Mean value surfaces with prescribed curvature form

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The Riemannian curvature of a Riemannian manifold is uniquely determined by the choice of the metric. The formulas for computing the curvature in terms of components of the metric, in isothermal coordinates, involve the Laplacian operator and therefore, the problem of finding a Riemannian metric for a given curvature form may be viewed as a potential theory problem. This problem has, generally speaking, a multitude of solutions. To specify the solution uniquely, we ask that the metric have the mean value property for harmonic functions with respect to some given point. This means that we assume that the surface is simply connected and that it has a smooth boundary. In terms of the so-called metric potential, we are looking for a unique smooth solution to a nonlinear fourth order elliptic partial differential equation with second order Cauchy data given on the boundary. We find a simple condition on the curvature form which ensures that there exists a smooth mean value surface solution. It reads: the curvature form plus the curvature form for plane hyperbolic (with the same coordinates) should be ≤ 0. The same analysis leads to results on the question of whether the canonical divisors in weighted Bergman spaces over the unit disk have extraneous zeros. Numerical work suggests that the above condition on the curvature form is essentially sharp.