

The index of a pair of pure states and the quantum Hall effect

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The index of a pair of projections on a Hilbert space was introduced in 1973 by Brown-Douglas-Filmore and connected to the integer quantum Hall effect by Avron-Seiler-Simon in 1994. For two orthogonal projections P, Q such that $P - Q$ is compact, $\text{index}(P, Q) = \dim \text{im} P \cap \ker Q - \dim \text{im} Q \cap \ker P$. It is manifestly an integer, and enjoys norm and compactness stability, much like the related Fredholm index. Such indices played a pivotal role in describing the quantization and stability properties in the quantum Hall effect; ASS94 related the Hall conductance to the index of a Fermi projection P and its Laughlin-flux-inserted projection U^*PU . What becomes of this story in the presence of interactions? To describe infinitely-many interacting electrons in infinite-volume, the Hilbert space is replaced by a unital C^* -algebra A (a CAR algebra), but there is no obvious notion of a Fredholm index. We introduce a new notion, the index of a pair of pure states (on A), prove its quantization, invariance and stability properties, and relate it to the (possibly fractional) Hall conductance. We comment that Kitaev's invertible states always have integer conductance. Joint with Bachmann and Tauber.