Taking Scope with Continuations and Dependent Types

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Dependencies are ubiquitously used and interpreted by natural language speakers.

- Plural unbound anaphora
- Donkey sentences
- Inverse linking constructions
- Possessive weak definites
- Long-distance indefinites
- Spray-load constructions

Unbound anaphora refers to instances where anaphoric pronouns occur outside the syntactic scopes of their quantifier antecedents

(1) Every man loves a woman. They (each) kiss them.

- The way to understand the second (anaphoric) sentence is that every man kisses the women he loves rather than those loved by someone else.
- The first sentence must introduce a dependency between each of the men and the women they love that can be elaborated upon in further discourse.

Inverse linking constructions refer to complex DPs which contain a quantified NP (QP), as in (2)

(2) a representative of every country

- ILC in (2) can be understood to mean that there is a potentially different representative for each country
  ✓ *every country* $>$ *a representative*
- The relational noun *representative* introduces a dependency between each of the countries and the representatives of that country.

Outline

- Semantic system with dependent types
  - Main features
    - Dependent types
    - Type-theoretic notion of context
    - Quantification over fibers
  - Common nouns (sortal and relational), QPs and predicates
- Applications - scopal phenomena
  - Inverse linking
  - Spray-load constructions
- Integrating dependent type semantics into a continuation-passing framework.

- The idea of having just one universe in first order models originated with Frege and is widely adopted in mathematics (as it fits well the mathematical/logical practice).
- But we can have more than just one type of elements (as is common practice in programming languages).
- The variables of our system are always typed: $x : X, y : Y, \ldots$
- Types are interpreted as sets: $\|X\|, \|Y\|, \ldots$

- Types can depend on the variables of other types: if x is a variable of the type X, we can have type $Y(x)$ depending on the variable x.
- The fact that Y is a type depending on X can be modeled as a function $\pi : \|Y\| \to \|X\|$

$$\begin{tikzcd} \|Y\| \arrow[d, "\pi"] \\ \|X\| \end{tikzcd}$$

so that each type $Y(x)$ is interpreted as the fiber $\|Y\|(a)$ of π over $a \in \|X\|$ (the inverse image of $\{a\}$ under π).

When we decide to have many (dependent) types, we need contexts to keep track of the typing of variables

$$\Gamma = x : X, y : Y(x), z : Z(x, y), u : U, \ldots$$

... and we consider formulas/expressions only in contexts.

Context is a partially ordered set of type declarations of the (individual) variables such that the declaration of a variable x of type X precedes the declaration of a variable y of type $Y(x)$.

Montage-Style Semantics

- Sortal nouns (e.g. *man*) are interpreted as one-place relations (expressions of type $\langle e, t \rangle$).
- Relational nouns (e.g. *representative*) are interpreted as two-place relations (expressions of type $\langle e, \langle e, t \rangle \rangle$).

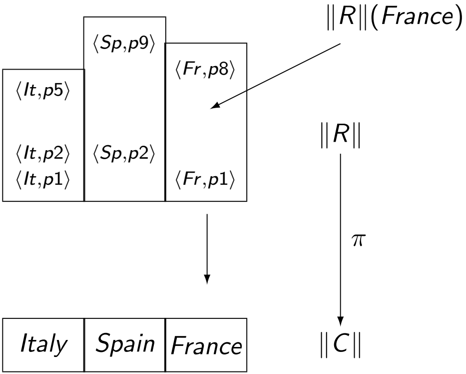
Dependent type analysis

- Sortal nouns (e.g. *man*) are interpreted as types.
- Relational nouns (e.g. *representative*) are interpreted as dependent types.

Semantics with DTs
Common nouns

If c is a variable of the type of countries C, there is a type $R(c)$ of the representatives of that country.

$$c : C, r : R(c)$$



If we interpret type C as the set $\|C\|$ of countries, then we can interpret R as the set of pairs:

$$\|R\| = \{\langle a, p\rangle : p \text{ is the person from the country } a\}$$

equipped with the projection $\pi : \|R\| \to \|C\|$.

The particular sets $\|R\|(a)$ of the representatives of the country a can be recovered as the fibers of this projection (the inverse images of $\{a\}$ under π):

$$\|R\|(a) = \{r \in \|R\| : \pi(r) = a\}.$$

The interpretation of the structure:

$$c : C, r : R(c)$$

gives us access to the sets (fibers) $\|R\|(a)$ of the representatives of the particular country a only.

To form the type of all representatives, we need to use Σ type constructor; $\Sigma_{c:C} R(c)$ is to be interpreted as the disjoint sum of fibers over elements in $\|C\|$:

$$\|\Sigma_{c:C} R(c)\| = \coprod_{a \in \|C\|} \pi^{-1}(a).$$

Polymorphic interpretation of quantifiers and predicates

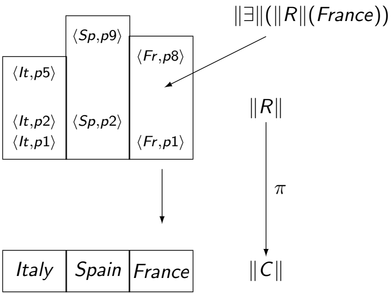
- Quantifiers and predicates are interpreted over various types (given in the context, e.g., *Country*, *Man*, ...), and not over the universe of all entities.
- A QP like *some country* is interpreted over the type *Country*, i.e. *some country* denotes the set of all non-empty subsets of the set of countries

$$\|\exists\|(\|Country\|) = \{X \subseteq \|Country\| : \ X \neq \emptyset\}.$$

Quantification over fibers

We can quantify over the fiber of the representatives of France, as in *some representative of France*:

$$\|\exists\|(\|R\|(France)) = \{X \subseteq \|R\|(France) : \; X \neq \emptyset\}.$$



Dependencies given in the context determine the relative scoping of quantifiers.

$$\Gamma = x : X, y : Y(x), z : Z(x, y), u : U, \ldots$$

✓ $Q_{1\,x:X} > Q_{2\,y:Y(x)}$
♯ $Q_{2\,y:Y(x)} > Q_{1\,x:X}$

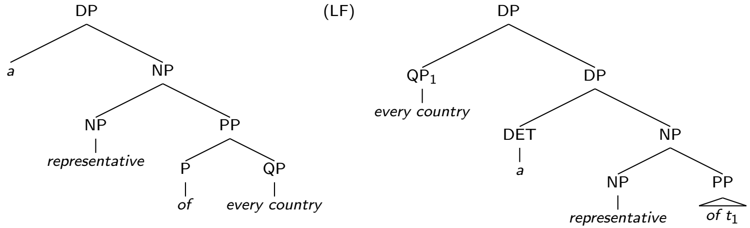
A global restriction on variables is that each occurrence of an indexing variable be preceded by a binding occurrence of that variable - free undeclared variables are illegal.

(2) a representative of every country

- ILC in (2) can be understood to mean that there is a potentially different representative for each country
  ✓ *every country* > *a representative* (inverse reading)
- ILC in (2) can be also understood to mean that there is some one person who represents all the countries
  ✓ *a representative* > *every country* (surface reading)

Standard LF-Movement Analysis

(SS) (LF)



An alternative non-movement analysis of inverse readings

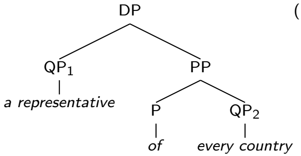
- Relational nouns (relational uses of sortal nouns) are modeled as dependent types.
- Here, *representative* (as in *a representative of every country*) is modeled as the dependent type $c : C, r : R(c)$. By quantifying over $c : C, r : R(c)$, we get the inverse ordering of quantifiers:

$$\forall_{c:C} \exists_{r:R(c)}.$$

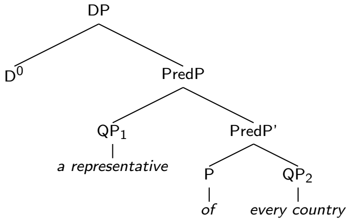
$$\sharp \quad \exists_{r:R(c)}\forall_{c:C}$$

- The interpretation where $\exists$ outscopes $\forall$ is not available because the indexing variable c (in $R(c)$) is outside the scope of the binding occurrence of that variable.
- By making the type of representatives dependent on (the variables of) the type of countries, our analysis forces the inversely linked reading without positing any extra scope mechanisms.

(IR-RA)



(IR-SC)

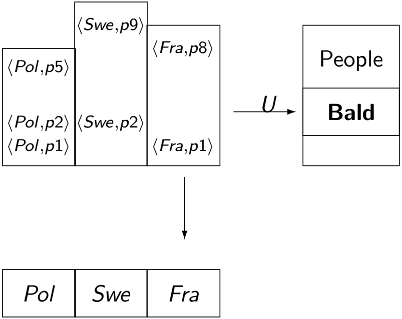


- The head nominal *representative* is modeled as the dependent type $c : C, r : R(c)$; the preposition *of* signals that *country* is a type on which *representative* depends; *country* is modeled as the type C.
- The complex DP *a representative of every country* is interpreted as the complex quantifier living on the set of all representatives

$$\|\forall_{c:C}\exists_{r:R(c)}\| = \{X \subseteq \|\Sigma_{c:C}R(c)\| :$$
$$\{a \in \|C\| : \{b \in \|R\|(a) : b \in X\} \in \|\exists\|(\|R\|(a))\} \in \|\forall\|(\|C\|)\}.$$

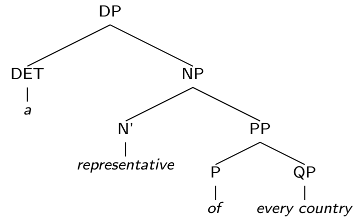
(3) **A representative of every country** is bald.

$$\|\forall_{c:C}\exists_{r:R(c)} Bald(r)\| = 1 \text{ iff } U^{-1}(\|Bald\|) \in \|\forall_{c:C}\exists_{r:R(c)}\|$$



Intuition: a person counts as a representative only in virtue of standing in a particular relationship with some country.

(SR)

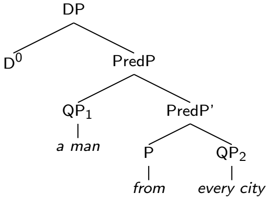


- The relational noun *representative* is now interpreted standardly as the predicate defined on $\|P(erson)\| \times \|C(ountry)\|$.
- The complex NP *representative of every country* is then interpreted as the type/set of individuals who represent all the countries $\{p : \{c : \langle p, c\rangle \in \|Represent\|\} \in \|\forall\|(\|C\|)\}$, and the DET *a* quantifies existentially over this set, yielding the surface ordering of quantifiers.

Problem: inverse scope readings are also available for ILCs involving sortal nouns, as in *a man from every city*.

Our solution is that a sortal noun like *man* can undergo a 'sortal-to-relational' shift, resulting in the (IR)-structure:

(IR-SC)



The relational use of the sortal noun *man* can be coerced by the presence of the locative preposition *from* - such prepositions specify the local position or origin of an entity and since entities do not occur at more than one place simultaneously, the dependency $c : C, m : M(c)$ is likely to be a preferred interpretation for *man from* (for any city, there is a set (fiber) of the men from that city).

Preposition puzzle: Why inverse readings are blocked with certain prepositions (e.g. *with*)?

(4) someone **with** every known skeleton key

- ILC in (4) can only be a statement about one person who happens to have every known skeleton key.

  ✓ *someone* $>$ *every known skeleton key*

  ♯ *every known skeleton key* $>$ *someone*

Solution: inverse readings are unavailable for ILCs with prepositions which induce dependencies corresponding to the surface ordering of the QPs.

- *a representative of (from) every country*:
  The 'dependent component' (*representative*) comes before the component on which it is dependent (*country*) - the dependency introduced, $c : C, r : R(c)$, forces the inversely linked interpretation.
- *a man with every key*:
  The potentially 'dependent component' (*key*) comes after the component on which it is dependent (*man*) - the dependency introduced, $m : M, k : K(m)$, corresponds to the surface ordering of the QPs.

$$\sharp \quad \forall_{k:K(m)} \exists_{m:M}$$

- By our global restriction on variables, the reading where $\forall$ outscopes $\exists$ is not available because the indexing variable m (in $K(m)$) is outside the scope of the binding occurrence of that variable.
- Thus, under the analysis proposed, the inverse interpretation is unavailable to the QP in the object position of *with*.

Difficulty: examples like *a problem with every account*

- *with* comes with a number of meanings, including:
  - 'having or possessing (something)',
  - 'accompanied by; accompanying',
- If the relation expressed is one of possession, as in our previous example, then the thing possessed depends on the possessor (as described above).
- If, however, the relation is that of accompanying, then the accompanying entity (problem) depends on the entity to be accompanied (account). Thus the dependency introduced is $a : A, p : P(a)$, forcing the inverse ordering of the QPs (in line with intuitions reported by native speakers).

(5) Maud draped a sheet over every chair

- Sentence (5) can be understood to mean that there is a potentially different sheet for every chair
  ✓ *every chair* $>$ *a sheet* (inverse reading)
- Sentence (5) can be also understood to mean that there is one sheet draped over all the chairs
  ✓ *a sheet* $>$ *every chair* (surface reading)

BUT

(6) Mary draped a chair with every sheet.

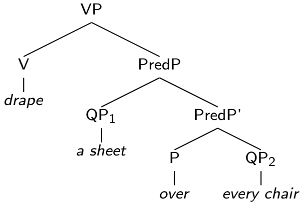
Sentence (6) exhibits frozen scope, i.e. only surface reading is possible
✓ *a chair* > *every sheet* (surface reading)

The inverse reading is disallowed in (6)
♯ *every sheet* > *a chair* (inverse reading)

Experimental work by Yining Nie, Structure vs competition: evidence from frozen scope in spray-load constructions, 2018.

Inverse reading

(VP-SC)



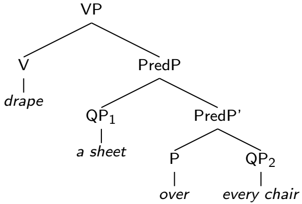
- The relational use of the sortal noun *sheet* is coerced by the presence of the locative preposition *over*, and the head nominal *sheet* is modeled accordingly as the dependent type $c : C, s : S(c)$.
- The predicate phrase *a sheet over every chair* is interpreted as the polyadic quantifier living on $\|\Sigma_{c:C} S(c)\|$ (an element of $\|\Sigma_{c:C} S(c)\|$ is a pair $\langle a, b\rangle$ such that $a \in \|C\|$ and $b \in \|S\|(a)$):

  $$\|\forall_{c:C}\exists_{s:S(c)}\| = \{R \subseteq \|\Sigma_{c:C} S(c)\| :$$

  $$\{a \in \|C\| : \{b \in \|S\|(a) : (a, b) \in R\} \in \|\exists\|(\|S\|(a))\} \in \|\forall\|(\|C\|)\}.$$

Surface reading

(VP-SC)



The predicate phrase *a sheet of every chair* is interpreted standardly as the polyadic quantifier living on $\|S\| \times \|C\|$

$$\|\exists_{s:S}\forall_{c:C}\| = \{R \subseteq \|S\| \times \|C\| :$$

$$\{a \in \|C\| : \{b \in \|S\| : (a, b) \in R\} \in \|\forall\|(\|C\|)\} \in \|\exists\|(\|S\|)\}.$$

Frozen scope puzzle: Why inverse readings are blocked with certain prepositions (e.g. *with*)?

(6) Mary draped a chair with every sheet.

Here, *with* comes with a meaning:

- indicating the instrument used to perform an action.
- indicating the material used for a purpose.

Inverse readings are only possible for constructions with (locative) prepositions that induce dependencies corresponding to the inverse ordering of the QPs.

(a) Mary gave a book to every student. $(\exists > \forall, \forall > \exists)$

BUT

(b) Mary gave a student every book. $(\exists > \forall, \sharp\forall > \exists)$

Interim conclusion

- In DP-internal small clauses and VP small clauses inverse readings are facilitated by dependencies.

Continuations

(7) **A representative of every country** missed a meeting.

✓ $\forall_{c:C}\exists_{r:R(c)}\ \exists_{m:M}$ (surface reading)

✓ $\exists_{m:M}\ \forall_{c:C}\exists_{r:R(c)}$ (inverse reading)

Interleaved interpretations are not possible for (7) (Larson's generalization, 1985).

Continuation semantics allows for the in situ analysis of (7).

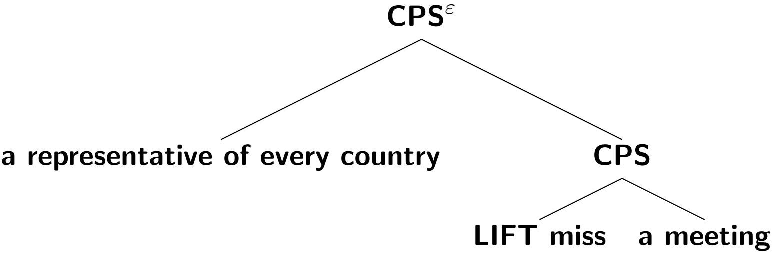
Integrating dependent type semantics into a continuation-passing framework

(7) **A representative of every country** missed a meeting.

- Predicate *miss* is defined on $\|P(erson)\| \times \|M(eeting)\|$. By taking the inverse image of this set under function U, $U^{-1}(\|P\| \times \|M\|)$, we get the predicate *miss* defined on the product of representatives and meetings $\|\Sigma R\| \times \|M\|$.
- In order to combine with QPs a predicate gets lifted ('continuized'), i.e., *miss* of type $\mathcal{P}(\Sigma R \times M)$ will be lifted to an expression of type $\mathcal{CP}(\Sigma R \times M)$.
- The two readings for (7) are then derived, using either (left or right) of the two **CPS** transforms.

Integrating dependent type semantics into a continuation-passing framework

(7) **A representative of every country** missed a meeting.



$$\mathbf{CPS}^{l}, \mathbf{CPS}^{r} : \mathcal{C}(\Sigma R) \times \mathcal{CP}(\Sigma R) \longrightarrow \mathcal{C}(\mathbf{t})$$

given, for $M \in \mathcal{C}(\Sigma R)$ and $N \in \mathcal{CP}(\Sigma R)$, by

$$\mathbf{CPS}^{l}(M, N) = \lambda c_{:\mathcal{P}(\mathbf{t})}.M(\lambda r_{:\Sigma R}.N(\lambda g_{:\mathcal{P}(\Sigma R)}.c(g\, r)))$$

and

$$\mathbf{CPS}^{r}(M, N) = \lambda c_{:\mathcal{P}(\mathbf{t})}.N(\lambda g_{:\mathcal{P}(\Sigma R)}.M(\lambda r_{:\Sigma R}.c(g\, r))).$$

Integrating dependent type semantics into a continuation-passing framework

(7) **A representative of every country** missed a meeting.

- One empirical constraint on a theory of inverse linking is the so-called Larson's generalization (1985): QPs external to ILCs cannot take scope between the embedded and containing QPs.
- *a meeting* cannot take scope in between *every country* and *a representative* - the two interleaved interpretations are not possible for (7).
- Under our analysis, the inseparability of the two nested QPs falls out immediately.

$$\textbf{CPS}(ev) : \mathcal{C}(X) \times \mathcal{C}(X \longrightarrow Y) \longrightarrow \mathcal{C}(Y).$$

unit (return): lifts elements of X to C-computations.

$$\eta_X : X \to C(X)$$

$$x \mapsto ev_x$$

$$ev_x : (x \longrightarrow \mathbf{t}) \longrightarrow \mathbf{t}$$

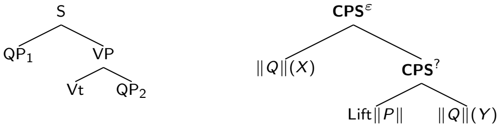
$$f \mapsto f(x)$$

For $M \in \mathcal{C}(X)$ and $R \in \mathcal{C}(X \longrightarrow Y)$,

$\textbf{CPS}^l(argument{-}first) : M, R \mapsto \lambda c : \mathcal{P}(Y).M(\lambda x.R(\lambda f : X \longrightarrow Y.c(fx)))$

$\textbf{CPS}^r(function{-}first) : M, R \mapsto \lambda c : \mathcal{P}(Y).R(\lambda f : X \longrightarrow Y.M(\lambda x.c(fx)))$

$(\mathcal{P}(X) = X \to \mathbf{t}$ and $\mathcal{C}(X) = \mathcal{P}\mathcal{P}(X))$



$$\mathbf{CPS}^{l} : \mathcal{C}(Y) \times \mathcal{CP}(X \times Y) \longrightarrow \mathcal{CP}(X).$$

We use **CPS** to combine $\mathcal{Q}_2$ in $\mathcal{C}(Y)$ (that interprets the QP$_2$) with a lift of a binary relation R^2 in $\mathcal{CP}(X \times Y)$ (that interprets the transitive verb) - we get a 'continuized' unary relation R^1 in $\mathcal{CP}(X)$.

Continuations

- An element of $\mathcal{CP}(X)$ can be thought of as a lifted relation.
- But since $\mathcal{C}(X) = \mathcal{P}(\mathcal{P}(X))$, we have that $\mathcal{CP}(X) = \mathcal{PC}(X)$, and hence an element of $\mathcal{CP}(X)$ can be also seen as a **(unary) relation on quantifiers:** a quantifier $\mathcal{Q}_1$ on X, i.e. $\mathcal{Q}_1$ in $\mathcal{C}(X)$, is in relation R^1 iff it can be put as the second quantifier so that the sentence/reading $\mathcal{Q}_2\mathcal{Q}_1R^2$ is true.

$$\mathbf{CPS}^{l} : \mathcal{C}(Y) \times \mathcal{CP}(X \times Y) \longrightarrow \mathcal{CP}(X).$$

For $N \in \mathcal{C}(Y)$ and $R \in \mathcal{CP}(X \times Y)$,

$$N, R \mapsto \lambda c : \mathcal{C}(X).N(\lambda y.c(\lambda x.r'(x, y))).$$

$$\mathbf{CPS}^{l} : \mathcal{C}(Y)\times\mathcal{CP}(X\times Y) \longrightarrow \mathcal{CP}(X).$$

For $N \in \mathcal{C}(Y)$ and $R \in \mathcal{CP}(X \times Y)$,
$N, R \mapsto \lambda c : \mathcal{C}(X).N(\lambda y.R(\lambda p : P(X \times Y).c(\lambda x.p(x, y))))$.
If $R = ev_{r'}, r' \in P(X \times Y)$,
then $\underline{\lambda c : \mathcal{C}(X).N(\lambda y.c(\lambda x.r'(x, y)))}$.

- Having R^1 and the interpretation $\mathcal{Q}_1$ of the first QP in the sentence, we can still get both readings of the sentence, $\mathcal{Q}_2\mathcal{Q}_1R^2$ and $\mathcal{Q}_1\mathcal{Q}_2R^2$, by applying right and left **CPS**'es respectively.
- **CPS**^{r} checks whether $\mathcal{Q}_1$ is in R^1.
  For $M \in \mathcal{C}(X)$ and $G = \lambda c : \mathcal{C}(X).N(\lambda y.c(\lambda x.r'(x,y)))$,
  CPS$^r(M,G) : \mathcal{C}(X) \times \mathcal{CP}(X) \longrightarrow \mathcal{C}(t)$
  $N(\lambda y.M(\lambda x.r'(x,y)))$
- **CPS**^{l} deduces from R^1, carrying the information about all quantifiers Q that can be placed in the second position, whether $\mathcal{Q}_1$ can be placed in the first position and yet have the sentence/reading $\mathcal{Q}_1\mathcal{Q}_2R^2$ true.
  $M(\lambda x.N(\lambda y.r'(x,y)))$

Continuations

For $M \in \mathcal{C}(X)$ and $G = \lambda c : \mathcal{C}(X).N(\lambda y.c(\lambda x.r'(x,y)))$,

$\mathbf{CPS}^{l}(M,G) : \mathcal{C}(X) \times \mathcal{CP}(X) \longrightarrow \mathcal{C}(t)$

$\{x \in X : \{y \in Y : \{x' \in X : P(x',y)\} \in ev_x\} \in N\}$

$\{x \in X : \{y \in Y : P(x,y)\} \in N\} \in M$

Continuations

For $M \in \mathcal{C}(X)$ and $G = \lambda c : \mathcal{C}(X).N(\lambda y.c(\lambda x.r'(x,y)))$,

$\mathbf{CPS}^l(M, G) : \mathcal{C}(X) \times \mathcal{CP}(X) \longrightarrow \mathcal{C}(t)$

$\lambda c' : \mathcal{P}(t).M(\lambda x.G(\lambda r : \mathcal{P}(x).c'(rx)))$

$M(\lambda x.\underline{\lambda c : \mathcal{C}(X)}.N(\lambda y.\underline{c}(\lambda x.r'(x,y)))\underline{(\lambda r : \mathcal{P}(x).(rx))})$

$M(\lambda x.N(\lambda y.(\lambda r : \mathcal{P}(x).(rx))(\lambda x'.r'(x',y))))$

$M(\lambda x.N(\lambda y.(\lambda x'.r'(x',y))x))$

$M(\underline{\lambda x.N(\lambda y.r'(x,y))})$

The end

Thank You for Your Attention!