A Marcus Wallenberg Symposium
in memory of SERGEY NABOKO

Analysis and Mathematical Physics
AMP2023

June 19 – 21, 2023, **Pärlan**

Stockholms universitet

Location:

Conference room **Pärlan**

floor 6, house 1, Albano complex of Stockholm University

Zoom link:

https://stockholmuniversity.zoom.us/j/67304359715

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Monday

June 19, 2023

Pärlan, house 1, Albano

9:00–9:10 Opening

9:10–9:50 Pavel Exner
A product formula related to Zeno dynamics

9:55–10:35 Jonathan Breuer
Eigenvalue asymptotics for coverings of Jacobi matrices on graphs

Coffee break

11:00–11:40 Evans Harrell
How large and small can the gaps between the eigenvalues of a quantum graph be?

11:45–12:25 Nils Dencker
The Instability of Differential Operators

12:30-12:55 Timon Weinmann
Lieb–Thirring Inequalities in Hyperbolic Space

Lunch break

14:00–14:40 Sergei Treil
Matrix weights and finite rank perturbations

14:45-15:25 Alan Sola
Clark measures for rational inner functions

Coffee break

15:50–16:15 Sergey Denisov
Jacobi matrices on trees, multiple orthogonal polynomials, and their strong asymptotics
(ZOOM https://stockholmuniversity.zoom.us/j/67304359715)

16:20–16:45 Marcin Moszynski
Nonsubordinacy and absolutely continuous spectrum of block Jacobi operators

16:50–17:15 Ayoub Harrat
Discrete spectrum estimation of a class of unbouded operators defined by infinite Jacobi matrices

17:20–17:40 Pavel Kurasov
Sergey Naboko as we remember him

Reception (Pärlan)

Tuesday

June 20, 2023

Pärlan, house 1, Albano

9:00–9:40 Rupert Frank
Endpoint Schatten class properties of commutators

9:45–10:25 Jacob Christiansen
Uniform approximation and polynomial preimages

Coffee break

10:50–11:30 Simon Larson
Discrete Schrödinger operators with decaying and oscillating potentials

11:35–12:15 Petr Siegl
Schrödinger operators with accretive potentials in weighted spaces

12:20-12:45 Matthias Hofmann
Computing eigenvalues of the discrete p-Laplacian via graph surgery

Lunch break

14:00–14:40 Anders Karlsson
The discrete analog of the Gaussian

14:45-15:10 Andrey Shkalikov
Spectral asymptotics for the systems of differential equations and applications
(ZOOM https://stockholmuniversity.zoom.us/j/67304359715)

15:15–15:40 Marco Marletta
Essential spectra and spectral pollution for Maxwell and Drude-Lorentz systems
(ZOOM https://stockholmuniversity.zoom.us/j/67304359715)

Coffee break

16:10–16:35 Sergey Simonov
Spectral multiplicity of Schrödinger operators on a star-graph with non-Kirchhoff interface condition
(ZOOM https://stockholmuniversity.zoom.us/j/67304359715)

16:40–17:05 Alexander V. Kiselev
A phase transition in a periodic tubular structure in an external magnetic field.
(ZOOM https://stockholmuniversity.zoom.us/j/67304359715)

Conference dinner at restaurant Kräftan

Wednesday

June 21, 2023

Pärlan, house 1, Albano

9:00–9:40 Luis Silva
Estimates for Green matrices of block Jacobi operators

9:45–10:25 Jonathan Rohleder
Laplacian eigenvalues and eigenfunctions: a non-standard variational principle

Coffee break

10:50–11:30 Håkan Hedenmalm
Hyperbolic Fourier series and the Klein-Gordon equation

11:35–12:15 Boris Shapiro
On spectral asymptotic of KMS-matrices

12:20-12:45 Oliver Petersen
Resonances on black hole spacetimes

Lunch break

14:00–14:40 Christoph Fischbacher
Complete non-selfadjointness for Schrödinger operators on the semi-axis

14:45-15:25 Rostyslav Kozhan
Multiple orthogonal polynomials on the unit circle

Coffee break

15:50–16:15 Volodymyr Mikhailets
Self-adjointness problem for Sturm-Liouville and $1D$ *Schrödinger operators with distribution coefficients*

16:20–17:00 Selim Sukhtaiev
The Maslov index in spectral theory: an overview

17:05–17:30 Peter Sarnak
Nonabelian Bloch wave theory?
(ZOOM https://stockholmuniversity.zoom.us/j/67304359715)

Closing

Eigenvalue Asymptotics for Coverings of Jacobi Matrices on Graphs

Jonathan Breuer
The Hebrew University of Jerusalem

Given a finite graph, $\mathcal{G}$, a Jacobi matrix, J, on $\mathcal{G}$ is the sum of its weighted adjacency matrix and a diagonal matrix. If $\widetilde{\mathcal{G}}$ is a graph covering of $\mathcal{G}$, one can lift J to a Jacobi matrix, $\widetilde{J}$ on $\widetilde{\mathcal{G}}$. Sequences of such coverings are naturally related to periodic Jacobi matrices on trees. In this talk, after describing this connection, we describe some bounds on the eigenvalues of $\widetilde{J}$ for sufficiently large coverings of $\mathcal{G}$, including an Alon-Boppana type bound on the second eigenvalue. This is joint work with Eyal Seelig.

Uniform approximation and polynomial preimages

Jacob S. Christiansen
Lund University

Let $\mathsf{E} \subset \mathbb{C}$ be an infinite compact set of logarithmic capacity equal to 1 and denote by T_n the minimax (or Chebyshev) polynomials of E, that is, the monic degree n polynomials which minimize the sup-norm on E. A classical result of Szegő states that $\|T_n\|_{\mathsf{E}} \geq 1$ for all n, a lower bound that doubles when $\mathsf{E} \subset \mathbb{R}$. More recently, Totik has proved that for real subsets, $\|T_n\|_{\mathsf{E}} \to 2$ if and only if E is an interval.

We shall pose the question if there are more subsets of $\mathbb{C}$ for which this limit is 2 and show the answer is in the affirmative for certain polynomial preimages. At the same time, we shall settle a conjecture of Widom concerning Jordan arcs. Interestingly, our proof relies on properties of the (orthogonal) Jacobi polynomials due to Bernstein. Related open problems will also be discussed.

The talk is based on joint work with B. Eichinger (TU Wien) and O. Rubin (Lund).

The instability of differential operators

Nils Dencker
Lund University

It came as a surprise when Hans Lewy in 1957 presented a non-vanishing smooth complex vector field that is not solvable. After all, the classical Cauchy-Kowalevskaya Theorem shows that any analytic PDE is solvable in the analytic category. The Lewy vector field is the tangential Cauchy-Riemann operator on the boundary of a strictly pseudo-convex domain in $\mathbf{C}^2$. Hörmander proved in 1960 that almost all linear PDE are not solvable, unless they satisfy the bracket condition which is a non-generic condition. The bracket condition has many consequences for differential equations, such as the spectral stability, the stability of the Cauchy problem, the kernel and the range of differential operators.

A fifty year development lead to the proof of the Nirenberg-Treves conjecture: that principal type differential operators are solvable if and only if condition (Ψ) is satisfied. This is a condition on the sign changes of the imaginary part of the principal symbol along the bicharacteristics of the real part. For non-principal type differential operators, conditions similar to (Ψ) on the subprincipal symbol are necessary for solvability. In this talk, we shall present sufficient conditions for solvability of non-principal type differential operators.

Jacobi matrices on trees, multiple orthogonal polynomials, and their strong asymptotics

Sergey A. Denisov
University of Wisconsin, Madison

The connection between the polynomials orthogonal on the real line and the Jacobi matrices proved to be important for both approximation theory and spectral theory. Not long ago, it was discovered that the general multiple orthogonal polynomials are related to Jacobi matrices on trees. In that talk, I will explain that connection and will show how the fixed-point theory can be used to obtain the strong asymptotics of these polynomials when the orthogonality measures satisfy the Szegő condition on the corresponding intervals of orthogonality. (Based on a series of papers with A. Aptekarev and M. Yattselev).

A product formula related to Zeno dynamics

Pavel Exner
Doppler Institute for Mathematical Physics and Applied Mathematics, Prague

This talk is not on a topic Sergey had worked on, but it belongs to the area where he made many weighty contributions, a crossroad between functional analysis and quantum physics. The main object is a new product formula which involves a unitary group generated by a positive self-adjoint operator and a continuous projection-valued function, with the focus on the case in which the projections involved are of infinite dimension. The question is not only mathematically interesting but has a direct physical motivation associated with the so-called Zeno effect which arises when an unstable quantum systems suffers frequently repeated measurements. We describe the background of the problem, sketch the proof of the formula, and present an example of a permanent position ascertaining which leads to an effective constraint given by the Dirichlet condition.

Complete non-selfadjointness for Schrödinger operators on the semi-axis

Christoph Fischbacher
Baylor University

This is a joint work with Sergey Naboko and Ian Wood. We investigate complete non-selfadjointness for all maximally dissipative extensions of a Schrödinger operator on a half-line with dissipative bounded potential and dissipative boundary condition. We show that all maximally dissipative extensions that preserve the differential expression are completely non-selfadjoint. However, it is possible for maximally dissipative extensions to have a one-dimensional reducing subspace on which the operator is selfadjoint. We give a characterization of these extensions and the corresponding subspaces and present a specific example. If time permits, I will also present recent work in progress (in collaboration with Andrés Patiño and Monika Winklmeier), where a similar problem is studied on the interval, which is more complex due to the higher defect indices.

Endpoint Schatten class properties of commutators

Rupert L. Frank
LMU Munich

We are interested in trace ideal properties of commutators $[\mathrm{sgn}D, f]$, as well as $[(-\Delta)^s, f]$ for $-d/2 < s \leq 1$, where f is a function on $\mathbb{R}^d$ and D is the Dirac operator. We show that these commutators belong to a certain weak Schatten class if and only if the function f belongs to an appropriate homogeneous Sobolev space, and in this case we determine the asymptotic behavior of the singular values. Our proofs use, among other things, the tool of Double Operator Integrals.

The talk is based on joint work with F. Sukochev and D. Zanin.

Discrete spectrum estimations of a class operators defined by unbounded Jacobi matrices

Ayoub HARRAT
University of Toulouse III-Paul Sabatier

By considering an unbounded Jacobi matrix with a diagonal $(d_k)_{k=1}^{\infty} \nearrow^{+\infty}$ and off-diagonals $(a_k)_{k=1}^{\infty}$, $(b_k)_{k=1}^{\infty}$ complex valued sequences dominated by $(d_k)_{k=1}^{\infty}$. In this talk, we aim to provide for the associated self-adjoint operator an estimation for its discrete spectrum of this kind of operators under certain conditions ensuring the discreteness of the spectrum. The investigation of the asymptotic behaviour for large eigenvalues of a class of Jacobi operators has been of interest for several authors. In 2004 J. Janas and S. Naboko have given the expression of eigenvalues with error estimate $O(k^{-3})$) and In 2009 M. Malejki has given another interesting result in this context. We will mention the famous Quantum Rabi model where Malejki's conditions are not satisfied and introduce our contribution.

How large and small can the gaps between the eigenvalues of a quantum graph be?

Evans Harrell
Georgia Institute of Technology

Gaps between eigenvalues, especially the fundamental gap $\lambda_2 - \lambda_2$ are well studied in spectral geometry and in the study of Schödinger operators on Euclidean sets, but in the context of quantum graphs (differential operators on metric graphs) the proofs do not always adapt, and some analogues of classic gap inequalities are known to fail in examples. I'll report on adaptations to quantum graphs of Cheeger inequalities and of universal (Payne-Pólya-Weinberger) bounds (joint work with Borthwick and Zhu), and on gap bounds for convex and single-well potentials (joint work with Ahrami, El Allali, and Kennedy).

Hyperbolic Fourier series and the Klein-Gordon equation

Håkan Hedenmalm
KTH Royal Institute of Technology

This reports on joint work with A. Bakan, A. Montes-R., D. Radchenko, M. Viazovska We develop a new type of series, dubbed hyperbolic Fourier series, which is an amalgam of two Fourier series where in one instance the variable t is replaced by 1/t. This has a strong connection with the Klein-Gordon equation, which governs relativistic bosons.

Computing eigenvalues of the discrete p-Laplacian via graph surgery

Matthias Hofmann
Texas A&M University

We discuss the dependence of the eigenvalues and eigenfunctions for the discrete signed p-Laplacian under perturbation by a cut parameter. In particular, we prove a formula for the derivative of the eigenvalues and show that the eigenvalues of the discrete signed p-Laplacian on the original graph can be characterized via extremal points of the perturbed system. In this context, we elaborate on how graph surgery can be used in order to compute eigenvalues of the discrete (signed) p-Laplacian by looking at some examples. The derivation formula is reminiscent of the formula for linear eigenvalue problems given by the Hellmann–Feynman theorem and our results extend previous results for the linear case p=2 attained by [Berkolaiko, Anal. PDE 6 (2013), no. 5, 1213-1233].

The discrete analog of the Gaussian

Anders Karlsson
University of Geneva and Uppsala University

The discrete analog of the Gaussian is $e^{-2t}I_x(2t)$, where I denotes the I-Bessel function. It is the building block of heat diffusion in continuous time on regular graphs. When the Gaussian is combined with Poisson summation, it leads to a collection of striking consequences. Similarly the discrete analogs give rise to remarkable relations. I will exemplify this involving in particular zeta functions, determinant of Laplacians, trigonometric sums, volume formulas. Based on joint works with Chinta, Jorgenson, Smajlovich, Friedli and Pallich.

A phase transition in a periodic tubular structure in an external magnetic field.

Alexander V. Kiselev
Univ. of Bath

I will consider an ε-periodic ($\varepsilon \to 0$) tubular structure (one might think of graphene nanotubes or some closely related model), modelled as a magnetic Laplacian on a metric graph, the latter assumed periodic along a single axis. We show that the corresponding Hamiltonian admits norm-resolvent convergence to an ODE on $\mathbb{R}$ which is fourth-order at a discrete set of values of the magnetic potential and second-order generically. In a vicinity of critical points we establish a mixed-order asymptotics. The rate of convergence is also estimated. This represents a physically viable model of a phase transition as the strength of the (constant) magnetic field increases.

The talk will be based on recent joint work with Kirill Ryadovkin from St. Petersburg.

Multiple Orthogonal Polynomials on the Unit Circle

Rostyslav Kozhan
Uppsala University

We investigate polynomials that satisfy simultaneous orthogonality conditions with respect to several measures on the unit circle. We generalize the direct and inverse Szegő recurrence relations, identify the analogues of the Verblunsky coefficients, and prove the Christoffel–Darboux formula. Joint work with M. Vaktnäs.

Discrete Schrödinger operators with decaying and oscillating potentials

Simon Larson
Chalmers University of Technology &
the University of Gothenburg

In this talk we will consider the nature of the essential spectrum of Schrödinger operators on $\ell^2(\mathbb{Z}_+)$ with oscillating and decaying potentials. The prototype case being the potential which at site n is given by

$$V(n) = \lambda \frac{\cos(\pi\omega n^\beta + \theta)}{n^\alpha},$$

where $\alpha, \beta > 0$, $\theta \in \mathbb{R}$ and $\omega, \lambda \in \mathbb{R} \setminus \{0\}$ are fixed. In general, the nature of the essential spectrum depends intricately on the parameters. However, we shall argue that if $1 < \beta < 2\alpha$ then the essential spectrum is purely absolutely continuous.

This is based on joint work with Rupert Frank.

Essential spectra and spectral pollution for Maxwell and Drude-Lorentz systems

Marco Marletta
Cardiff University

The methods required to study spectral pollution for dissipative Maxwell (and related) systems turn out to be closely connected to the methods required to study the essential spectrum for such systems. These are very different from the methods used for Schrödinger equations: the presence of dissipation - in the form of conductivity - means that the essential spectrum can be changed by changing the coefficients in the system on any arbitrarily small, non-empty open set. We use instead a reduction of the system to a triangular block operator matrix, together with the concept of limiting essential spectrum developed by Sabine Boegli in 2015. We show that any spectral pollution is confined either to the real axis or to a segment of the imaginary axis. Any computed eigenvalue whose real and imaginary parts are both non-zero is genuine.

This talk describes joint work with various co-authors including Giovanni Alberti, Sabine Boegli, Francesco Ferraresso, Christiane Tretter, Ian Wood and the late Malcolm Brown.

Self-adjointness problem for Sturm-Liouville and 1D Schrödinger operators with distribution coefficients

Vladimir Mikhailets

Institute of Mathematics of the National Academy of Sciences of Ukraine,
Institute of Mathematics of the Czech Academy of Sciences

We introduce and investigate symmetric Sturm-Liouville operators on the line under minimal assumptions on the regularity of the coefficients. Two Povzner-Wiengoltz-type theorems will be presented. Conditions on measure-valued potential which provide that 1D Schrödinger operator is self-adjoint are found.

Nonsubordinacy and absolutely continuous spectrum of block Jacobi operators

Marcin Moszyński

Uniwersity of Warsaw, Poland
Faculty of Mathematics Informatics and Mechanics

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There is a lot of results for classical scalar self-adjoint Jacobi operators J in the Hilbert space $l^2(\mathbb{N})$ showing that some kinds of asymptotic information on generalized eigenvectors of J imply important spectral information for the operator. The theorem on spectral absolute continuity of J, based on the notion of subordinated solution and being the main part of the Gilbert-Pearson-Khan subordination theory (see [KP]), is one of the main abstract results of this kind. Roughly speaking, the theorem says that non-existence of subordinated solutions for λ-s in a "region" G of $\mathbb{R}$ results in the absolute continuity of J in G.

The talk presents a partial generalization of such result for the case of block Jacobi matrices, i. e., for self-adjoint block Jacobi operators acting in the $l^2(\mathbb{N}, \mathbb{C}^d)$ space, with arbitrary block size $d \geq 1$ (see [MS]).

The problem seems important as the subordination theory so far has no analogs for block Jacobi matrices.

The talk based on the joint work with Grzegorz Świderski

References:

[KP] S. Khan and D. B. Pearson, *Subordinacy and spectral theory for infinite matrices*, Helv. Phys. Acta **65** (1992) no. 4, 505–527.

[MS] M. Moszyński and G. Świderski, *Nonsubordinacy and absolutely continuous spectrum of block Jacobi matrices*, arXiv:2301.00204.

Resonances on black hole spacetimes

Oliver Petersen
KTH Stockholm

Solutions to linear wave equations on bounded domains in Euclidean space (with suitable boundary conditions) can be expressed in terms of eigenfunctions and oscillations given by the corresponding eigenvalues. As shown by Vasy in 2013, solutions to linear wave equations in expanding universes with a single black hole can similarly be expressed (up to an exponentially decaying term) in terms of resonant states and oscillations given by resonance frequencies, if the rotation speed of the black hole is suffciently small. In this talk, I will present a very recent result removing Vasy's restriction on the rotation speed. This is joint work with A. Vasy.

Laplacian eigenvalues and eigenfunctions: a non-standard variational principle

Jonathan Rohleder
Stockholm University

In this talk we discuss a non-standard variational principle for the eigenvalues of the Neumann and Dirichlet Laplacians on bounded planar domains. The novelty is that the minimizers are gradients of eigenfunctions instead of the eigenfunctions themselves. We present applications to the hot spots conjecture and to eigenvalue inequalities.

Nonabelian Bloch Wave Theory?

Peter Sarnak
Institute for Advanced Study and
Department of Mathematics, Princeton University

With simple examples on infinite graphs we explain the challenges of a nonabelian Bloch wave theory and some progress towards a theory of bands and gaps.

On the spectral asymptotic of the KMS-matrices

Boris Shapiro
Stockholm University

KMS-matrices (baptised efter Kac-Mürdock-Szegö) is a natural generalization of the Töplitz matrices in which instead of being constant the entries on each diagonal are varying according to a certain function assigned to this diagonal. In the Hermtian case important results about the asymptotic of the spectra of the sequence of principal minors of any such matrix have been obtained in the original paper by Kac-Mürdock-Szegö. Already the case of 3-diagonal non-Hermitian KMS-matrices presents a substantial difficulty. We will be talking about some theoretical and some numerical results in the latter case. (The speaker has discussed this topic with the late S. Naboko during his visits to Stockholm.)

Spectral Asymptotics for the Systems of Differential Equations and Applications

Andrei Shkalikov
Moscow University

We shall present in the talk new results on asymptotic representations for the fundamental system of solutions of the equation

$$y' + Q(x)y = \lambda A(x)y, \quad x \in [0,1],$$

with respect to $\lambda \to \infty$. Here Q and A are $n \times n$ matrices. In particular, we present explicit formulae for the first k terms of the asymptotic expansions under minimal assumptions on the smoothness of the matrices Q and A. As application we get results on the unconditional convergence of the Fourier series in the eigenfunctions of the operators generated by the systems of differential equations and by the scalar differential high order operators with distribution coefficients.

Schrödinger operators with accretive potentials in weighted spaces

Petr Siegl
Graz University of Technology, Austria

We analyze Schrödinger operators with accretive potentials in weighted spaces. We find conditions on potentials and weights for which the Dirichlet realization, introduced by generalized form methods, has non-empty resolvent set. We establish a domain and graph norm separation property, as well as sufficient conditions for the compactness and Schatten class of the resolvent. Moreover, we investigate the relation between discrete spectra and eigenfunctions of operators in standard and weighted spaces. As applications, we discuss the completeness of eigensystems or operator matrices with

Estimates for Green matrices of Block Jacobi operators

Luis O. Silva
Universidad Nacional Autónoma de México

In this talk, I obtain decay bounds for Green matrices and generalized eigenvectors of block Jacobi operators when the real part of the spectral parameter lies in a gap of the operator's essential spectrum. It is found that the cases when the spectral parameter is in a finite gap and when it is in an infinite gap differ substantially from each other. The cases of commutative and noncommutative blocks are considered. Finally, examples of block Jacobi operators illustrate the results.

This talk is based on joint works with S. Naboko and J. Janas.

Spectral multiplicity of Schrödinger operators on a star-graph with non-Kirchhoff interface condition

Sergey Simonov
St. Petersburg Department of V. A. Steklov Mathematical Institute of the Russian Academy of Sciences;
St. Petersburg State University;
Alferov Academic University of the Russian Academy of Sciences

We consider a star-graph with a finite number n of finite or infinite edges and Schrödinger operators with self-adjoint interface conditions at the inner vertex of the form $Au(0) + Bu'(0) = 0$ (where $u(0) = (u_1(0), ..., u_n(0))^t$, $u'(0) = (u_1'(0), ..., u_n'(0))^t$, $A, B \in M^{n\times n}$, $AB^* = BA^*$ and $\operatorname{rank}(A, B)$ is maximal). We study local multiplicity of point and singular continuous spectra in terms of the spectral data of decoupled Dirichlet operators on edges. This is joint work with Harald Woracek.

Clark measures for rational inner functions

Alan Sola
Stockholm University

I will present results from a series of papers with J.T. Anderson, L. Bergqvist, K. Bickel, and J. Cima on Clark measures associated with rational inner functions in polydisks. Focussing on the two-dimensional case, I will present a precise description of such measures in terms of their smooth supports and bounded weights.

The Maslov index in spectral theory: an overview

Selim Sukhtaiev
Auburn University

This talk is centered around a symplectic approach to eigenvalue problems for systems of ordinary differential operators (e.g., Sturm-Liouville operators, canonical systems, and quantum graphs), multidimensional elliptic operators on bounded domains, and abstract self-adjoint extensions of symmetric operators in Hilbert spaces. The symplectic view naturally relates spectral counts for self-adjoint problems to the topological invariant called the Maslov index. In this talk, the notion of the Malsov index will be introduced in analytic terms and an overview of recent results on its role in spectral theory will be given.

Matrix weights and finite rank perturbations.

Sergei Treil
Brown University

The matrix-valued (and operator-valued, especially trace-class-valued) measures provide a natural language in the perturbation theory. They appeared in the earlier days of the spectral theory (de Branges, Kuroda), and were used, in particular, for one of the proofs of the Kato–Rosenblum theorem about preservation of the absolutely continuous spectrum.

Turns out that they are also quite useful for the investigation of the singular parts of the spectrum. Namely, he classical Aronszajn–Donoghue theorem states that the singular parts of the spectral measures of a self-adjoint operator and its rank one perturbation (by a cyclic vector) are mutually singular. While simple direct sum type examples would indicate that such result is impossible for the scalar spectral measures, it holds if one introduces the notion of *vector* mutual singularity of matrix-valued measures.

Two weight estimates with matrix weights and the matrix A_2 condition appear natively in this context, and will be used to prove the Aronszajn–Donoghue type theorem for finite rank perturbations.

I'll also discuss the Aleksanrov's disintegration theorem for matrix Clark measures, as well as the Hausdorff dimension of the exceptional set of perturbations, where the Aronszajn–Donoghue theorem for scalar spectral measures fails.

The talk is based on a joint work with C. Liaw.

Lieb–Thirring Inequalities in Hyperbolic Space

Timon Ruben Weinmann
Imperial College London

Consider a Schrödinger operator $-\Delta - V$ on $L^2(\mathbb{R}^n)$, where V is a real-valued function. In their celebrated 1976 paper, Lieb and Thirring proved that

$$\operatorname{Tr}(-\Delta - V)_-^\gamma \leq L_{\gamma,n} \int_{\mathbb{R}^n} V_+^{\gamma+n/2} dx$$

holds with finite constants $L_{\gamma,n}$ as long as $\gamma > \max(0, 1 - n/2)$. In the case $n = 3, \gamma = 0$ this bound is known as the Cwikel-Lieb-Rozenblum (CLR) inequality and was proven as early as 1972. The second critical case $n = 1, \gamma = 1/2$ was settled by Weidl in 1996. The inequality is known to fail for $n = 2, \gamma = 0$.

We use a lifting argument to extend the Lieb–Thirring inequality to hyperbolic spaces, obtaining

Theorem. Let $\gamma \geq 1/2$ and $n \geq 2$. Then there exists a constant $L_{\gamma,n}$ such that for every real-valued function V defined on $\mathbb{H}^n$ whose positive part fulfils $V_+ \in L^{\gamma+n/2}(\mathbb{H}^n, y^{-2}dxdy)$, the eigenvalues $\lambda_k = (n-1)^2/4 - \mu_k$ of the operator $-\Delta_{\mathbb{H}^n} - V$ satisfy

$$\sum_k \mu_k^\gamma \leq L_{\gamma,n} \int_{\mathbb{H}^n} V_+^{\gamma+n/2} y^{-2} dxdy.$$

The talk is based on a joint paper with Ari Laptev and Alexey Ilyin.