Existence of matrix-valued analytic functions with prescribed zeroes

Marcus Carlsson

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There is a famous theorem in complex analysis by Weierstrass which in a simplified form states that given an arbitrary open set $\Omega \in \mathbb{C}$, and given a discrete subset N, one can find an analytic function f such that $\{z \in \Omega : f(z) = 0\} = N$. We will generalize this theorem to matrix-valued analytic functions in the following sense. For $n \in \mathbb{N}$ let M_n denote the set of $n \times n$-matrices. Given Ω and N as above there exists a M_n-valued analytic function on Ω such that $\{z \in \Omega : \det(f(z)) = 0\} = N$.