Band, Divya Goel, Jonathan Rohleder and Aviya Strauss.

Standard Laplacian on Metric Cayley Graphs

Mina Farag

Stockholm University

A metric graph analogue of Cayley graphs is introduced. It turns out that the standard Laplacian on metric Cayley graphs is unitarily equivalent to a direct sum over the irreducible representations of the Laplacian defined on flower graphs with pendant edges and flux-type vertex conditions depending on the action of the irreducible representation on the generators. The number of petals and leaves will depend on the generators and their order. If time allows it, we will then discuss additional results concerning flower graphs with fluxes. It is shown that for such graphs, the spectral gap is increasing as the flux varies from 0 to π. It is also shown that the corresponding secular polynomials are generally irreducible. This is based on joint work with Pavel Kurasov.

Sharp Polynomial Decay Bounds for Multidimensional Periodic Schrödinger Operators

Christoph Fischbacher

Baylor University

In this talk, I will present our results on discrete periodic Schrödinger operators in arbitrary dimensions in the large coupling regime. We establish that both the Lieb-Robinson velocity and the asymptotic velocity decay at an inverse polynomial rate in the coupling, with the precise exponent determined by the period of the underlying potential. In particular, we show sharp polynomial decay rates that capture the precise dependence on the periodic structure.

Nonlocal boundary conditions for symmetric hyperbolic systems

Penelope Gehring

Stockholm University

The Atiyah–Patodi–Singer index theorem for Dirac operators on compact Riemannian manifolds with boundary is a cornerstone result that has stimulated extensive research on nonlocal boundary conditions for first-order elliptic operators. In contrast, the study of nonlocal boundary conditions in the Lorentzian setting – and hence for hyperbolic operators – is a more recent development.

In this talk, we focus on spacetimes with timelike boundary and an important class of first order operators, the symmetric hyperbolic systems. In this setting, we examine how nonlocal boundary conditions can be imposed on the timelike boundary in order to obtain solutions to the associated initial boundary value problem. We will do so by reducing this geometric problem to a purely analytical question in the realm of semigroup theory.

This is joint work with Christian Bär.

Asymptotic Properties of Atomic Hamiltonians

Michal Jex
Czech Technical University in Prague

We present sharp upper bounds for the behaviour of the ground state eigenfunctions of N-electron atoms with general charge Z. This result is generalisation of the result recently accepted for publication about Helium atom (Hundermark-Jex-Lange 2026). The proof relies on careful energy estimates and use of non-conical regions. Furthermore, we show why using conical regions can not lead to sharp bounds.

Special values of spectral zeta functions of graphs and spaces

Anders Karlsson
University of Geneva / Uppsala University

In this talk I will discuss spectral zeta functions of graphs and their special values at integers. These values appear in a number of topics, for example the volumes of spheres. An asymptotic analysis will ultimately lead to a combinatorial interpretation, not involving any limits, of Euler's values of the Riemann zeta function and the corresponding values for Dirichlet L-functions. I will also make some remarks about special values of the zeta functions of metric graphs. Based on joint works with J. Jorgenson, L. Smajlović, F. Friedli, M. Pallich, and D. Müller.

Numerical ranges of real linear operators

Damian Kołaczek
University of Agriculture in Krakow

Real linear operators are continuous additive operators and can be expressed as a sum of linear operator and antilinear operator in a unique way. Here we discuss the concept of numerical range for real linear operators in both Hilbert and Banach space cases and its relation to spectra of such operators. We compare various similarities and differences between numerical radii and numerical ranges in (complex) linear and real linear setting. Our main result is showing that the numerical range of real linear operator on at least two-dimensional complex Hilbert space is always a convex set, which generalize classic Toeplitz-Hausdorff theorem. We also present some new results for the numerical ranges of antilinear operators. The talk is based on recent joint work with Vladimir Müller (Institute of Mathematics, Czech Academy of Sciences).

Zeros of Multiple Orthogonal Polynomials

Rostyslav Kozhan
Uppsala University

We demonstrate how the Christoffel transform serves as a natural tool to analyze the location and interlacing property of zeros of orthogonal polynomials. We apply this approach to multiple orthogonal polynomials of type I and type II for Angelesco, AT, and Nikishin systems on the real line. Furthermore, we extend this methodology to the unit circle, where we analyze both multiple orthogonal and paraorthogonal polynomials for Angelesco and AT systems. Joint work with Marcus Vaktnäs.

Eigenvalue asymptotics for strong δ-interactions supported on curves with corners

Noah Körner
Carl von Ossietzky Universität Oldenburg

Let $\Gamma \subset \mathbb{R}^2$ be a piecewise smooth closed curve with corners. We discuss the asymptotic behavior of the individual eigenvalues of the two-dimensional Schrödinger operator $-\Delta - \alpha\delta_\Gamma$ for $\alpha \to \infty$, where δ_Γ is the Dirac δ-distribution supported by Γ. It is shown that the asymptotics of several first eigenvalues is determined by the corner openings only, while the main term in the asymptotic expansion for the other eigenvalues is the same as for smooth curves. Under an additional assumption on the corners of Γ (which is satisfied, in particular, if Γ has no acute corners), a more detailed eigenvalue asymptotics is established in terms of a one-dimensional effective operator on the boundary.

From quasicrystals to crystallines

Pavel Kurasov
Stockholm University

In this talk we discuss how to prepare discrete sets that generalise ordinary crystals, but are not quasicrystals. Such sets are physically relevant only if they are **uniformly discrete** – any two atoms are always separated by a small but non zero distance, and **relatively dense** – any sufficiently large ball always contain atoms from the set. The sets satisfying both properties are called **Delaunay sets**.

Classical (periodic) crystals provide numerous examples of such sets. Another important class of examples is provided by nowadays classical **quasicrystals**.

Quasicrystal is a Delaunay set Λ such that one of the following (equivalent) conditions hold:

- $\Lambda - \Lambda \subset \Lambda + F$, where F is a finite set (Meyer, 1970);
- Λ is a cut and project set (Meyer, 1974);
- $\Lambda - \Lambda$ is a Delaunay set (Galiulin, 1989).

The purpose of this talk is to popularise **crystallines** – a new generalisation of classical crystals, which is not a quasicrystal. A discrete set Λ is crystalline if the counting measure

$$\mu = \sum_{\lambda \in \Lambda} \delta_\lambda$$

is a tempered distribution and its Fourier transform is given by a discrete sum of delta functions as well

$$\hat{\mu} = \sum_{s \in S} b_s \delta_s .$$

We shall be interested in **crystallines** characterised by the additional property that $|\hat{\mu}| = \sum_{s \in S} |b_s| \delta_s$ is a tempered distribution.

Existence of such sets has been established only recently in our joint work with Peter Sarnak. Multidimensional crystallines are given in Meyer, Alon-Kummer,-K.-Vinzant; and Lawton-Tsikh.

Based on joint work with Y.Meyer and P.Sarnak.

To the Birman-Krein-Visic Theory

Mark Malamud
St.-Petersburg University

Let A be a closed non-negative symmetric densely defined operator in a Hilbert space $\mathfrak{H}$ and let $\mathfrak{H}_1 := \operatorname{ran}(I + A)$. By the Stone – Friedrichs theorem the set $\operatorname{Ext}_A(0,\infty)$ of all nonnegative selfadjoint extensions $\widetilde{A} = \widetilde{A}^*$ of A is nonempty. M. Krein established that the set $\operatorname{Ext}_A(0,\infty)$ forms an operator segment with two endpoints: the maximal (the Friedrichs) and the minimal (the Krein) extensions $\widehat{A}_F$ and $\widehat{A}_K$. They are uniquely characterized by means of the inequalities: $\widehat{A}_K \le \widetilde{A} \le \widehat{A}_F$, $\widetilde{A} \in \operatorname{Ext}_A(0,\infty)$, which are understood in the form sense.

Krein's theory has substantially been completed by M. Vicik and M. Birman.

If A is positive definite, then $\widehat{A}_K$ admits a representation $\widehat{A}_K = \widehat{A}'_K \oplus (\mathbb{O} \upharpoonright \mathfrak{N}_0)$ where $\mathfrak{N}_0 := \ker A^*$. The operator $\widehat{A}'_K$ is called the reduced Krein extension.

M.Krein discovered the following implication

$$(I_{\mathfrak{H}} + \widehat{A}_F)^{-1} \in \mathfrak{S}_\infty \implies (I_{\mathfrak{M}_0} + \widehat{A}'_K)^{-1} \in \mathfrak{S}_\infty. \qquad (0.1)$$

First we will discuss the following its improvement (see [1]):

$$P_1(I_{\mathfrak{H}} + A)^{-1} \in \mathfrak{S}(\mathfrak{H}_1) \quad \Longleftrightarrow \quad (I_{\mathfrak{M}_0} + \widehat{A}'_K)^{-1} \in \mathfrak{S}(\mathfrak{M}_0). \qquad (0.2)$$

Here P_1 is the orthoprojection in $\mathfrak{H}$ onto $\mathfrak{H}_1 = \operatorname{ran}(I + A)$, $\mathfrak{M}_0 = \mathfrak{N}_0^\perp$, and $\mathfrak{S}$ is any symmetrically normed ideal (including ideals $\mathfrak{S}_p$, Σ_p, Σ_p^0, etc.).

We will discuss the following problems:

(i) Additional conditions on A that ensure the following equivalence

$$\lambda_n\big(\widehat{A}_F)\big) = a^{-1} n^{1/p} \left(1 + o(1)\right) \iff \lambda_n\big(\widehat{A}'_K\big) = a^{-1} n^{1/p} \left(1 + o(1)\right).$$

which completes (0.2) in the case of $\mathfrak{S} = \Sigma_p$

(ii) The Birman problem and the validity of the inverse implication to (0.1).

(iii) Solution to the abstract Alonso-Simon problem.

(iv) Improvement of certain results by Birman and Grubb regarding the LSB-property of A and its application to elliptic BV problems in unbounded domains.

The talk is based on results published in [1]-[3] and their recent developments.

References

1. M. M.Malamud, To Birman–Krein–Vishik Theory, Doklady Mathematics, V. 107, No. 1 (2023), pp. 44–48.
2. M.M. Malamud, On the Birman problem on positive symmetric operators with compact inverse, Func. Anal. Appl., V.57, No 2 (2023), p. 111–116.
3. M. Malamud, Explicit Solution to the Birman Problem for the $2D$-Laplace operator, Russian Jour. of Math. Phys., 2024, V. 31, No.3, p. 495-504.

Landscape Functions And Agmon-Type Estimates For Magnetic Schrödinger Operators On Infinite Graphs

Delio Mugnolo

FernUniversität in Hagen

We develop a theory of landscape functions to derive Agmon-type exponential decay estimates for magnetic Schrödinger operators on possibly infinite weighted graphs. Given a bounded, non-negative source ρ, the associated non-magnetic landscape function $u = (\Delta + V)^{-1}\rho$ defines an effective potential ρ/u that governs the spatial distribution of magnetic eigenfunctions. We prove that any eigenpair (E, ϕ) of the full magnetic operator $\Delta_\alpha + V$ decays exponentially away from the potential well with respect to a natural Agmon pseudometric. Crucially, our landscape function is defined using the non-magnetic operator, yet controls magnetic eigenfunctions via semigroup domination. Our framework extends existing finite-graph theories recently developed by Filoche-Mayboroda-Tao to infinite settings, incorporates magnetic fields, and significantly improves system-size dependence to provide uniform decay estimates. We present applications to Schrödinger operators with confining potentials and Anderson-type models on $\mathbb{Z}^d$. This is joint work with Liza Schonlau (Bonn) and Matthias Täufer (Valenciennes).

Characterization of spectral measures and hyperrigidity

Paweł Pietrzycki
Jagiellonian University

One of the features of a Borel spectral measure on $\mathbb{R}$ is the multiplicativity of the corresponding Stone-von Neumann functional calculus. In particular, if E is a Borel spectral measure on $\mathbb{R}$ with compact support, then the following identities hold

$$\Big(\int_{\mathbb{R}} xE(\mathrm{d}x)\Big)^n = \int_{\mathbb{R}} x^n E(\mathrm{d}x), \quad n = 1, 2, \ldots. \tag{0.3}$$

Hence, all operator moments of E are determined by the first one, and according to the spectral theorem there is a one-to-one correspondence between Borel spectral measures on $\mathbb{R}$ and their first operator moments. This is no longer true for general Borel semispectral measures on $\mathbb{R}$. It turns out, however, that the single equality in (0.3) with $n = 2$ guarantees spectrality.

It turns out that, from a mathematical and physical point of view, it is important to investigate the relationship between semispectral and spectral measures. In the classical von Neumann description of quantum mechanics selfadjoint operators or, equivalently, Borel spectral measures on the real line represent observables. This approach is insufficient in describing many natural properties of measurements, such as measurement inaccuracy. Therefore, in standard modern quantum theory, the generalization to semispectral measures is widely used.

In 2006 Kiukas, Lahti and Ylinen asked the following general question. *When is a positive operator measure projection valued?* A version of this question formulated in terms of operator moments was posed in [1]. *Let T be a selfadjoint operator and F be a Borel semispectral measure on the real line with compact support. For which positive integers $p < q$ do the equalities $T^k = \int_{\mathbb{R}} x^k F(\mathrm{d}x)$, $k = p, q$, imply that F is a spectral measure?* In this talk, we show that the answer is affirmative if p is odd and q is even, and negative otherwise.

Motivated both by the fundamental role of the classical Choquet boundary in classical approximation theory, and by the importance of approximation in the contemporary theory of operator algebras, Arveson introduced hyperrigidity as a form of approximation that captures many important operator-algebraic phenomena. We discuss the relationship between hyperrigidity and the aforementioned spectral measure characterisation. This talk is based on joint work with Jan Stochel.

[1] P. Pietrzycki, J. Stochel, Subnormal nth roots of quasinormal operators are quasinormal, *J. Funct. Anal.* **280** (2021), 109001.

[2] P. Pietrzycki, J. Stochel, Two-moment characterization of spectral measures on the real line, *Canad. J. Math.* vol. 75 (4) (2023), 1369-1392

[3] P. Pietrzycki, J. Stochel, Hyperrigidity I: singly generated commutative C^*-algebras, to appear in *the Israel Journal of Mathematics*

Eigenvalues of self-adjoint extensions for defect larger than one

Jakob Reiffenstein

Stockholm University

Finding eigenvalues of a self-adjoint operator often amounts to determining zeros or singularities of analytic functions. Our recent work provides the theoretical background for this principle in the context of Krein's formula, which parametrizes all self-adjoint extensions of an underlying symmetric operator S. The analytic functions in question are the Weyl function $m(\lambda)$ and the parameter $\tau(\lambda)$. They are both matrix- or operator-valued if the defect of S is larger than one. In this talk I present a complete characterization of the eigenvalues of any given self-adjoint extension in terms of *generalized values* of m and τ.

Based on joint work with A. Luger.

Boundary Value Problems for Dirac Operators on Graphs

Alberto Richtsfeld
Stockholm University

We study the Dirac operator on metric graphs within the Bär-Ballmann framework for boundary value problems of first-order differential operators. This approach yields an index formula for general first-order operators on metric graphs. Focusing on self-adjoint boundary conditions for the Dirac operator on a complex line bundle, we derive an explicit spectral description in the case of equilateral graphs. We examine two distinguished classes of boundary conditions. The first is induced by permutation matrices and corresponds to decompositions of the graph into directed trails; the resulting spectrum determines key features of the decomposition, including the trail lengths. The second is defined via a novel incidence matrix. Here the spectrum is controlled by polynomials encoding information about the cycle structure of the directed graph, allowing, for instance, the direct computation of its girth.

Optimization of the lowest Neumann eigenvalues for parallelograms

Jonathan Rohleder
Stockholm University

We consider the problem of optimization of the two lowest non-zero eigenvalues of the Neumann Laplacian on bounded Euclidean domains under a perimeter constraint. In the planar case, we discuss the solution of this problem within the toy class of parallelograms. The results are contained in joint works with Corentin Léna and Vladimir Lotoreichik.

Extrema of determinants of Laplacians on tori

Julie Rowlett
Chalmers University of Technology

The search for extremal geometries is a central theme in several areas of mathematics. In this talk, we address the following question: among all n-dimensional orthogonal tori of unit volume, which one maximizes the determinant of the Laplacian? This determinant is obtained via the zeta regularization method, originally introduced by Ray and Singer to define an analytic analogue of the topological invariant, the Reidemeister torsion. While this determinant is connected to topology, geometry, analysis, and number theory, it is also important in physics. For example, Stephen Hawking, observed that the zeta-function regularization technique could be used to regularize quadratic path integrals on a curved background space-time.

This talk is based on joint work with Fabio Francesconi and will be broadly accessible to mathematicians, physicists, and their students!

Inequalities between the eigenvalues of elliptic operators

Timotheus Schmatzler
Stockholm University

In this talk we discuss inequalities between the Dirichlet and the Neumann eigenvalues of elliptic operators on bounded domains. One question of particular interest concerns the number of Neumann eigenvalues μ_k of a given operator below the first Dirichlet eigenvalue λ_1: For the Laplacian $-\Delta$, the inequality $\mu_2 < \lambda_1$ holds on any bounded Lipschitz domain in $\mathbb{R}^d$, and in simply connected domains in $\mathbb{R}^2$ this was recently improved to $\mu_3 \leq \lambda_1$. We present a generalization of this inequality to certain elliptic operators with coefficients.

The index of a pair of pure states and the quantum Hall effect

Jacob Shapiro

Princeton University

The index of a pair of projections on a Hilbert space was introduced in 1973 by Brown-Douglas-Filmore and connected to the integer quantum Hall effect by Avron-Seiler-Simon in 1994. For two orthogonal projections P, Q such that $P - Q$ is compact, $\text{index}(P, Q) = \dim \text{im} P \cap \ker Q - \dim \text{im} Q \cap \ker P$. It is manifestly an integer, and enjoys norm and compactness stability, much like the related Fredholm index. Such indices played a pivotal role in describing the quantization and stability p roperties in the quantum Hall effect; ASS94 related the Hall conductance to the index of a Fermi projection P and its Laughlin-flux-inserted projection U^*PU. What becomes of this story in the presence of interactions? To describe infinitely-many interacting electrons in infinite-volume, the Hilbert space is replaced by a unital C^*-algebra A (a CAR algebra), but there is no obvious notion of a Fredholm index. We introduce a new notion, the index of a pair of pure states (on A), prove its quantization, invariance and stability properties, and relate it to the (possibly fractional) Hall conductance. We comment that Kitaev's invertible states always have integer conductance. Joint with Bachmann and Tauber.

Combining graph and ergodic theories to introduce and compute the Lyapunov exponent of a quantum Hamiltonian

Uzy Smilansky
Weizmann Institute

In the first part of the lecture I shall provide a brief review of the method:

Given a quantum Hamiltonian H as a Hermitian matrix of an arbitrary large but finite dimension N. A graph on N vertices is associated with the matrix. Graph theory guarantees that an initial quantum state expressed as a vector in the space of the graph directed edges, evolves in time by a unitary matrix U – the quantum analogue of a discrete classical Poincaré map. U is written in terms of H and it is a function of the energy. The intimate connection between H and U is evident from the known theorem that the spectrum of H is the set of E values where U has 1 as an eigenvalue (if and only if). The semi-classical evolution is written in terms of a Markovian matrix with elements which are the absolute square of the corresponding elements of U. Ergodic theory provides an expression for the Lyapunov exponent per trajectory on the graph. The mean Lyapunov exponent (with respect to the distribution of all graph trajectories) and its variance follow.

In the second part I shall describe the application of the formalism to five random matrix ensembles: The adjacency matrices of random d-regular graphs, GOE , GUE and the tri-diagonal Dumitriu-Edelman ensembles. We computed the mean values of the Lyapunov exponents, variances, and thermal averages both numerically and by deriving approximate expressions.

The spectral distributions of the stochastic evolution matrices, and in particular their gap distributions were also computed.

I shall conclude with a summary and a list of open problems.