From quasicrystals to crystallines

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In this talk we discuss how to prepare discrete sets that generalise ordinary crystals, but are not quasicrystals. Such sets are physically relevant only if they are **uniformly discrete** – any two atoms are always separated by a small but non zero distance, and **relatively dense** – any sufficiently large ball always contain atoms from the set. The sets satisfying both properties are called **Delaunay sets**.

Classical (periodic) crystals provide numerous examples of such sets. Another important class of examples is provided by nowadays classical **quasicrystals**.

Quasicrystal is a Delaunay set Λ such that one of the following (equivalent) conditions hold:

- $\Lambda - \Lambda \subset \Lambda + F$, where F is a finite set (Meyer, 1970);
- Λ is a cut and project set (Meyer, 1974);
- $\Lambda - \Lambda$ is a Delaunay set (Galiulin, 1989).

The purpose of this talk is to popularise **crystallines** – a new generalisation of classical crystals, which is not a quasicrystal. A discrete set Λ is crystalline if the counting measure

$$\mu = \sum_{\lambda \in \Lambda} \delta_\lambda$$

is a tempered distribution and its Fourier transform is given by a discrete sum of delta functions as well

$$\hat{\mu} = \sum_{s \in S} b_s \delta_s.$$

We shall be interested in **crystallines** characterised by the additional property that $|\hat{\mu}| = \sum_{s \in S} |b_s| \delta_s$ is a tempered distribution.

Existence of such sets has been established only recently in our joint work with Peter Sarnak. Multidimensional crystallines are given in Meyer, Alon-Kummer,-K.-Vinzant; and Lawton-Tsikh.

Based on joint work with Y.Meyer and P.Sarnak.