Exercise 1

A population consists of two individuals located at $s_1 = (0,0)'$ and $s_2 = (1,1)'$. Let Z denote a stationary and isotropic Gaussian spatial process with mean $\mu = 3$ and variance $\sigma^2 = 10$ having a Mátern covariance function with $\kappa = 0.5$ and $\phi = 1$.

(a) Use the function `cov.spatial` in the R package `geoR` to compute the covariance $C(u_{12}) = \text{Cov}(Z(s_1), Z(s_2))$. What is the corresponding correlation $\rho(u_{ij})$?

(b) Let $\boldsymbol{Z} = (Z(s_1), Z(s_2))'$. Create a 2×2 matrix `Sigma` in R corresponding to the covariance matrix $\boldsymbol{\Sigma}$ of $\boldsymbol{Z}$.

(c) Use the function `mvrnorm{MASS}` to generate 1000 realizations of $\boldsymbol{Z}$ and create a scatterplot of the realizations.

(d) Use the `grf{geoR}` function to perform exactly the same simulation as in (c). Create a scatterplot showing the 1000 realizations from both methods on top of each other.

(e) Assume now that $Y(s_i) = Z(s_i) + \epsilon_i$ with $\epsilon_i \overset{\text{iid}}{\sim} N(0, 2^2)$. How could we quickly modify the call to `grf` to simulate from this model?

(f) Use `grf{geoR}` to generate a realization of the above Gaussian spatial process for a regular grid of 100 points on $[0,1] \times [0,1]$ and illustrate the result using the `image` function.
Hint: You can use the options `n=100` and `grid="reg"` in the call of `grf`.

Exercise 2 (Dioxin-contaminated soil samples)

In 1971, a truck transporting dioxin-contaminated residues dumped an unknown quantity of waste in a rural area of Missouri. In November 1983, the United States Environmental Protection Agency (EPA) collected soil samples in several areas and measured the TCDD (tetrachlorodibenzo-p-dioxin) concentration (in μg/kg) in each sample to determine the spread to other areas. The data available in the file `tcdd.txt` consists of measurements of 31 samples of TCDD, within $D = 350 \times 55$ foot rectangle.

(a) Load the data into R and convert them to a `geodata` object using the `as.geodata` function.

(b) Create descriptive plots of the data as available using the `plot` function for `geodata` objects.
Hint: You may want to study the following tutorial.

(c) We assume a trend in the direction orthogonal to the highway, i.e. the y coordinate. Use `likfit{geoR}` to fit a Gaussian model with exponential covariance function and trend

$$\mu((s_x, s_y)') = \beta_1 + \beta_2 s_y^2.$$

Is the effect of s_y^2 significant in the model?

(d) Add the estimated curve $\hat{\mu}(s_y)$ to a plot of $(s_{iy}, y_i), i = 1, \ldots, 31$.

(e) Create a smoothed risk map of TCDD pollution using universal Kriging.