

Constraint Maps and Free Boundaries

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Constraint maps arise as minima of the Dirichlet energy under image constraints, which can be considered a natural extension of the obstacle problem into a vectorial setting. The presence of the obstacle gives rise to a free boundary, which is the interface between the contact and non-contact sets in the domain. As the simplest example, one can consider a geodesic connecting two prescribed points but constrained to lie outside a given open set. In this case, the closure of the open set is the obstacle, and the free boundary is given by the points where the geodesic enters and exits the obstacle. This is the problem for a 1-dimensional domain, but the target can be of any dimension. For two-dimensional domains, the problem already becomes interesting as it produces minimal surfaces with obstacles. For three- or higher-dimensional domains, the problem becomes significantly more challenging as it gives rise to singularities that have rarely been studied in the context of a free boundary problem. Historically, this problem was studied from the perspectives of harmonic maps and minimal surfaces, yet it left a fundamental issue open for decades concerning the interplay between the singularities and the free boundaries. This open problem was resolved recently by my collaborators and me, using a set of new approaches based on insights from free boundary problems. In this talk, I will give some examples of constraint maps in several different dimensions and show under which circumstances the free boundary can repel or embrace the singularities. The talk will be based on recent joint work with Alessio Figalli (ETH), André Guerra (Cambridge), and Henrik Shahgholian (KTH).