Combining graph and ergodic theories to introduce and compute the Lyapunov exponent of a quantum Hamiltonian

Uzy Smilansky

Weizmann Institute

In the first part of the lecture I shall provide a brief review of the method:

Given a quantum Hamiltonian H as a Hermitian matrix of an arbitrary large but finite dimension N. A graph on N vertices is associated with the matrix. Graph theory guarantees that an initial quantum state expressed as a vector in the space of the graph directed edges, evolves in time by a unitary matrix U – the quantum analogue of a discrete classical Poincaré map. U is written in terms of H and it is a function of the energy. The intimate connection between H and U is evident from the known theorem that the spectrum of H is the set of E values where U has 1 as an eigenvalue (if and only if). The semi-classical evolution is written in terms of a Markovian matrix with elements which are the absolute square of the corresponding elements of U. Ergodic theory provides an expression for the Lyapunov exponent per trajectory on the graph. The mean Lyapunov exponent (with respect to the distribution of all graph trajectories) and its variance follow.

In the second part I shall describe the application of the formalism to five random matrix ensembles: The adjacency matrices of random d-regular graphs, GOE , GUE and the tri-diagonal Dumitriu-Edelman ensembles. We computed the mean values of the Lyapunov exponents, variances, and thermal averages both numerically and by deriving approximate expressions.

The spectral distributions of the stochastic evolution matrices, and in particular their gap distributions were also computed.

I shall conclude with a summary and a list of open problems.