Logic and Algorithms in Computational Linguistics 2017
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Parsing sluicing

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Plan

- Problem: **Parsing**: finding structure
- Algorithm: **Logic**: proof search as parsing
- Application: **Sluicing**: a challenge for parsing

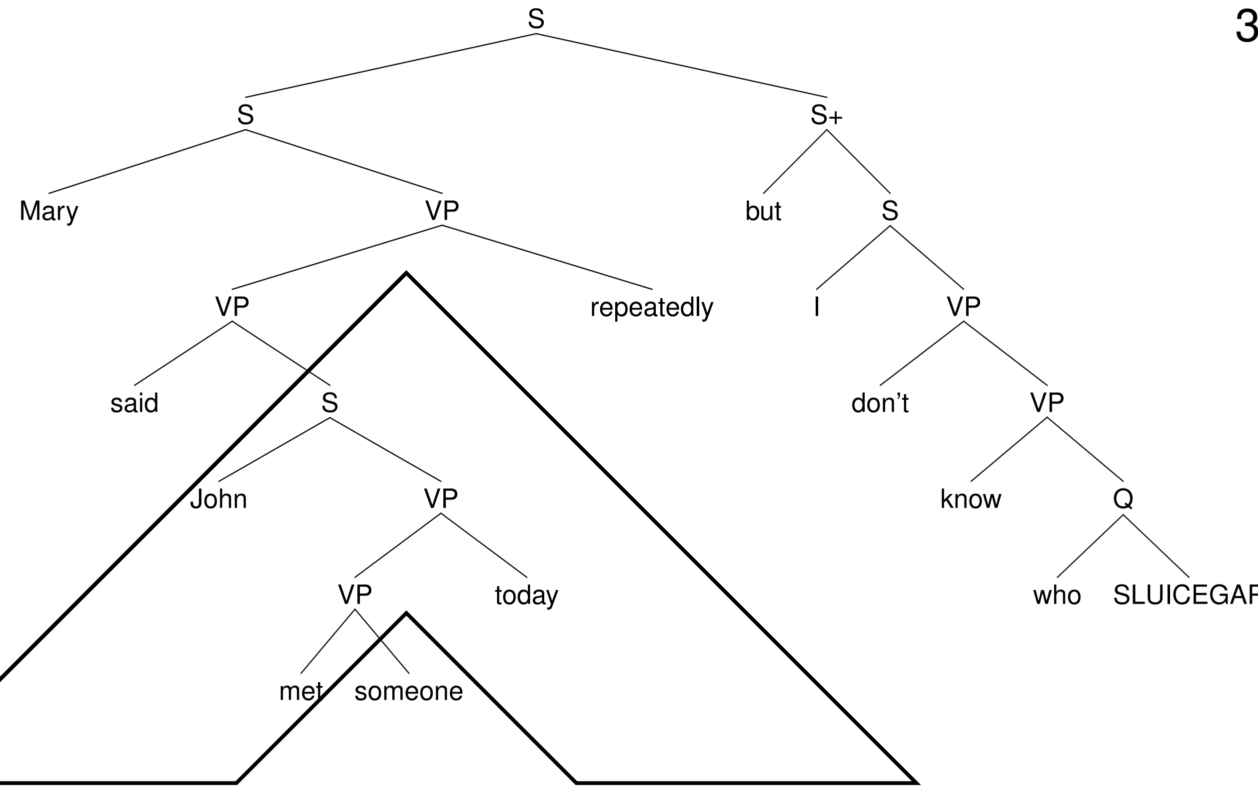
  (1) Ann spoke to someone,
  but I don't know [who __]

- The logical approach naturally gives rise to levels of parsing difficulty that cleanly puts sluicing on a higher level

Preview

- Unlike some ellipsis, sluicing arguably must be syntactic
- Why it's extra difficult to parse: antecedent is not just a constituent, but a constituent *with a piece removed*
- But the piece isn't missing in the antecedent
- Sluicing antecedent in (1): *Ann spoke to ~~someone~~*

Dialog between description, logic, and parsing difficulty



(2) Mary said [John met (someone) today] repeatedly, but I don’t know who ~~John met __ today~~.

Levels of parsing difficulty imposed by the grammar

Each level requires adding new schemas to the search

- Function/argument combination: *Ann (saw Bill)*
- Covert movement (Q scope): *Someone saw everyone*
- Overt movement: *Who did Ann see __?*
- Parasitic scope: *Ann and Bill read the same books*

- Sluicing

Lambek's substructural logic NL: the logic of external merge

Without Exchange, '$\supset$' splits into '$\backslash$' and '$/$'

- **Formulas:** $\mathcal{F} \quad = \quad \mathsf{DP} \quad | \quad \mathsf{S} \quad | \quad \mathcal{F}\backslash\mathcal{F} \quad | \quad \mathcal{F}/\mathcal{F}$
- **Structures:** $\mathcal{S} \quad = \quad \mathcal{F} \quad | \quad \mathcal{S} \bullet \mathcal{S}$
- **Sequents:** $\mathcal{S} \vdash \mathcal{F}$
- **Logical rules:**

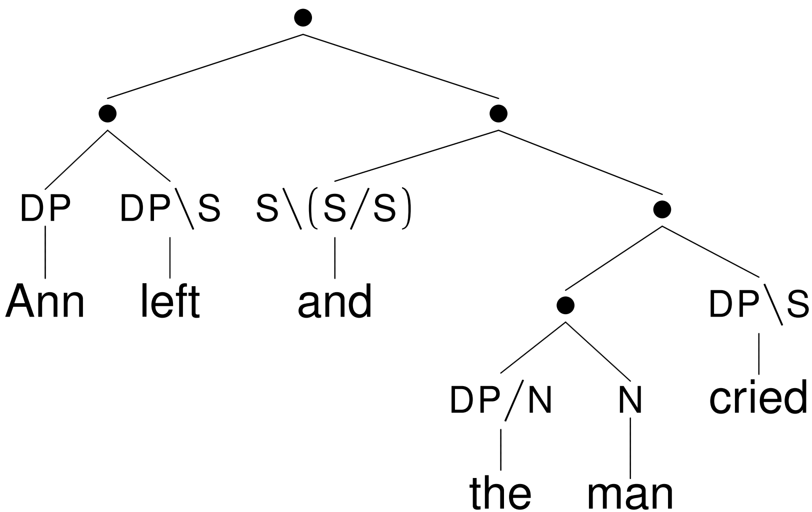
$$\frac{\Gamma \vdash A \quad \Sigma[B] \vdash C}{\Sigma[\Gamma \bullet A\backslash B] \vdash C}\ \backslash L \qquad\qquad \frac{A \bullet \Gamma \vdash B}{\Gamma \vdash A\backslash B}\ \backslash R$$

$$\frac{\Gamma \vdash A \quad \Sigma[B] \vdash C}{\Sigma[B/A \bullet \Gamma] \vdash C}\ /L \qquad\qquad \frac{\Gamma \bullet A \vdash B}{\Gamma \vdash B/A}\ /R$$

Intuitionistic (single formula in the consequent); implicative fragment; Structural rules: none! (Cut baked in)

How context notation works in inference rules

- Capital Greek letters (Δ, Γ, Σ) stand for complete structures
- '$\Sigma[\Delta]$' $\equiv$ Σ containing a distinguished instance of Δ
- '$\Sigma[\Gamma \bullet A \backslash B]$' matches the structure below in two ways:
  - [Ann • DP\S] • (and ((the • man) • cried))
  - (Ann • left) • (and • [(the • man) • DP\S])

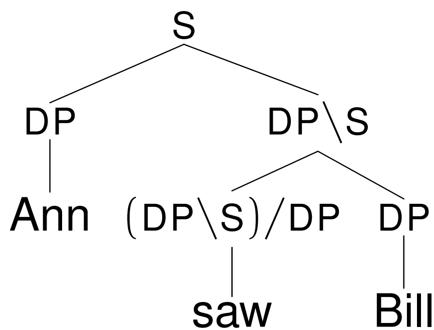


An example derivation of *Ann saw Bill*

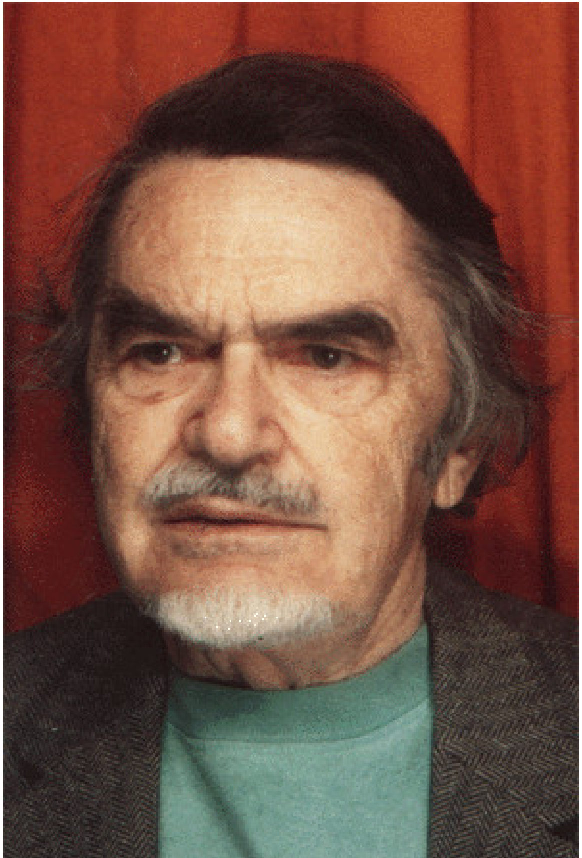
(3)

$$\dfrac{\dfrac{\mathsf{DP} \vdash \mathsf{DP} \qquad \dfrac{\mathsf{DP} \vdash \mathsf{DP} \qquad \mathsf{S} \vdash \mathsf{S}}{\mathsf{DP} \bullet \mathsf{DP}\backslash\mathsf{S} \vdash \mathsf{S}}\,\backslash L}{\mathsf{DP} \bullet ((\mathsf{DP}\backslash\mathsf{S})/\mathsf{DP} \bullet \mathsf{DP}) \vdash \mathsf{S}}\,/L}{\text{Ann} \bullet (\text{saw} \bullet \text{Bill}) \vdash \mathsf{S}}\,\text{LEX}$$

(4)



Gentzen sequent presentation; payoff: simple decidability [=termination] argument

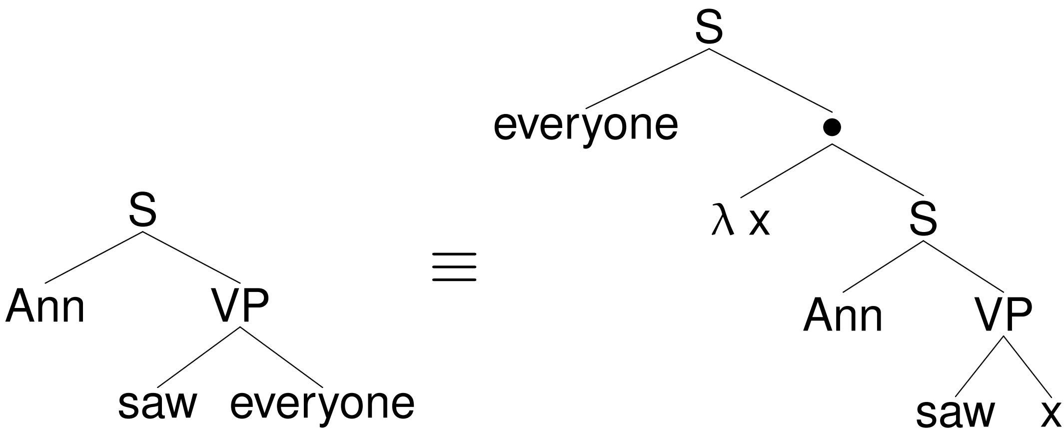


Joachim Lambek

Quantifier Raising as a logical inference

- Montague 1973: Quantifying In: (3065 citations)
- May 1978,1985: Quantifier Raising (QR): (3286 citations)

$$\text{Montague}\downarrow \qquad \frac{\text{everyone}(\lambda x.\text{Ann saw } x) \vdash S}{\text{Ann saw everyone } \vdash S} \qquad \uparrow \text{May}$$

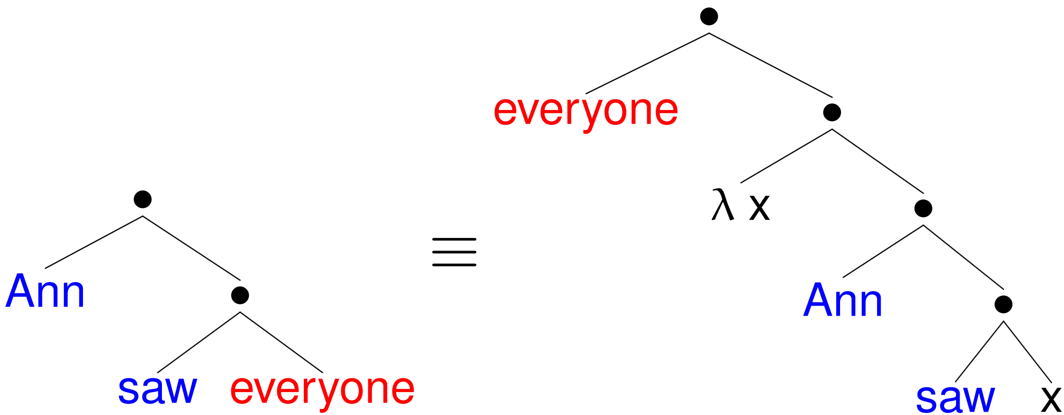


Richard Montague

Robert May

Quantifier Raising as a structural rule

Quantifier Raising: $\Sigma[\Delta] \equiv \Delta \bullet \lambda x \Sigma[x]$



The λ-calculus

- **Variables:** $\mathcal{V} \;=\; x \;|\; y \;|\; z \;|\; \ldots$
- **Terms:** $\mathcal{T} \;=\; \mathcal{V} \;|\; \lambda\mathcal{V}\mathcal{T} \;|\; \mathcal{T}\mathcal{T}$

Exx: x, y, λxx, λxy, $((\lambda xy)z$, $((\lambda x(xx))(\lambda x(xx)))$, ...

Reduction:

$$((\lambda\alpha M)N) \rightsquigarrow_\beta M\{\alpha \mapsto N\}$$

Example:

$$(((\lambda x(\lambda y(yx)))a)b) \rightsquigarrow ((\lambda y(ya))b) \rightsquigarrow (ba)$$

$\mathbf{NL}_{\mathrm{QR}}$: NL with Quantifier Raising

- **Variables:** $\mathcal{V} \quad = \quad x \quad | \quad y \quad | \quad z \quad | \quad \ldots$
- **Formulas:** $\mathcal{F} \quad = \quad \mathsf{DP} \quad | \quad \mathsf{S} \quad | \quad \mathcal{F}\backslash\mathcal{F} \quad | \quad \mathcal{F}/\mathcal{F}$
- **Structures:** $\mathcal{S} \quad = \quad \mathcal{F} \quad | \quad \mathcal{S} \bullet \mathcal{S} \quad | \quad \mathcal{V} \quad | \quad \lambda\mathcal{V}\mathcal{S}$
- **Sequents:** $\mathcal{S} \vdash \mathcal{F}$
- **Logical rules:**

$$\frac{\Gamma \vdash A \quad \Sigma[B] \vdash C}{\Sigma[\Gamma \bullet A\backslash B] \vdash C}\ \backslash L \qquad \frac{A \bullet \Gamma \vdash B}{\Gamma \vdash A\backslash B}\ \backslash R$$

$$\frac{\Gamma \vdash A \quad \Sigma[B] \vdash C}{\Sigma[B/A \bullet \Gamma] \vdash C}\ /L \qquad \frac{\Gamma \bullet A \vdash B}{\Gamma \vdash B/A}\ /R$$

- **Structural rule:** $\Sigma[\Delta] \equiv_{\mathrm{QR}} \Delta \bullet \lambda x \Sigma[x]$

x chosen fresh; rule to be adjusted slightly below

$$
\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{c}\vdots\\ \text{Ann} \bullet ((\text{gave} \bullet \textsc{dp}) \bullet \text{cookies}) \vdash \textsc{s}\end{array}}{\textsc{dp} \bullet \lambda x(\text{Ann} \bullet ((\text{gave} \bullet x) \bullet \text{cookies})) \vdash \textsc{s}}\,\color{red}{\text{QR}}}{\lambda x(\text{Ann} \bullet ((\text{gave} \bullet x) \bullet \text{cookies})) \vdash \textsc{dp}\backslash\textsc{s}}\,\backslash R \qquad \textsc{s} \vdash \textsc{s}}{\textsc{s}/(\textsc{dp}\backslash\textsc{s}) \bullet \lambda x(\text{Ann} \bullet ((\text{gave} \bullet x) \bullet \text{cookies})) \vdash \textsc{s}}\,\backslash L}{\dfrac{\text{everyone} \bullet \lambda x(\text{Ann} \bullet ((\text{gave} \bullet x) \bullet \text{cookies})) \vdash \textsc{s}}{\text{Ann} \bullet (\text{gave} \bullet (\text{everyone}) \bullet \text{cookies}) \vdash \textsc{s}}\,\color{red}{\text{QR}}}\,\text{LEX}
$$

- Reading bottom up (direction of proof search):
- First, QR *everyone*
- more reasoning
- Then, QR the DP trace back into the original position
- QR versus QI? Both essential!

Meaning: Curry-Howard labeling for $\mathbf{NL}_{\mathrm{QR}}$

$$\frac{}{a : A \vdash a : A}\ \text{Axiom}$$

$$\frac{\Gamma \vdash a{:}A \qquad \Sigma[b{:}B] \vdash M{:}C}{\Sigma[\Gamma \bullet f{:}(A\backslash B)] \vdash M\{b \mapsto f(a)\}{:}C}\ \backslash L \qquad \frac{a{:}A \bullet \Gamma \vdash N{:}B}{\Gamma \vdash (\lambda a N){:}(A\backslash B)}\ \backslash R$$

'$M\{b \mapsto f(a)\}$' means 'the term just like M, but with each occurrence of b replaced by $f(a)$'.

Summary of Curry Howard and example

- identity axioms are labeled with atomic symbols
- formulas are labeled with lambda terms
- the computat'l content of L inferences is function application
- the computat'l content of R inferences is lambda abstraction
- structural rules do not affect the Curry-Howard labeling

$$
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b:\mathsf{DP}\vdash b:\mathsf{DP}
\qquad
\dfrac{a:\mathsf{DP}\vdash a:\mathsf{DP}\qquad p:\mathsf{S}\vdash p:\mathsf{S}}{a:\mathsf{DP}\bullet f:\mathsf{DP}\backslash\mathsf{S}\vdash fa:\mathsf{S}}\,\backslash L
}{a:\mathsf{DP}\bullet(g:(\mathsf{DP}\backslash\mathsf{S})/\mathsf{DP}\bullet b:\mathsf{DP})\vdash gba:\mathsf{S}}\,/L
}{b:\mathsf{DP}\circ\lambda x(a:\mathsf{DP}\bullet(g:(\mathsf{DP}\backslash\mathsf{S})/\mathsf{DP}\bullet x))\vdash gba:\mathsf{S}}\,\mathsf{QR}
}{\lambda x(a:\mathsf{DP}\bullet(g:(\mathsf{DP}\backslash\mathsf{S})/\mathsf{DP}\bullet x))\vdash\lambda b.gba:\mathsf{DP}\backslash\!\!\backslash\mathsf{S}}\,\backslash\!\!\backslash R
\qquad
q:\mathsf{S}\vdash q:\mathsf{S}
}{Q:\mathsf{S}/\!\!/(\mathsf{DP}\backslash\!\!\backslash\mathsf{S})\circ\lambda x(a:\mathsf{DP}\bullet(g:(\mathsf{DP}\backslash\mathsf{S})/\mathsf{DP}\bullet x))\vdash Q(\lambda b.gba):\mathsf{S}}\,/\!\!/L
}{a:\mathsf{DP}\bullet(g:(\mathsf{DP}\backslash\mathsf{S})/\mathsf{DP}\bullet Q:\mathsf{S}/\!\!/(\mathsf{DP}\backslash\!\!\backslash\mathsf{S}))\vdash Q(\lambda b.gba):\mathsf{S}}\,\mathsf{QR}
}{a:\mathsf{Ann}\bullet(saw:\mathsf{saw}\bullet everyone:\mathsf{everyone})\vdash everyone(\lambda b.saw(b)(a)):\mathsf{S}}\,\mathsf{LEX}
$$

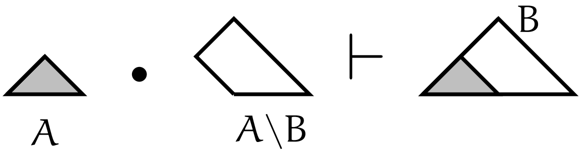
Next level: overt movement

Two steps:

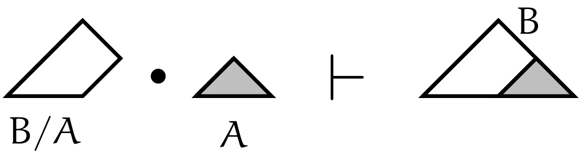
- First step: motivate second logical mode
  (5) Ann saw who? who: $\mathrm{Q} /\!\!/ (\mathrm{DP} \backslash\!\!\backslash \mathrm{S})$
  who **surrounded** by its argument

  (6) Who did ann see __? who: $\mathrm{Q} / (\mathrm{DP} \backslash\!\!\backslash \mathrm{S})$
  who **followed** by its argument
- Second step: recognize the presence of gaps ('__')
  - Gaps as silent pronouns (units in the logic)
  - Silent lambdas present in the final conclusion
  - Structural rule: $\Gamma[\Delta] \Rightarrow \lambda x \Gamma[\Delta \bullet x]$
  - Here: proof search strategy that preserves decidability: absorb into logical rules

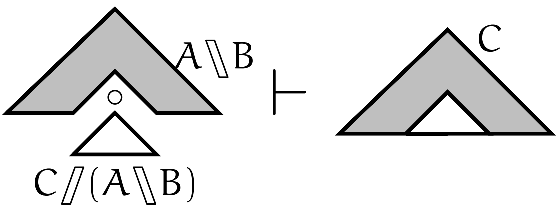
Movement: two modes of syntactic combination



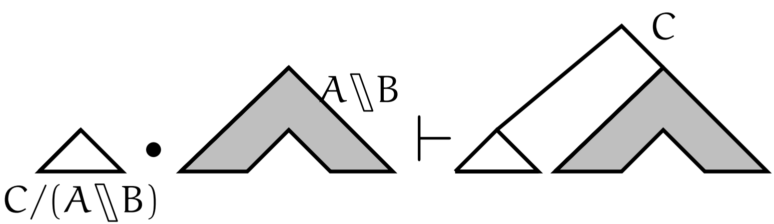
external merge, \: *Ann left*



external merge, /: *saw Ann*



covert movement: *Ann saw who?*



overt movement:
Who did Ann see __?

Adding a second mode to $\mathbf{NL}_{\mathrm{QR}}$: $\mathbf{NL}_{\lambda}$

- **Variables:** $\mathcal{V} \;=\; x \mid y \mid z \mid \ldots$
- **Formulas:** $\mathcal{F} = \;\mathsf{DP} \mid \mathsf{S} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid \mathcal{F}\backslash\!\!\backslash\mathcal{F} \mid \mathcal{F}/\!\!/\mathcal{F}$
- **Structures:** $\mathcal{S} \;=\; \mathcal{F} \mid \mathcal{S}\bullet\mathcal{S} \mid \mathcal{S}\circ\mathcal{S} \mid \mathcal{V} \mid \lambda\mathcal{V}\mathcal{S}$
- **Sequents:** $\mathcal{S} \vdash \mathcal{F}$
- **Logical rules:**

$$\frac{\Gamma\vdash A \quad \Sigma[B]\vdash C}{\Sigma[\Gamma\bullet A\backslash B]\vdash C}\backslash L \quad \frac{A\bullet\Gamma\vdash B}{\Gamma\vdash A\backslash B}\backslash R \qquad \frac{\Gamma\vdash A \quad \Sigma[B]\vdash C}{\Sigma[\Gamma\circ A\backslash\!\!\backslash B]\vdash C}\backslash\!\!\backslash L \quad \frac{A\circ\Gamma\vdash B}{\Gamma\vdash A\backslash\!\!\backslash B}\backslash\!\!\backslash R$$

$$\frac{\Gamma\vdash A \quad \Sigma[B]\vdash C}{\Sigma[B/A\bullet\Gamma]\vdash C}/L \quad \frac{\Gamma\bullet A\vdash B}{\Gamma\vdash B/A}/R \qquad \frac{\Gamma\vdash A \quad \Sigma[B]\vdash C}{\Sigma[B/\!\!/A\circ\Gamma]\vdash C}/\!\!/L \quad \frac{\Gamma\circ A\vdash B}{\Gamma\vdash B/\!\!/A}/\!\!/R$$

- **Structural rule:** $\Sigma[\Delta] \equiv_{\lambda} \Delta\circ\lambda x\Sigma[x]$

First presented in print in Barker 2007, Parasitic Scope. *L&P*

Decidable proof search with covert movement

$$\dfrac{\dfrac{\lambda x\Gamma[x] \vdash A \qquad \Sigma[B] \vdash C}{\Sigma[B /\!\!/ A \circ \lambda x\Gamma[x]] \vdash C}\,/\!\!/L}{\Sigma[\Gamma[B /\!\!/ A]] \vdash C}\,\lambda \quad \equiv \quad \dfrac{\lambda x\Gamma[x] \vdash A \qquad \Sigma[B] \vdash C}{\Sigma[\Gamma[B /\!\!/ A]] \vdash C}\,/\!\!/L_\lambda$$

$$\dfrac{\dfrac{\Gamma[A] \vdash B}{A \circ \lambda x\Gamma[x] \vdash B}\,\lambda}{\lambda x\Gamma[x] \vdash A \backslash\!\!\backslash B}\,\backslash\!\!\backslash R \quad \equiv \quad \dfrac{\Gamma[A] \vdash B}{\lambda x\Gamma[x] \vdash A \backslash\!\!\backslash B}\,\backslash\!\!\backslash R_\lambda$$

Example of covert movement for *Ann saw everyone*:

$$\dfrac{\dfrac{\vdots}{\dfrac{\mathsf{DP} \bullet ((\mathsf{DP}\backslash \mathsf{S})/\mathsf{DP} \bullet \mathsf{DP}) \vdash \mathsf{S}}{\lambda x(\mathsf{DP} \bullet ((\mathsf{DP}\backslash \mathsf{S})/\mathsf{DP} \bullet x)) \vdash \mathsf{DP} \backslash\!\!\backslash \mathsf{S}}\,/\!\!/R_\lambda} \qquad \mathsf{S} \vdash \mathsf{S}}{\mathsf{DP} \bullet ((\mathsf{DP}\backslash \mathsf{S})/\mathsf{DP} \bullet \mathsf{S} /\!\!/ (\mathsf{DP} \backslash\!\!\backslash \mathsf{S})) \vdash \mathsf{S}}\,\backslash\!\!\backslash L_\lambda$$

Decidability argument

$$\frac{\Gamma \vdash A \quad \Sigma[B] \vdash C}{\Sigma[\Gamma \bullet A\backslash B] \vdash C}\backslash L \quad \frac{A \bullet \Gamma \vdash B}{\Gamma \vdash A\backslash B}\backslash R \qquad \frac{\Gamma \vdash A \quad \Sigma[B] \vdash C}{\Sigma[\Gamma \circ A\mathbin{\backslash\!\!\backslash} B] \vdash C}\mathbin{\backslash\!\!\backslash} L \quad \frac{A \circ \Gamma \vdash B}{\Gamma \vdash A\mathbin{\backslash\!\!\backslash} B}\mathbin{\backslash\!\!\backslash} R$$

$$\frac{\Gamma \vdash A \quad \Sigma[B] \vdash C}{\Sigma[B/A \bullet \Gamma] \vdash C}/L \quad \frac{\Gamma \bullet A \vdash B}{\Gamma \vdash B/A}/R \qquad \frac{\Gamma \vdash A \quad \Sigma[B] \vdash C}{\Sigma[B/\!\!/A \circ \Gamma] \vdash C}/\!\!/L \quad \frac{\Gamma \circ A \vdash B}{\Gamma \vdash B/\!\!/A}/\!\!/R$$

Covert movement: $$\frac{\lambda x\Gamma[x] \vdash A \qquad \Sigma[B] \vdash C}{\Sigma[\Gamma[B/\!\!/A]] \vdash C}/\!\!/L_\lambda \quad \frac{\Gamma[A] \vdash B}{\lambda x\Gamma[x] \vdash A\mathbin{\backslash\!\!\backslash} B}\mathbin{\backslash\!\!\backslash} R_\lambda$$

- The structural rule has been absorbed into the logical rules
- Every logical rule
  - has the subformula property (lambdas are structural)
  - premises contain one fewer connective than conclusion
- Decidability follows

Reconstruction example involving quantificational binding

(7) [I found out] which friendof hers everyone saw

(8) $(\textbf{which}(\lambda x(\textbf{everyone}(\textbf{he}(\lambda y(\lambda z((x(\textbf{friend.of}\,y))(\lambda a((\textbf{saw}\,a)z)))))))))$

```
                  DP |- DP
                  N |- N
                N/DP * DP |- N _/L
                      DP |- DP
                        DP |- DP
                        S |- S
                      DP * DP\S |- S _\L
                    DP * ((DP\S)/DP * DP) |- S _/L
                  DP * (DP\S)/DP |- DP\\S _\R_rgap
                  S |- S
                S/(DP\\S) * (DP * (DP\S)/DP) |- S _/L
              ((S/(DP\\S))/N * (N/DP * DP)) * (DP * (DP\S)/DP) |- S _/L
            ((S/(DP\\S))/N * (N/DP * DP)) * ((I * ((DP\S)/DP * B)) * C) |- DP\\S _\\R_l
          ((S/(DP\\S))/N * ((N/DP * (I * C)) * C)) * (((I * ((DP\S)/DP * B)) * C) * B)
          DP\\S |- DP\\S
        ((S/(DP\\S))/N * (N/DP * (DP\\S)//(DP\\(DP\\S)))) * ((I * ((DP\S)/DP * B)) * C)
        S |- S
      ((S/(DP\\S))/N * (N/DP * (DP\\S)//(DP\\(DP\\S)))) * (S//(DP\\S) * (DP\S)/DP) |- S
    (I * ((N/DP * (DP\\S)//(DP\\(DP\\S))) * B)) * ((S//(DP\\S) * (DP\S)/DP) * B) |- ((S
    Q |- Q
  (Q//(((S/(DP\\S))/N)\\S) * (N/DP * (DP\\S)//(DP\\(DP\\S)))) * (S//(DP\\S) * (DP\S)/DP
```

Remnant movement

Requires postulating silent functional heads like Acc or W.
From an example worked out in detail by Stabler:

(9) believe it : $((\mathrm{I}(\lambda x((\mathrm{I}x)\textbf{it})))(\lambda x(\textbf{believe}\,x)))$: WP

(10) (Reduces to **believe**(**it**))

```
        DP\\VP |- DP\\VP
          DP |- DP
          AccP |- AccP
        DP * DP\AccP |- AccP _\L
      DP * ((DP\AccP)/(DP\\VP) * DP\\VP) |- AccP _/L
    DP * (DP\AccP)/(DP\\VP) |- (DP\\VP)\\AccP _\R_rgap
          DP |- DP
          VP |- VP
        VP/DP * DP |- VP _/L
      VP/DP |- DP\\VP _\R_rgap
      WP |- WP
    VP/DP * (DP\\VP)\WP |- WP _\L
  VP/DP * (((DP\\VP)\WP)/((DP\\VP)\\AccP) * (DP * (DP\AccP)/(DP\\VP))) |- WP _/L
```

Reasoning about time cost

- Proof depth n is equal to the number of logical connectives
- n proportional to the number of words
- Simple proof search strategy:
  - choose a constituent structure (group the words)
  - try to match every rule with every connective
- Worst-case time cost is factorial: $n * (n-1) * (n-2) * ... * 1$
- Practical time cost: much better (future work)
- One reason for optimism: Lambek equivalent to CFG
- Goal for today: figure the cost imposed by each rule
- Lambek rules: given that we've already selected a connective, constructing the premise has constant cost
- $\backslash\!\backslash R_\lambda$: cost of beta reduction; using cleverness, constant
- $/\!/ L_\lambda$: need to choose Γ, i.e., need to choose how big the scope domain will be. Worst case: every node is a potential scope-taking position: order n possibilities to consider

Lambek rules: constant cost per connective
Covert movement: factor of n

Decidable proof search, adding overt movement

Same strategy: absorb gap postulation into the logical rules:

$$(11) \qquad \frac{\Gamma[A \bullet \Delta] \vdash B}{\Gamma[\Delta] \vdash A \backslash\!\!\backslash B}\, \backslash\!\!\backslash R_{lgap} \qquad\qquad \frac{\Gamma[\Delta \bullet A] \vdash C}{\Gamma[\Delta] \vdash A \backslash\!\!\backslash B}\, \backslash\!\!\backslash R_{rgap}$$

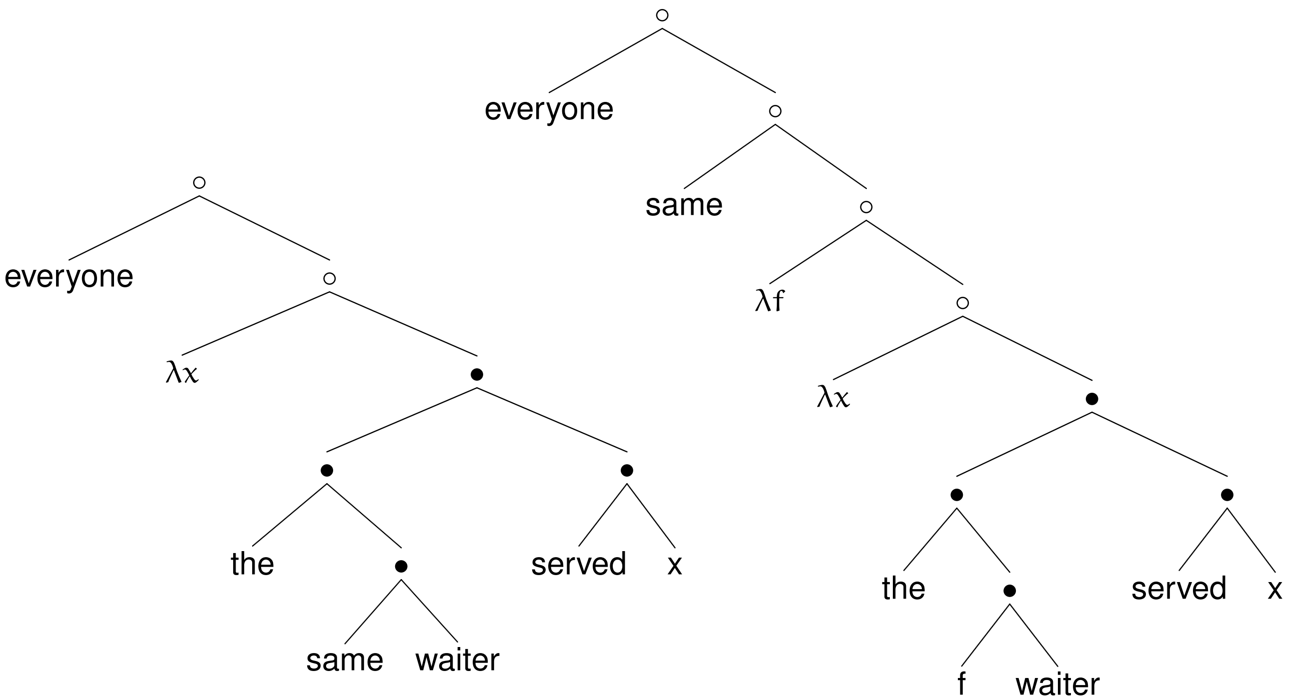
Derivation of *Who did Ann see __?* (ignoring *did* for simplicity):

$$(12) \qquad \cfrac{\cfrac{\cfrac{\cfrac{\vdots}{\text{Ann} \bullet (\text{see} \bullet \mathsf{DP}) \vdash \mathsf{S}}}{\text{Ann} \bullet \text{see} \vdash \mathsf{DP} \backslash\!\!\backslash \mathsf{S}}\, \backslash\!\!\backslash R_{gap} \qquad \mathsf{Q} \vdash \mathsf{Q}}{\mathsf{Q}/(\mathsf{DP} \backslash\!\!\backslash \mathsf{S}) \bullet (\text{Ann} \bullet \text{see}) \vdash \mathsf{Q}}\, /L}{\text{who} \bullet (\text{Ann} \bullet \text{see}) \vdash \mathsf{Q}}\, \mathsf{LEX}$$

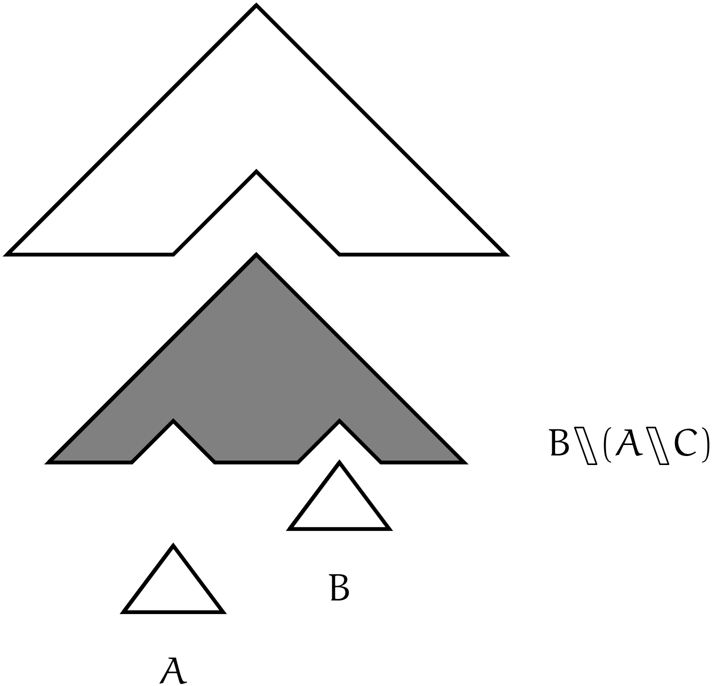
- Decidability argument continues to hold
- Cost: gap could be anywhere: possible sites $\sim n$

Decidable implementation not complete: parasitic scope

(13) The same waiter served everyone. [Heim, Stump]



Parasitic scope in schematic format



A = Antecedent; B = parasitic scope-taker

Ann told (A = everyone) the (B = same) story.

$\lambda y \lambda x(\text{ann} \bullet ((\text{told} \bullet y) \bullet (\text{the} \bullet (x \bullet \text{story})))) \vdash \text{DP}\backslash\!\!\backslash(\text{DP}\backslash\!\!\backslash \text{S})$

Other phenomena with a parasitic scope analysis

(14) a. Anaphora: Morrill, Fadda & Valentín 2011
b. *he*: $(\mathrm{DP}\backslash\!\backslash \mathrm{S}) /\!/ (\mathrm{DP}\backslash\!\backslash(\mathrm{DP S}))$
c. Everyone thinks he is smart.
d. $\text{everyone} \circ (\text{he} \circ \lambda y \lambda x (x \bullet (\text{thinks} \bullet (y \bullet (\text{is} \bullet \text{smart}))))) \vdash \mathrm{S}$

(15) a. *Average*: Kennedy and Stanley 2009
b. The average American has 2.3 kids.
c. $2.3 \circ (\text{avg} \circ \lambda f \lambda n ((\text{the} \bullet (f \bullet \text{Am'n})) \bullet (\text{has} \bullet (n \bullet \text{kids}))))$

(16) a. Fancy coordination: Kubota & Levine (various papers)
b. I said the same thing to Terry on Mon and to Kim on Tue.
c. $\neq$ I said the same thing to Terry on Monday and ~~I said the same thing~~ to Kim on Tuesday.

(17) a. Remnant comparatives: Pollard and Smith 2013
b. Ann owes Bill more than Clara.

Challenge: two points of discontinuity

$$\frac{\lambda y \lambda x \Gamma[x][y] \vdash A \qquad \Sigma[\Pi \circ B] \vdash C}{\Sigma[\Gamma[B /\!\!/ A][\Pi]] \vdash C}\ \text{PARA}$$

(18) They saw the same dog.

```
        N |- N
          N |- N
            DP |- DP
              DP |- DP
              S |- S
            DP * DP\S |- S _\L
          DP * ((DP\S)/DP * DP) |- S _/L
        DP * ((DP\S)/DP * (DP/N * N)) |- S _/L
      DP * ((DP\S)/DP * (DP/N * (N/N * N))) |- S _/L
    I * (((DP\S)/DP * (DP/N * (N/N * N))) * B) |- DP\\S _\\R_lam
  I * ((((DP\S)/DP * ((DP/N * ((I * (N * B)) * C)) * C)) * (B * B)) * C) |- (N/
    DP |- DP
    S |- S
  DP ** DP\\S |- S
DP * ((DP\S)/DP * (DP/N * ((DP\\S)//((N/N)\\(DP\\S)) * N))) |- S _//L_para]
```

Conjecture: two points of discontinuity suffice for natural lg.

Diagnosis

- In order to serve as antecedent, must take scope
- If a phrase doesn't force scope-taking, must find it anyway
- Two degrees of freedom: how big is the scope domain (Γ)
- and which constituent is the antecedent (Π).
- Cost for each potential application of the rule: n^2

Score: Lambek: constant
covert movement: n
overt movement: n
parasitic scope: n^2

Sluicing as parasitic scope

SLUICEGAP: $\lambda k \lambda P.kPP : ((DP\backslash\backslash S)\backslash\backslash S)/\!/((DP\backslash\backslash S)\backslash\backslash((DP\backslash\backslash S)\backslash\backslash S))$

(16) Someone left, but I don't know who SLUICEGAP.

The continuation of *someone* relative to the clause *someone left* (i.e., $\lambda x(x \bullet \text{left})$) provides the semantic value for the sluice gap:

$$\dfrac{\dfrac{\dfrac{\dfrac{(\text{someone} \circ DP\backslash\backslash S) \bullet (\text{bidk} \bullet (\text{who} \bullet DP\backslash\backslash S)) \vdash S}{DP\backslash\backslash S \circ \lambda y((\text{someone} \circ y) \bullet (\text{bidk} \bullet (\text{who} \bullet DP\backslash\backslash S))) \vdash S}\,\lambda}{\lambda y((\text{someone} \circ y) \bullet (\text{bidk} \bullet (\text{who} \bullet DP\backslash\backslash S))) \vdash (DP\backslash\backslash S)\backslash\backslash S}\,\backslash\backslash R}{DP\backslash\backslash S \circ \lambda z\lambda y((\text{someone} \circ y) \bullet (\text{bidk} \bullet (\text{who} \bullet z))) \vdash (DP\backslash\backslash S)\backslash\backslash S}\,\lambda}{\lambda z\lambda y((\text{someone} \circ y) \bullet (\text{bidk} \bullet (\text{who} \bullet z))) \vdash (DP\backslash\backslash S)\backslash\backslash((DP\backslash\backslash S)\backslash\backslash S)}\,\backslash\backslash R$$

$$\dfrac{\dfrac{\dfrac{DP \bullet DP\backslash S \vdash S}{DP \circ \lambda x(x \bullet \text{left}) \vdash S}\,\lambda}{\lambda x(x \bullet \text{left}) \vdash DP\backslash\backslash S}\,\backslash\backslash R \qquad S \vdash S}{\lambda x(x \bullet \text{left}) \circ (DP\backslash\backslash S)\backslash\backslash S \vdash S}\,\backslash\backslash L$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\cdots \qquad \cdots}{\lambda x(x \bullet \text{left}) \circ (((DP\backslash\backslash S)\backslash\backslash S)/\!/((DP\backslash\backslash S)\backslash\backslash((DP\backslash\backslash S)\backslash\backslash S)) \circ \lambda z\lambda y((\text{someone} \circ y) \bullet (\text{bidk} \bullet (\text{who} \bullet z)))) \vdash S}\,/\!/L}{\lambda x(x \bullet \text{left}) \circ (\text{SLUICEGAP} \circ \lambda z\lambda y((\text{someone} \circ y) \bullet (\text{bidk} \bullet (\text{who} \bullet z)))) \vdash S}\,\text{LEX}}{\lambda x(x \bullet \text{left}) \circ \lambda y((\text{someone} \circ y) \bullet (\text{bidk} \bullet (\text{who} \bullet \text{SLUICEGAP}))) \vdash S}\,\lambda}{(\text{someone} \circ \lambda x(x \bullet \text{left})) \bullet (\text{bidk} \bullet (\text{who} \bullet \text{SLUICEGAP})) \vdash S}\,\lambda}{(\text{someone} \bullet \text{left}) \bullet (\text{bidk} \bullet (\text{who} \bullet \text{SLUICEGAP})) \vdash S}\,\lambda$$

bidk = but-I-don't-know

Arguments that sluicing is syntactic

Deep versus surface anaphora

(19) [Holds up a cigarette] I don't know why not.

(20) [Holds up an earing] ??I don't know whose.

Claim: Verb phrase elipsis doesn't require a linguistic antecedent (deep anaphora), but sluicing does (surface anaphora)

Case matching: the case of the WH element in the sluice must match the case of the wh correlate.

(4)	Er	will	jemandem	schmeicheln,	aber	sie	wissen	nicht,	{*wen / wem}.
	he	wants	someone.DAT	flatter	but	they	know	not	{who.ACC / who.DAT}

‘He wants to flatter someone, but they don't know who.’

(5)	Er	will	jemanden	loben,	aber	sie	wissen	nicht,	{wen / *wem}.
	he	wants	someone.ACC	praise	but	they	know	not	{who.ACC / who.DAT}

‘He wants to praise someone, but they don't know who.’

Problem: the implementation doesn't cover sluicing

Solution: add a special-case derived inference rule for sluicing:

$$\frac{\lambda x \lambda y \Gamma[\Delta \circ y][z] \vdash A \qquad \Sigma[(\lambda x \Pi[x]) \circ B] \vdash C}{\Sigma[\Gamma[B /\!\!/ A][\Pi[\Delta]]] \vdash C} \text{ SLUICING}$$

(21) John left, but I don't know who (else).

```
        DP\\S |- DP\\S :Ax
        Q |- Q :Ax
      Q/(DP\\S) * DP\\S |- Q :/L
          DP |- DP :Ax
          S |- S :Ax
        DP ** DP\\S |- S :\\L
        S |- S :Ax
      (DP ** DP\\S) * S\S |- S :\L
    (DP ** DP\\S) * ((S\S)/Q * (Q/(DP\\S) * DP\\S)) |- S :/L
            DP |- DP :Ax
            S |- S :Ax
          DP * DP\S |- S :\L
        DP ** ((C * I) * DP\S) |- S :b
      (C * I) * DP\S |- DP\\S :\\R
      S |- S :Ax
    ((C * I) * DP\S) ** (DP\\S)\\S |- S :\\L
  (DP * DP\S) * ((S\S)/Q * (Q/(DP\\S) * ((DP\\S)\\S)//((DP\\S)\\((DP\\S)\\S)))) |- S :Sluicing

((sg (\x (\y ((bidk (who x)) (y John))))) (\z (left z)))
```

Comparing cases

$$\frac{\lambda x \Gamma[x] \vdash A \qquad \Sigma[B] \vdash C}{\Sigma[\Gamma[B /\!/ A]] \vdash C} /\!/ L_\lambda$$

$$\frac{\lambda y \lambda x \Gamma[x][y] \vdash A \qquad \Sigma[\Pi \circ B] \vdash C}{\Sigma[\Gamma[B /\!/ A][\Pi]] \vdash C} \text{PARA}$$

$$\frac{\lambda x \lambda y \Gamma[\Delta \circ y][z] \vdash A \qquad \Sigma[(\lambda x \Pi[x]) \circ B] \vdash C}{\Sigma[\Gamma[B /\!/ A][\Pi[\Delta]]] \vdash C} \text{SLUICING}$$

- For normal scope-taking, look for Γ containing $B /\!/ A$
- For parasitic scope-taking, look also for a Π
- For sluicing, look also for Δ within Π
- Select Γ, or select Γ and Π, or select Γ, Π, and Δ.
- Cost per application of rule: n, n^2, n^3

Conclusions

- How precisely does cost per rule applicaton fit into overall time cost?
- What is the logic (including a sound and complete model theory) for the overt movement rules?
- Are there any natural language phenomena that require additional special case rules?
- Parasitic scope (pronominal anaphora and *same*), and sluicing are both efffortless for native speakers. Is it more efficient to do a rough parse, and then do search followed by checking, rather than an exaustive parse?

THANKS!

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