Boundary Value Problems for Dirac Operators on Graphs

Alberto Richtsfeld
Stockholm University

We study the Dirac operator on metric graphs within the Bär-Ballmann framework for boundary value problems of first-order differential operators. This approach yields an index formula for general first-order operators on metric graphs. Focusing on self-adjoint boundary conditions for the Dirac operator on a complex line bundle, we derive an explicit spectral description in the case of equilateral graphs. We examine two distinguished classes of boundary conditions. The first is induced by permutation matrices and corresponds to decompositions of the graph into directed trails; the resulting spectrum determines key features of the decomposition, including the trail lengths. The second is defined via a novel incidence matrix. Here the spectrum is controlled by polynomials encoding information about the cycle structure of the directed graph, allowing, for instance, the direct computation of its girth.