Workshop of the GAMM Activity Group APPLIED OPERATOR THEORY

Stockholm University, 19–21 May 2022

Email organizers: gamm-ot21@math.su.se

With financial support from Stockholm University
and Swedish Research Council VR

Timetable

LOCATION: Kräftriket, house 5, lecture hall 15 (ground floor)

THURSDAY (19 MAY 2022) AFTERNOON

14:00 – 14:30 Annemarie Luger: Characterizations of (generalized) poles of matrix $\mathcal{N}_\kappa$-functions

14:30 – 15:00 Christian Emmel: Realizations of meromorphic functions of bounded type

15:00 – 15:30 Pavel Kurasov: Magnetic boundary control: inverse problems for quantum graphs with loops

---

16:00 – 16:30 Amru Hussein: Non-self-adjoint boundary conditions on graphs and domains

16:30 – 17:00 Pedro J. Miana: Catalan generating functions for bounded operators

---

From 18:30 **Conference dinner in Restaurant Rodolfino (Stora Nygatan 1)**

FRIDAY (20 MAY 2022) MORNING

09:00 – 09:30 Pavel Exner: Discrete spectrum of soft quantum waveguides

09:30 – 10:00 Riccardo Adami: Towards nonlinear quantum hybrids. The problem of ground states.

10:00 – 10:30 Edison Jair Leguizamon Quinche: On the problem of lower-bounded one-dimensional Schrödinger oprators with singular perturbations

---

11:00 – 11:30 Nausica Aldeghi: Monotonicity of mixed Dirichlet-Neumann eigenvalues on piecewise smooth domains

11:30 – 12:00 Dale Frymark: Self-adjointness of the 2D Dirac operator with singular interactions supported on star-graphs

12:00 – 12:30 Vladimir Lotoreichik : Self-adjointness for the MIT bag model on a cone

FRIDAY (20 MAY 2022) AFTERNOON

14:00 – 14:30 Martin Grothaus: Strong mixing with rate of convergence for operator semi-groups with non-sectorial generator

14:30 – 15:00 Maximilian Mertin: A geometric scaling method for anisotropic fibre lay-down dynamics

15:00 – 15:30 Benedikt Eisenhuth: Essential m-dissipativity and hypocoercivity for generators of infinite dimensional degenerate stochastic differential equations

---

16:00 – 16:30 Simon Wittmann: Mosco convergence of gradient forms with non-convex interaction potentials

16:30 – 17:00 General assembly

SATURDAY (21 MAY 2022) MORNING

09:00 – 09:30 Christian Seifert: On Observability and Null-Controllability of Systems in Banach spaces

09:30 – 10:00 Katharina Klioba: On Joint Convergence Rates for Approximation of Random Evolution Equations

10:00 – 10:30 Matthias Täufer: On controllability of the Schrödinger equation in $\mathbb{R}^d$

11:00 – 11:30 Riko Ukena: Spectral approximation for aperiodic Schödinger operators

11:30 – 12:00 Christian Wyss: Enclosing the pseudospectrum via inverted numerical ranges and discretization

Abstracts in running order

Characterizations of (generalized) poles of matrix $\mathcal{N}_\kappa$-functions

Annemarie Luger

Stockholms universitet

(Generalized) Poles of matrix-valued functions do exhibit behaviour different from the scalar case, for instance can there be points that are both poles and zeros of the function.
We focus here on generalized poles of matrix-valued Generalized Nevanlinna functions, $Q \in \mathcal{N}_\kappa(\mathbb{C}^{n\times n})$. In the neighbourhood of a point $\alpha \in \mathbb{C}$ the function can be written in the form

$$Q(z) = H(z) + \Gamma^{+}(A - z)^{-1}\Gamma,$$

where H is analytic in this neighbourhood, A is a self-adjoint operator in a Pontryagin-space $\mathcal{K}$ and $\Gamma \in \mathcal{B}(\mathbb{C}^n, \mathcal{K})$ a bounded operator.
Hence for α there is both an inner structure, reflecting the spectral properties of A at α, e.g., lengths of Jordan chains, and external properties, such as growth of Q close to the point.
In this talk we are connecting these two by a characterization of the inner structure in terms of the asymptotic behaviour of Q.

Realizations of meromorphic functions of bounded type

Christian Emmel

Abstract

Realizations of analytic functions are a way of writing the function in terms of the resolvent of an self-adjoint relation acting on a Krein space (an indefinite inner product space). Examples for such correspondences are given by *Herglotz-Nevanlinna functions*, which are exactly those functions, which admit an realization in a Hilbert space, or more generally by *Generalized Nevanlinna functions* which are characterized by realizations in a Pontryagin space (i.e., an indefinite inner product space with finitely many negative squares).

In this talk we consider the rather broad class of analytic functions that are meromorphic in the upper halfplane and of bounded type there. For these functions it is shown that there exists a realization in a Krein space.

This talk is based on joint work with Annemarie Luger

Magnetic boundary control: inverse problems for quantum graphs with loops

Pavel Kurasov

Dept. of Math., Stockholm Univ.

The classical Boundary Control method is extended by including magnetic fields. It is applied to solve inverse problems for Schrödinger operators on metric graphs with cycles using spectral data depended on the magnetic fluxes through the cycles. Our main goal is to characterise the minimal set of contact points that have to be used.

Non-self-adjoint boundary conditions on graphs and domains

Amru Hussein

Classical boundary conditions for elliptic operators include for instance Dirichlet, Neumann and Robin boundary conditions. Going from scalar valued to vector valued functions raises the complexity of the problem considerably. An early pathbreaking approach in this direction has been developed at the begining of the 20th century by the classical Birkhoff-Tamarkin theory for ordinary differential operators. A key issue is how to parametrize boundary conditions and how to ensure basic spectral properties. Here, I consider the model case of a Laplacian on a finite metric graph subject to general non-self-adjoint matching conditions imposed at the graph's vertices. A regularity criterium related to the Cayley transform of boundary conditions is discussed and spectral properties of such operators are investigated, in particular similarity transforms to self-adjoint operators and the generation of C0-semigroups. Concrete examples are discussed exhibiting that non-self-adjoint boundary conditions can yield to unexpected spectral features. How this can be transfered to the situation of partial differential operators on domains is outlined.
The talk is based on joint works with David Krejcirik (Czech Technical University in Prague), Petr Siegl (Queen's University Belfast) and Delio Mugnolo (FernUniversität Hagen).

Catalan generating functions for bounded operators

Pedro J. Miana
Universidad de Zaragoza, Spain

In this talk we study the solution of the quadratic equation $TY^2 - Y + I = 0$ where T is a linear and bounded operator on a Banach space X. We describe the spectrum set and the resolvent operator of Y in terms of operator T. In the case that $4T$ is a power-bounded operator, we show that a solution (named Catalan generating function) is given by the Taylor series

$$C(T) := \sum_{n=0}^{\infty} C_n T^n$$

where the sequence $(C_n)_{n\geq 0}$ is the well-known Catalan numbers. We express $C(T)$ by means of an integral representation which involves the resolvent operator $(\lambda - T)^{-1}$. Some particular examples to illustrate our results are given, in particular an iterative method defined for square matrices T which involves Catalan numbers. This is a joint pre-publicacition with Natalia Romero from the Universidad de La Rioja.

Discrete spectrum of soft quantum waveguides

Pavel Exner

Doppler Institute for Mathematical Physics
and Applied Mathematics, Prague

The topic of this talk are 'soft' quantum waveguides described by Schrödinger operators with an attractive potential in the form of a channel of a fixed profile built along a smooth curve in $\mathbb{R}^\nu$. In the case when $\nu = 2$ and the curve is infinite and not straight, but asymptotically straight in a suitable sense, we derive using Birman-Schwinger principle a sufficient condition for the discrete spectrum of such an operator to be nonempty; this happens, in particular, when the potential well defining the channel profile is deep and narrow enough. Under a restriction on the waveguide torsion, this results extends to curves in $\mathbb{R}^3$. We also address the question about ground state optimization in the situation when the generating curve in $\mathbb{R}^2$ has the shape of a loop without self-intersections. Some related results and problems are also mentioned.

Mailing addresses:
Department of Theoretical Physics, NPI
Czech Academy of Sciences
Hlavní 130, CZ-25068 Řež near Prague;
Department of Physics, FNSPE
Czech Technical University
Břehová 7, CZ-11519 Prague

E-mail: exner@ujf.cas.cz

Towards nonlinear quantum hybrids. The problem of ground states.

Riccardo Adami

In recent years the topic of Nonlinear Quantum Graphs has met a considerable interest, especially for what concerns the problem of the existence of Ground States, that proved connected to topological properties of the graphs. We propose an extension of the investigation to connected structures made not only of one-dimensional edges, but also of pieces of different dimensions, and give some preliminary results. The model we discuss was proposed in the nineties by Exner and Seba in order to describe a microwave cavity with antenna, and consists of a plane with a halfline attached to it. This is a joint work with Filippo Boni (Naples), Raffaele Carlone (Naples), and Lorenzo Tentarelli (Turin).

On the problem of lower-bounded one-dimensional Schrödinger operators with singular perturbations

Edison Jair Leguizamon Quinche

Departamento de Matemáticas, Universidad de los Andes

ej.leguizamon@uniandes.edu.co

Abstract

In the theory of Schödinger operators it is well known that δ and δ' interactions on discrete sets can be obtained in a suitable self-adjoint extension of a second order differential operator. Our talk will be about the principal results when we consider a lower bounded potential on the real line and how this new operator change its spectrum with respect to the original. Finally we will give a brief introduction about the δ and δ' interactions on non-discrete sets and the possible approaches to this problem.

Monotonicity of mixed Dirichlet-Neumann eigenvalues on piecewise smooth domains

Nausica Aldeghi

We investigate the monotonicity of the lowest mixed Dirichlet–Neumann eigenvalues of the Laplacian and the Schrödinger operator, with respect to the part of the boundary where a Dirichlet boundary condition is imposed, on various piecewise smooth bounded domains. We show how the resulting eigenvalue inequalities depend heavily on the geometry and regularity of the domains, and in particular on the shape of the single smooth parts of the boundary.

Self-adjointness of the 2D Dirac operator with singular interactions supported on star-graphs

Dale Frymark

Nuclear Physics Institute, Czech Academy of Sciences

We consider the two-dimensional Dirac operator with Lorentz-scalar delta-shell interactions on each edge of a star-graph. An orthogonal decomposition is performed which shows such an operator is unitarily equivalent to an orthogonal sum of half-line Dirac operators with off-diagonal Coulomb potentials. This decomposition reduces the computation of the deficiency indices to determining the number of eigenvalues of a one-dimensional spin-orbit operator in the interval (-1/2,1/2).
If the number of edges of the star graph is two or three, these deficiency indices can then be analytically determined for a range of parameters. For higher numbers of edges, it is possible to numerically calculate the deficiency indices. Among others, examples are given where the strength of the Lorentz-scalar interactions directly change the deficiency indices while other parameters are all fixed and where the deficiency indices are (2,2), neither of which have been observed in the literature to the best knowledge of the authors. The distinguished self-adjoint extension can also be characterized in most situations. This is joint work with Vladimir Lotoreichik.

Self-adjointness for the MIT bag model on a cone

Vladimir Lotoreichik
Nuclear Physics Institute, Czech Academy of Sciences

We will discuss the massless Dirac operator D on an unbounded three-dimensional circular cone. We define this operator on four-component H^1-functions satisfying the MIT bag boundary conditions on the boundary of the cone. The closed Dirac operator D is symmetric and the natural question arises whether it is self-adjoint or it has non-trivial deficiency indices. We prove that D is self-adjoint for convex cones and provide a numerical evidence for its self-adjointness also for non-convex cones. The situation is different from the Dirichlet Laplacian on the cone (defined on H^2-functions with vanishing trace), where there is a transition between self-adjointness and deficiency indices $(1, 1)$ for the opening angle of the cone exceeding the critical value. As a by-product of our analysis we obtain a Hardy inequality for D. These results are obtained in collaboration with Biagio Cassano.

Strong mixing with rate of convergence for operator semi-groups with non-sectorial generator

Martin Grothaus
Mathematics Department / TU Kaiserslautern EMAIL:
grothaus@mathematik.uni-kl.de

Motivated by problems from Industrial Mathematics we further developed the concepts of hypocoercivity. The original concepts needed Poincaré inequalities and where made for finite dimensional and linear state spaces. In between we can treat as a state space manifolds or even infinite dimensional state spaces. The condition giving micro- and macroscopic coercivity we could relax from Poincaré to weak Poincaré inequalities. In this talk an overview and many examples are given.

A geometric scaling method for anisotropic fibre lay-down dynamics

Maximilian Mertin

TU Kaiserslautern

We study Langevin-type models, that describe a fibre filament on a (nonmoving) conveyor belt in the industrial production of nonwoven materials, on Euclidean space or some Riemannian manifold. In particular, one is interested in anisotropic 3D fibre lay-down models, since they incorporate reasonable preferential directions. Previous works argue that those models should converge to the 2D models. We use a new geometric approach via stratifolds to prove in broad generality Mosco convergence of the generalised Dirichlet forms associated to the anisotropic fibre lay-down models. In this way, we give a strict geometric meaning to the expected dimension reduction.

Essential m-dissipativity and hypocoercivity for generators of infinite dimensional degenerate stochastic differential equations

Benedikt Eisenhuth

Abstract. In this talk we analyze infinite-dimensional non-linear degenerate stochastic differential equations in terms of their infinitesimal generators. We establish essential m-dissipativity of these generators in $L^2(\mu^\Phi)$, where μ^Φ is a measure with density $e^{-\Phi}$ w.r.t. an infinite dimensional Gaussian measure. Using general resolvent methods we construct corresponding μ^Φ-standard right processes with infinite lifetime and weakly continuous paths providing weak solutions to such infinite-dimensional non-linear degenerate stochastic differential equations. Moreover, we identify the transition semigroup of the process with the C_0-semigroup $(T_t)_{t\geq 0}$ generated by the essential m-dissipative generator of the equation. Afterwards, we use the abstract Hilbert space hypocoercivity method, developed by Grothaus and Stilgenbauer to derive hypocoercivity of $(T_t)_{t\geq 0}$ and the transition semigroup, respectively. Finally, we apply our results to stochastic reaction-diffusion and Cahn-Hilliard type equations.

Benedikt Eisenhuth
Department of Mathematics
P.O. Box 3049
67653
Kaiserslautern
e-mail: `eisenhuth@mathematik.uni-kl.de`

Title

Mosco convergence of gradient forms with non-convex interaction potentials

Speaker *Simon Wittmann*

Abstract

Given a countable family of symmetric Markovian semigroups $(T_t^N)_{t\geq 0}$ on variable L^2 spaces, indexed by $N \in \mathbb{N}$, the question of convergence for $N \to \infty$, i.e. the identification of a semigroup $(T_t)_{t\geq 0}$ in the limiting L^2 space, can be relevant to different fields of application and is of particular interest to scaling limits of stochastic dynamical interface or semiflexible polymer models. The surface tension of a physical interface or the stiffness of a semiflexible polymer is qualitatively expressed in terms of an operator L on $H := L^2([0,1]^{dim}, \mathrm{d}x)$, most commonly with Dirichlet boundary conditions and $L = \Delta$, or mixed powers of the Laplace such as $L = \Delta + \Delta^2$ considered in [3]. A non-degenerate, centred Gaussian measure μ on a Souslean, locally convex topological vector space E, where $H \subseteq E$ densely and the covariance of μ is associated to the inverse of L, then serves as the equilibrium distribution for the static counterpart of our model. The semigroup of interest is the one associated on $L^2(E,\mu)$ to the classical minimal Dirichlet form of gradient type $(\mathcal{E}, \mathcal{D}(\mathcal{E}))$ with tangential space H, i.e. the one satisfying all relations

$$\mathcal{D}(\mathcal{E}) = \Big\{ u \in L^2(E,\mu) \,\Big|\, \sup_{t>0} \frac{1}{t}\big(u - T_t u, u\big)_{L^2(E,\mu)} < \infty \Big\},$$

the cylindrical smooth functions $\mathcal{F}C_b^\infty$ are a form core,

$$\lim_{t\downarrow 0} \frac{1}{t}\big(u - T_t u, u\big)_{L^2(E,\mu)} = \int_E \langle \nabla u, \nabla u \rangle_H \,\mathrm{d}\mu \quad \text{for } \in \mathcal{D}(\mathcal{E}). \tag{1}$$

We give a non-log-concave perturbation result of a statement, well-known in the Gaussian case: Weak measure convergence $\mu_N \Rightarrow \mu$ on E implies the strong convergence of semigroups $(T_t^N)_{t\geq 0} \to (T_t)_{t\geq 0}$ in the sense specified by [1], if $(T_t^N)_{t\geq 0}$ is related to μ_N through the analogue of (1). Combining a classic disintegration argument with the concept of Finite Elements we are able to show the Mosco convergence of perturbed forms $\mathcal{E}^N(u) := \int_E |\nabla u|_H^2 e^{-V_N} \,\mathrm{d}\mu_N$, $N \in \mathbb{N}$, towards $\mathcal{E}(u) := \int_E |\nabla u|_H^2 e^{-V} \,\mathrm{d}\mu$, which is equivalent to the strong convergence of perturbed semigroups. The distorting potentials V, V_N, $N \in \mathbb{N}$, we can allow for, include non-convex functions on E, as our example with

$$V_N(h) = V(h) = \int_{[0,1]^{dim}} (f \circ h)(x)\,\mathrm{d}x, \quad h \in H,\ f : \mathbb{R} \to \mathbb{R} \text{ is of BV},$$

in the case $E = H$ shows.

On Observability and Null-Controllability of Systems in Banach spaces

Christian Seifert

In this talk we are interested in two types of systems. Firstly, we consider

$$\dot{x}(t) = -Ax(t) + Bu(t) \quad (t > 0), \quad x(0) = x_0$$

in a Banach space X, where $-A$ generates a C_0-semigroup and $B \in \mathcal{L}(U, X)$ for a Banach space U. We study the question of null-controllability, i.e. whether, for given final time $T > 0$, for all initial conditions $x_0 \in X$ there exists a so-called control function $u \in L_2((0,T);U)$ (where $r \in [1,\infty]$) such that $x(T) = 0$.
Second, we consider

$$\dot{x}(t) = -Ax(t) \quad (t > 0), \quad x(0) = x_0, \quad y(t) = Cx(t) \quad (t \geq 0)$$

in Banach spaces X and Y, where $C \in \mathcal{L}(X, Y)$. Here, we study the question of final state observability, i.e. whether, for given final time $T > 0$, there exists $C_{\mathrm{obs}} \geq 0$ such that for all initial conditions $x_0 \in X$ we have

$$\|x(T)\| \leq C_{\mathrm{obs}} \|y\|_{L_r((0,T);Y)}$$

where $r \in [1, \infty]$. These two properties are related via duality and we will show sufficient criteria for the final state observability, which in turn yields null-controllability (via duality).
The talk is based on joint works with Clemens Bombach, Michela Egidi, Fabian Gabel, Dennis Gallaun, Jan Meichsner and Martin Tautenhahn.

On Joint Convergence Rates for Approximation of Random Evolution Equations

Katharina Klioba

Solving random evolution equations numerically requires a discretisation in space, in time, and of random parameters. Methods to treat these three problems seperately are well-known, including rates of convergence. In this talk, conditions are presented under which these rates of convergence are conserved for the fully discretised solution. Focusing on spatial discretisation, a quantified version of the Trotter-Kato theorem corresponding to the weak formulation is presented. On a semigroup level, this corresponds to approximating form-induced semigroups on separable Hilbert spaces by restricting the form to simpler, often finite-dimensional, approximating spaces. Rates of strong convergence are obtained on dense subspaces under a joint condition on properties of both the form and the approximating spaces. As a standard application, results are discussed for the heat equation with random coefficients. This is joint work with Christian Seifert.

On controllability of the Schrödinger equation in $\mathbb{R}^d$

Matthias Täufer

FernUniversität in Hagen

In control theory for the Schrödinger equation, one investigates whether the time-dependent Schrödinger equation can be driven from any prescribed initial state at time 0 to any prescribed target state at time T by adding an appropriate inhomogeneity ("the control") to the equation. This control is only allowed to act on a subset ω of $\mathbb{R}^d$ and the central question is to decide whether a control set ω leads to controllability or not. The case of dimension $d = 1$ has been recently solved by Jérémy Martin and Karel Pravda-Starov. We present some preliminary results in higher dimensions.

Spectral approximation for aperiodic Schrödinger operators

Riko Ukena (with Julian Großmann, Fabian Gabel, Dennis Gallaun, Marko Lindner)

Hamburg University of Technology

We analyse discrete Schrödinger operators $H_{\lambda,\alpha,\theta} : \ell^p(\mathbb{Z}) \to \ell^p(\mathbb{Z})$, $p \in [1, \infty]$, with Sturmian potential, namely,

$$(H_{\lambda,\alpha,\theta}x)_n = x_{n+1} + x_{n-1} + \lambda v_{\alpha,\theta}(n)x_n \,, \quad n \in \mathbb{Z} \,,$$

where

$$v_{\alpha,\theta}(n) = \chi_{[1-\alpha,1)}(n\alpha + \theta \mod 1)$$

with coupling constant $\lambda \in \mathbb{R}$, irrational slope $\alpha \in [0, 1]$ and $\theta \in [0, 1)$. The famous Fibonacci Hamiltonian arises when choosing $\alpha = \frac{1}{2}(\sqrt{5} - 1)$.
We discuss how spectral properties of aperiodic Schrödinger operators can be approximated via their periodic counterparts. Moreover, we locate eigenvalues that arise from a restriction to $\ell^p(\mathbb{Z}_+)$ with Dirichlet boundary conditions. This talk is based on `https://arxiv.org/abs/2104.00711`.

Enclosing the pseudospectrum via inverted numerical ranges and discretization

Christian Wyss

The pseudospectrum of a linear operator is a superset of the spectrum with certain nice properties. It is stable under perturbations and contains additional information, e.g., about resolvent estimates and the transient behaviour of linear systems. We present new enclosures for the pseudospectrum in terms of numerical ranges of inverses of the operator. Our method can be used to compute the pseudospectrum numerically using finite-dimensional approximations of the original operator. We prove convergence results for suitable approximation schemes, including finite element discretizations.
This is joined work with Andreas Frommer, Birgit Jacob, Lukas Vorberg and Ian Zwaan.
