Switcher Semantics and Belief Sentences

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Compositionality and belief sentences, I

With extensions as semantic values, the semantics of English isn't compositional if

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and

(2) Karl believes that Samuel Clemens is a novelist

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The intuition is that they can, and that this requires a framework that is at least intensional.

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The intuition is that they can, and this requires modes of presentation. But can we believe modes of presentation?

Compositionality and belief sentences, III

With intuitive synonymy-meanings as semantic values, the semantics of English isn't compositional if

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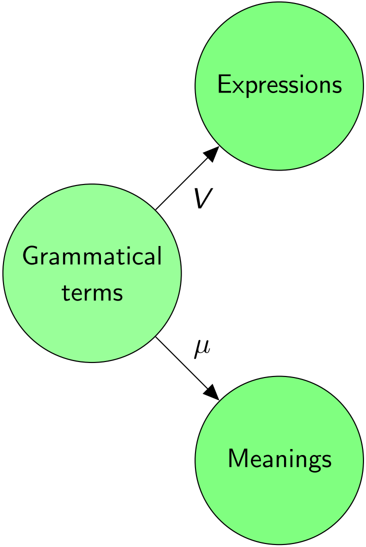
In this case, there is no difference: if *the speaker* uses the two nouns as synonymous, then the same belief is ascribed.

A Hodgean framework for syntax and semantics



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vii) a *semantic function* μ from grammatical terms to meanings

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(PC) For every n-ary operation $\sigma_i \in \Sigma_L$ there is a function $r_i \in R$ such that for all terms $t_1, \ldots, t_n$ for which σ_i is defined,

$$\mu(\sigma_i(t_1, \ldots, t_n)) = r_i(\mu(t_1), \ldots, \mu(t_n))$$

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Here it is assumed that the *Domain principle* holds: if t is μ meaningful and u is a subterm of t, then u is μ meaningful.

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I shall here use the second method.

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Let Ψ_S be a *selection function* for S. Ψ_S takes as arguments triples (μ_i, σ_j, k), where $\mu_i \in S$, $\sigma_j \in \Sigma_L$, and $1 \leq k \leq n$, where n is the arity of σ_j. For each argument for which it is defined it gives as value a semantic function $\mu_m \in S$.

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We assume a *generalized domain principle*: if $\sigma_j(t_1, \ldots, t_n)$ is μ_i meaningful, and if $\Psi_S(\mu_i, \sigma_j, k) = \mu_m$, then t_k is μ_m meaningful.

General compositionality, II

We can state the principle of general compositionality:

(PGC) For every pair (μ_i, σ_j), where $\mu_i \in S$ and $\sigma_j \in \Sigma_L$, there is a meaning operation $r_{i,j}$ such that for any terms $t_1, \dots, t_n$, if μ_i is defined for $\sigma_j(t_1, \dots, t_n)$, it holds that

$$\mu_i(\sigma_j(t_1, \dots, t_n)) = r_{i,j}(\mu_{m_1}(t_1), \dots, \mu_{m_n}(t_n))$$

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If $S = \{\mu_1\}$, we get standard compositionality as a special case.

Switcher semantics, I

Given a grammatical term algebra (T_L, A_L, Σ_L), a *switcher semantics* $\mathbf{S} = (S, \mu, \Psi)$ is a triple of a set S of semantic functions, a designated member μ of S, and a switching function Ψ, such that

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i) $\mathbf{S}$ is general compositional

ii) switching takes place; i.e. for some $\sigma \in \Sigma_L$ and some argument place i of σ, $\Psi(\mu, \sigma, i) \neq \mu$.

Switcher semantics, II

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This can happen if s occurs in the scope of temporal adverb such as ‘always’.

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It has been applied, in the algebraic format, to quotation, developing Frege's idea (Pagin and Westerståhl 2010).

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In Montague Grammar, the *denotation* of a complex expression can depend on the *sense* of one or more of its parts.

But denotation is not strictly needed as a separate semantic function, since it is the value of sense at an index.

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This perspective is applied here.

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We will need both structures and propositions, and a way of getting from structures to propositions. The main idea here is to combine structured and unstructured meanings by means of embedding structured meanings in a possible-worlds framework.

Structures and propositions

Let intensions be functions from worlds to extensions, where the extension of a sentence at a world w is a truth value in $\{0, 1\}$, the extension of singular term at w is an object in w and the extension of a (time-insensitive) one-place predicate in w is a set of objects in w. Similarly for many-place predicates.

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Let $r'_p(\ldots, \ldots, \ldots)$ be a function of three arguments such that

$$r'_p(x, y, w) = \begin{cases} 1 \text{ iff } x(w) \in y(w) \\ 0 \text{ iff } x(w) \notin y(w), \text{ but } x, y, w \text{ are of the right types} \\ \text{undefined otherwise} \end{cases}$$

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Let $r_p(x, y) = \lambda w(r'_p(x, y, w))$

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Let σ_p be a syntactic operation that forms grammatical terms that evaluate to subject-predicate combinations:

$$V(\sigma_p(\text{`John'}, \text{`runs'})) = \text{`John runs'}$$

Let μ be a semantic function that gives intensions as values, and that is compositional for the linguistic fragment without belief sentences. Clearly,

$$\mu(\sigma_p(t_i, t_j)) = r_p(\mu(t_i), \mu(t_j))$$

Structure and application

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We will need a larger domain O, such that $M \subseteq O$ and if $r \in R$ and $o_1, \ldots, o_n \in O$, and r is defined for $E(o_1), \ldots, E(o_n)$, then $\langle r, o_1, \ldots, o_n \rangle \in O$.

From structured meanings to unstructured

To define E:

(E) i) $E(o) = o$, if $o \in M$

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Now we can define another semantic function μ' from μ:

(μ') i) $\mu'(t) = \mu(t)$, if t is atomic

ii) $\mu'(\sigma_i(t_1, \ldots, t_n)) = \langle r_i, \mu'(t_1), \ldots, \mu'(t_n) \rangle$

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μ' is a compositional semantic function, where $\lambda \overline{x} \langle r_i, \overline{x} \rangle$ is the meaning operation corresponding to σ_i.

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μ' is a compositional semantic function, where $\lambda \overline{x} \langle r_i, \overline{x} \rangle$ is the meaning operation corresponding to σ_i.

We can show by induction that for all t, $E(\mu'(t)) = \mu(t)$.

Belief syntax

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We assume that all syntactic operations, and μ', are defined for well-formed terms containing σ_b.

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$$E'(o) = \begin{cases} o & \text{if } o \in M \\ r_b(E'(o_1), o_2) & \text{if } o = \langle r_b, o_1, o_2 \rangle \\ r(E'(o_1), \ldots, E'(o_n)) & \text{if } o = \langle r, o_1, \ldots, o_n \rangle, r \neq r_b \end{cases}$$

Belief alternatives

The account uses a Hintikka-style possible worlds framework, with the accessibility relation B:

$B(x, w, w')$ holds between a subject x and worlds w and w' iff w' is an x-belief-alternative to w.

Belief alternatives

The account uses a Hintikka-style possible worlds framework, with the accessibility relation B:

$B(x, w, w')$ holds between a subject x and worlds w and w' iff w' is an x-belief-alternative to w.

The idea is that everything x believes in w is true in w'.

Modes of presentation

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The idea is that the meaning of a sentence

(5) x believes that p

is a function that gives the value 1 for a world w *only if* $\pi(x, o, w)$ holds relative to E', where o is the structured semantic output of $\ulcorner p \urcorner$, with respect to E'.

The meaning operation

The meaning operation r_b that corresponds to σ_b is given as follows:

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This blocks unwanted substitutions: It may be that $E'(o_1) = E'(o_2)$, but $\pi(x, o_1, w)$ while *not* $\pi(x, o_2, w)$.

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According to (Ψ), $\mathbf{S}$ switches from μ to μ' when encountering the belief operation σ_b, at the second argument position, and makes no other switches.

That is, everything within the scope of the belief operator gets a structured interpretation.

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$\mathbf{S}_b$ is a (general compositional) switcher semantics for belief sentences. As it is presented, it is defined for iterated (notional) belief attributions.

It is also extended to relational attributions, and we shall have a brief look at that at the end.

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Assume that $\mu(p) = \{w : \text{ pelicans fly at } w\}$, and let $\overline{p}$ be the structured meaning resulting from ‘pelicans fly’. We get this result, informally rendered:

(7) $\mu(t_{(6)})$ = the set of worlds w such that $\langle r_b, ma, \overline{p}\rangle$ models a belief state of John’s in w and it holds in every belief alternative w' of John’s in w that $\overline{p}$ models a belief state of Mary’s in w' and it holds in every belief alternative w'' of Mary’s in w' that pelicans fly in w''.

Example, full derivation

(8)
(i) $\mu(t_{(6)}) =$

(ii) $r_b(J, \langle r_b, \mu'(ma), \mu'(p)\rangle) =$

(iii) $\{w : \pi(J, \langle r_b, \mu'(ma), \mu'(p)\rangle, w)\} \wedge \{w' : B(J, w, w')\} \subseteq E'(\langle r_b, \mu'(ma), \mu'(p)\rangle) =$

(iv) $\{w : \pi(J, \langle r_b, \mu'(ma), \mu'(p)\rangle, w)\} \wedge \{w' : B(J, w, w')\} \subseteq r_b(E'(\mu'(ma)), \mu'(p)) =$

(v) $\{w : \pi(J, \langle r_b, \mu'(ma), \mu'(p)\rangle, w) \wedge \{w' : B(J, w, w')\} \subseteq \{w_2 : \pi(\mu'(ma), \overline{p}, w_2) \wedge \{w_3 : B(\mu'(ma), w_2, w_3)\} \subseteq E'(\overline{p})\}\} =$

(vi) $\{w : \pi(J, \langle r_b, M, \mu(p)\rangle, w) \wedge \{w' : B(J, w, w')\} \subseteq \{w_2 : \pi(M, \mu'(p), w_2) \wedge \{w_3 : B(M, w_2, w_3)\} \subseteq \mu(p))\}\} =$

(vii) $\{w : \pi(J, \langle r_b, M, \mu(p)\rangle, w) \wedge \forall w_4(B(J, w, w_4) \rightarrow \pi(M, \mu'(p), w_4) \wedge \forall w_5(B(M, w_4, w_5) \rightarrow w_5 \in \{w_6 : \text{ pelicans fly in } w_6\})))\} =$

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where set inclusion is reduced to universal quantification in steps (vii) and (viii). The fact that $E'(\mu'(p)) = \mu(p)$ is used in the step to (vi).

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(11) John believes that Mary will come. (Of course, John does not yet know her name.)

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(10) exemplifies what is called *quantifying-in* (to belief contexts). I call these three kinds *relational attributions*.

The Church-Kaplan approach

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This is a main idea feature of the approach of Alonzo Church (1951) and David Kaplan (1968).

The Church-Kaplan approach, II

Kaplan, like Quine 1956, treated belief contexts as a kind of quotation context. Following Church, he introduced Δ as a denotation relation and proposed (12b) as a formalization of (12a):

(12) a. Nine is such that Hegel believed it to be greater than five.

b. $\exists\alpha(\Delta(\alpha, \text{nine})$ & Hegel $\mathbf{B}$ $\ulcorner\alpha$ is greater than five$\urcorner))$

Here '**B**' expresses the belief relation, as a relation between persons and sentences. The quantifier introduces an expression α that denotes the number nine and forms part of a sentence that Hegel, according to (12b), believes.

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In the present format, with 'm' an intension variable, with 'h' for Hegel and 'G' for the *greater than* predicate, we get:

(13) $\exists m(m(w) = 9 \ \wedge \ \forall w'(B(\text{Hegel}, w, w') \ \rightarrow \ \langle m(w'), 5\rangle \in \mu(G)(w')))$

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We will need the (underspecified) structured meaning

$$o_h = \langle r_2, \mu(G), m, 5 \rangle,$$

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Using this, the *de re* belief attribution (12a) should come out as true at the actual world @ iff

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In the semantics, it actually does.

Thanks!