Linear Algebraic Representation of Knowledge State of Agent

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Outline

Introduction

- One of the most important aspects of multi-agent communication is changes of agent's knowledge or belief (cf. Gärdenfors 2003)
- Nowadays, such changes are well-discussed in terms of modal logic, as Dynamic Epistemic Logic (DEL)
- We show a computational tool of DEL for multi-agent communication

George de La Tour: Le Tricheur à l'as de carreau

φ: a has the ace of diamonds.
$B_x\varphi$: x believes that φ.

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George de La Tour: Le Tricheur à l'as de carreau

Issues

In the picture, we can see many aspects of belief change of agents triggered by an informing action by others.

- Liar
- Belief revision
- Reliability of news source
- Mutual belief
- Channel/ Whisper /Announcement
- Awareness

How can we formalize these troublesome situations in logic, or in an efficient, scalable and reliable computation system?

Outline

Linear Algebraic Approach to Kripke Semantics

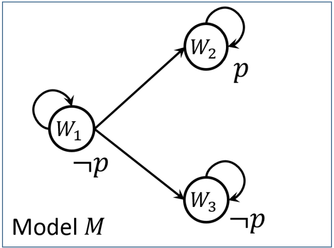
Historical development:

- Frame properties (Lemmon & Scott 1977)
- Boolean matrix approach for
  - bisimulation for modal logic (Fitting 2003).
  - belief revision & fusion of belief logic (Liau 2004).
  - DEL with communication channels (Tojo 2013, Hatano, Sano & Tojo 2015).
- Real-valued matrix approach for belief revision & update of belief logic (Fusaoka et al. 2007)
- Relational algebraic approach for modal logic of knowledge (Berghammer & Schmidt 2006).

Linear Algebraic Approach to Kripke Semantics

Define a Kripke Model $\mathfrak{M} = (W, R, V)$ by:

$$\begin{aligned} W &= \{ w_1, w_2, w_3 \}, \\ R &= \{(w_1, w_1), (w_1, w_2), (w_1, w_3), (w_2, w_2), (w_3, w_3)\}, \\ V(p) &= \{ w_2 \}. \end{aligned}$$



Boolean Matrix $\quad 1 + 1 = 1$

$$\text{From}\ \begin{array}{c} \\ w_1 \\ w_2 \\ w_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & \boxed{\text{To}} & w_3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ w_1 \\ w_2 \\ w_3 \end{array} \begin{array}{c} p \\ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{array}$$

Matrix of R $\qquad$ Matrix of $V(p)$

Syntax & Semantics

- PROP $= \{ p, q, \dots \}$ is a finite set of propositional variables.

$$\mathsf{Form}_{\mathbf{ML}} \ni A ::= p \mid \neg A \mid (A \vee A) \mid \Diamond A$$

- Given any $\mathfrak{M} = (W, R, V)$ and any $w \in W$,

$$\begin{array}{lll}
\mathfrak{M}, w \models p & \text{iff} & w \in V(p), \\
\mathfrak{M}, w \models \neg A & \text{iff} & \mathfrak{M}, w \not\models A, \\
\mathfrak{M}, w \models A \vee B & \text{iff} & \mathfrak{M}, w \models A \text{ or } \mathfrak{M}, w \models B, \\
\mathfrak{M}, w \models \Diamond A & \text{iff} & \text{for some } v \in W : wRv \text{ and } \mathfrak{M}, v \models A.
\end{array}$$

Matrix Representation of Kripke Semantics

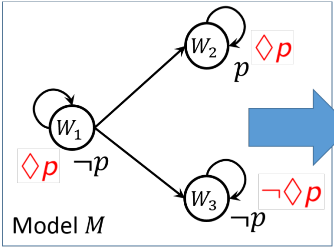
- Accessibility relation $R \mapsto$ a square matrix R^M
- Valuation $V(p) \mapsto$ a column vector $(V(p))^M$

A column vector $\|A\|_{\mathfrak{M}}$ is defined by:

$$\begin{array}{lcl} \|p\|_{\mathfrak{M}} & := & (V(p))^M, \\ \|\neg A\|_{\mathfrak{M}} & := & \overline{\|A\|_{\mathfrak{M}}}, \\ \|A \vee A\|_{\mathfrak{M}} & := & \|A\|_{\mathfrak{M}} + \|A\|_{\mathfrak{M}}, \\ \|\Diamond A\|_{\mathfrak{M}} & := & R^M \|A\|_{\mathfrak{M}}. \end{array}$$

$$\overline{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Example: the column vector of $\Diamond p$



Boolean Matrix

$$\begin{array}{cc} & \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{array}{c} w_1 \\ w_2 \\ w_3 \end{array} & \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad \begin{array}{cc} & p \\ \begin{array}{c} w_1 \\ w_2 \\ w_3 \end{array} & \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{array}$$

Matrix of R Matrix of $V(p)$

$$\|\Diamond p\| := R^M \|p\| = R^M (V(p))^M = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

$$\|\Box p\| = \|\neg\Diamond\neg p\| = \overline{\|\Diamond\neg p\|} = \overline{R\|\neg p\|} = \overline{R\overline{\|p\|}}$$

Kripke semantics becomes an extended truth table calculation.

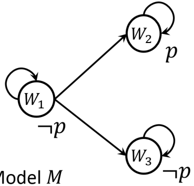
Matrix Representation of Frame Properties

Name	Formula		Matrix Reformulation
Reflexive	T	$\Box p \to p$	$R = R + \mathbf{E}$
Symmetric	B	$p \to \Box \Diamond p$	$R = {}^tR$ (or $R = {}^tR + R$)
Transitive	4	$\Box p \to \Box\Box p$	$R = R^2 + R$
Serial	D	$\Box p \to \Diamond p$	$R\,{}^tR = R\,{}^tR + E$ (or $\mathbf{1} = R\mathbf{1}$)
Euclidean	5	$\Diamond p \to \Box\Diamond p$	$R = {}^tRR + R$

- $\mathbf{E}$: a unit square matrix
- $\mathbf{1}$: a column vector of all 1s
- ${}^tR$: the transposition of the matrix R

$$\mathbf{E} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow {}^tR = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

Example: Verification of a Frame Property



Model M

$$\begin{array}{c|ccc} & w_1 & w_2 & w_3 \\ \hline w_1 & 1 & 1 & 1 \\ w_2 & 0 & 1 & 0 \\ w_3 & 0 & 0 & 1 \end{array}$$

Matrix of R

$$\begin{array}{c|c} & p \\ \hline w_1 & 0 \\ w_2 & 1 \\ w_3 & 0 \end{array}$$

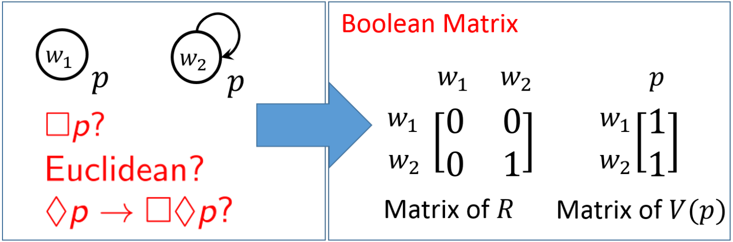
Matrix of $V(p)$

Let us check whether R is transitive ($R = R^2 + R$).

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} + \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R is transitive.

Difficulty of The Ordinary Approach

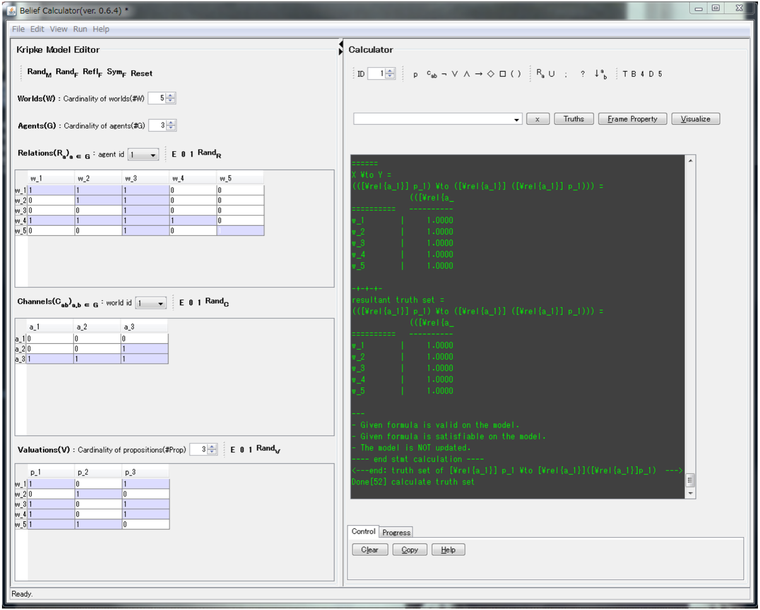


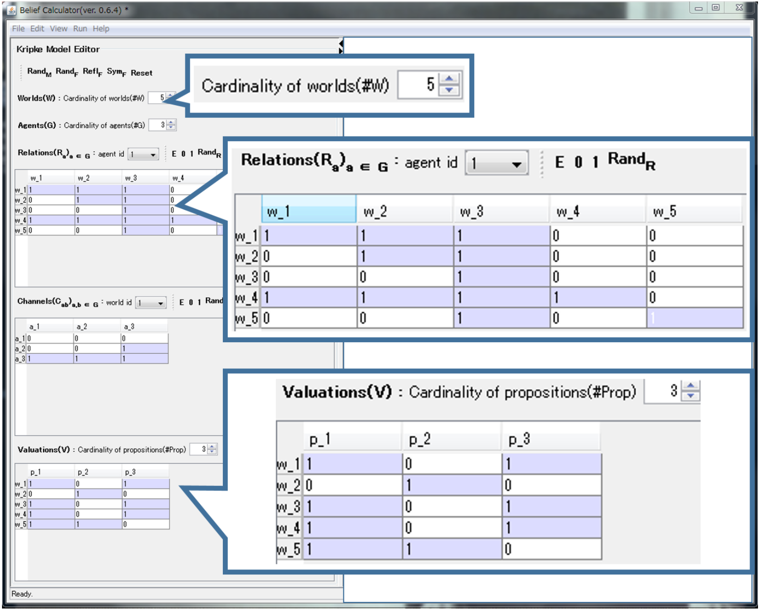
- The verification of the Euclideanness property

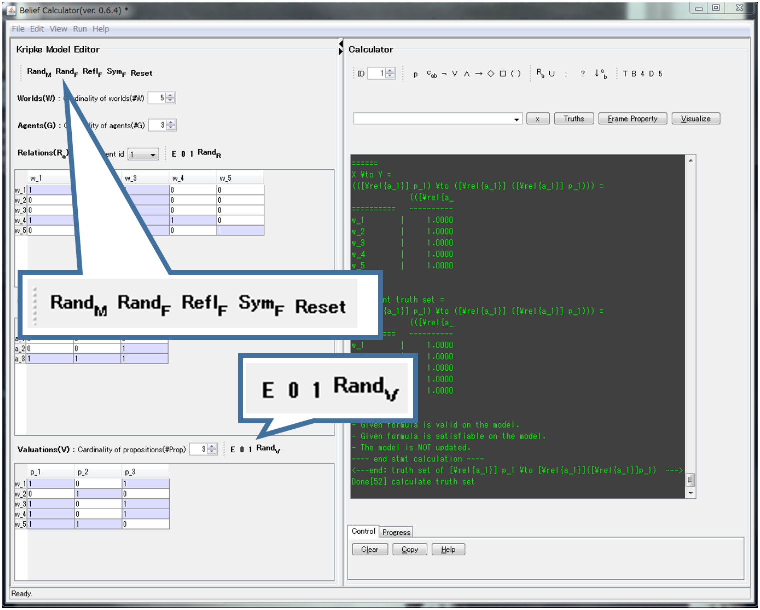
  $\Rightarrow\ R = {}^{t}RR + R$

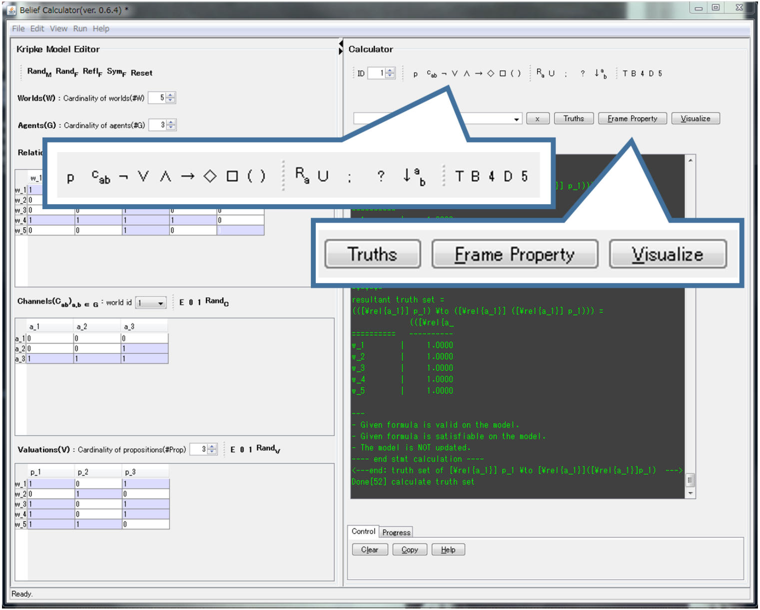
- The truth of a formula with the nested modal operators.

  $\Rightarrow\ \|\Diamond p \to \Box\Diamond p\| = \overline{R\|p\|} + \overline{R\,\overline{R\|p\|}} = \mathbf{1}$

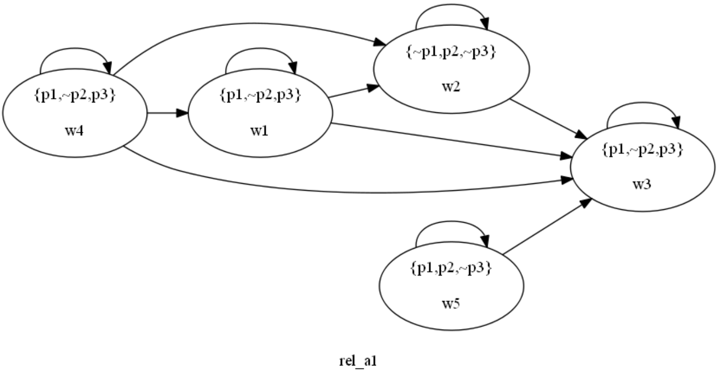

Belief Calculator(ver. 0.6.4) *
File Edit View Run Help
Kripke Model Editor
Rand_M Rand_F Refl_F Sym_F Reset
Worlds(W) : Cardinality of worlds(#W) 5
Agents(G) : Cardinality of agents(#G) 3
Relations(R_a)_a ∈ G : agent id 1 E 0 1 Rand_R
Channels(C_ab)_a,b ∈ G : world id 1 E 0 1 Rand_C
Valuations(V) : Cardinality of propositions(#Prop) 3 E 0 1 Rand_V
Calculator
ID 1
Truths
Frame Property
Visualize
X ¥to Y =
resultant truth set =
- Given formula is valid on the model.
- Given formula is satisfiable on the model.
- The model is NOT updated.
---- end stmt calculation ----
Done[52] calculate truth set
Control Progress
Clear Copy Help
Ready.

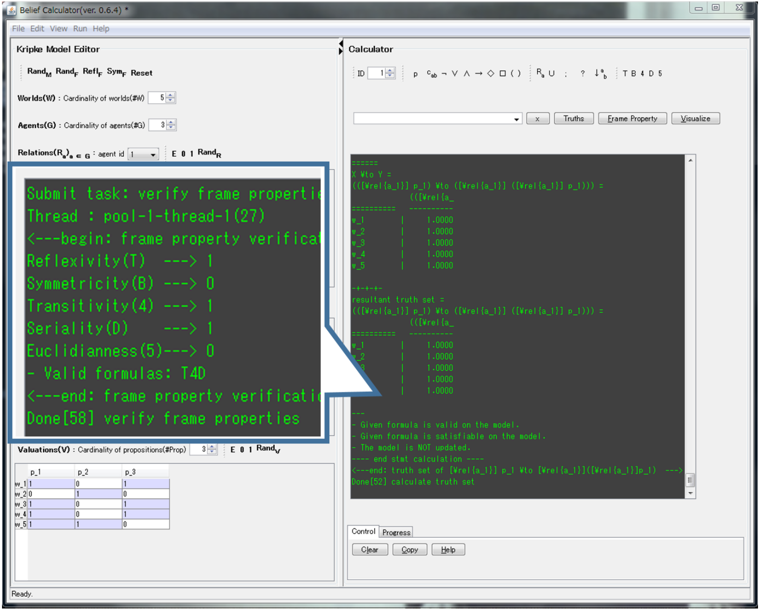

Belief Calculator(ver. 0.6.4) *
File Edit View Run Help
Kripke Model Editor
RandM RandF ReflF SymF Reset
Worlds(W) : Cardinality of worlds(#W)
Agents(G) : Cardinality of agents(#G)
Cardinality of worlds(#W) 5
Relations(Ra)a ∈ G : agent id 1 E 0 1 RandR
w_1 w_2 w_3 w_4 w_5
w_1 1 1 1 0 0
w_2 0 1 1 0 0
w_3 0 0 1 0 0
w_4 1 1 1 1 0
w_5 0 0 1 0 1
Channels(Cab)a,b ∈ G : world id 1 E 0 1 Rand
a_1 a_2 a_3
a_1 0 0 0
a_2 0 0 1
a_3 1 1 1
Valuations(V) : Cardinality of propositions(#Prop) 3
p_1 p_2 p_3
w_1 1 0 1
w_2 0 1 0
w_3 1 0 1
w_4 1 0 1
w_5 1 1 0
Ready.

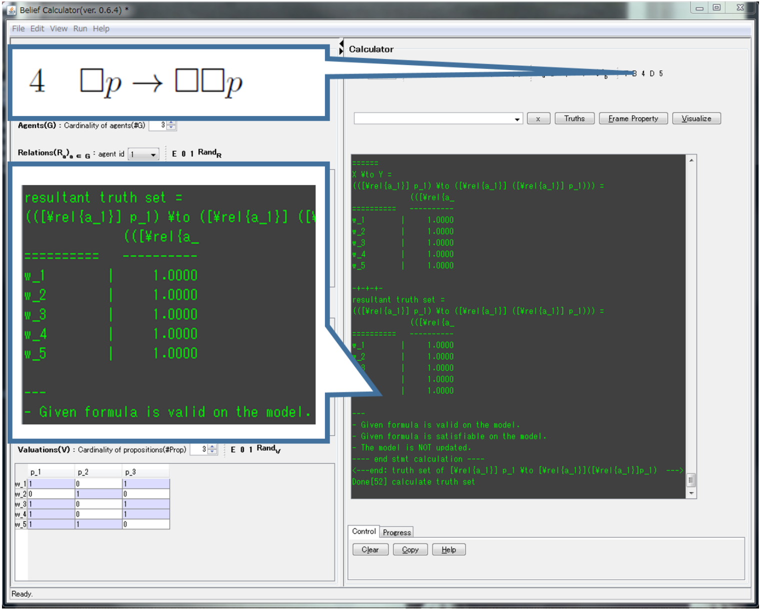

Belief Calculator(ver. 0.6.4) *
File Edit View Run Help
Kripke Model Editor
Calculator
Worlds(W) :
Agents(G) :
Relations(R_a)
Valuations(V) : Cardinality of propositions(#Prop)
Truths
Frame Property
Visualize
Control
Progress
Clear
Copy
Help
Ready.
Rand_M Rand_F Refl_F Sym_F Reset
E 0 1 Rand_V



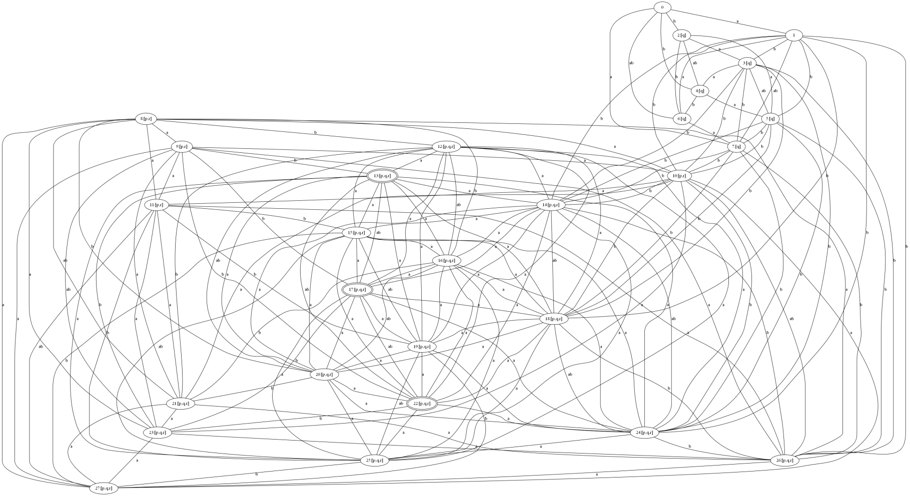
Belief Calculator(ver. 0.6.4) *
File Edit View Run Help
Kripke Model Editor
Rand_M Rand_F Refl_F Sym_F Reset
Worlds(W) : Cardinality of worlds(#W) 5
Agents(G) : Cardinality of agents(#G) 3
Channels(C_ab)_a,b ∈ G : world id 1 E 0 1 Rand_C
Valuations(V) : Cardinality of propositions(#Prop) 3 E 0 1 Rand_V
Calculator
ID 1
p c_ab ¬ ∨ ∧ → ◇ □ ( ) R_a U ; ? ↓^a_b T B 4 D 5
x Truths Frame Property Visualize
resultant truth set =
w_1 | 1.0000
w_2 | 1.0000
w_3 | 1.0000
w_4 | 1.0000
w_5 | 1.0000
- Given formula is valid on the model.
- Given formula is satisfiable on the model.
- The model is NOT updated.
---- end stmt calculation ----
Done[52] calculate truth set
Control Progress
Clear Copy Help
Ready.


{p1,~p2,p3}
w4
{p1,~p2,p3}
w1
{~p1,p2,~p3}
w2
{p1,~p2,p3}
w3
{p1,p2,~p3}
w5
rel_a1

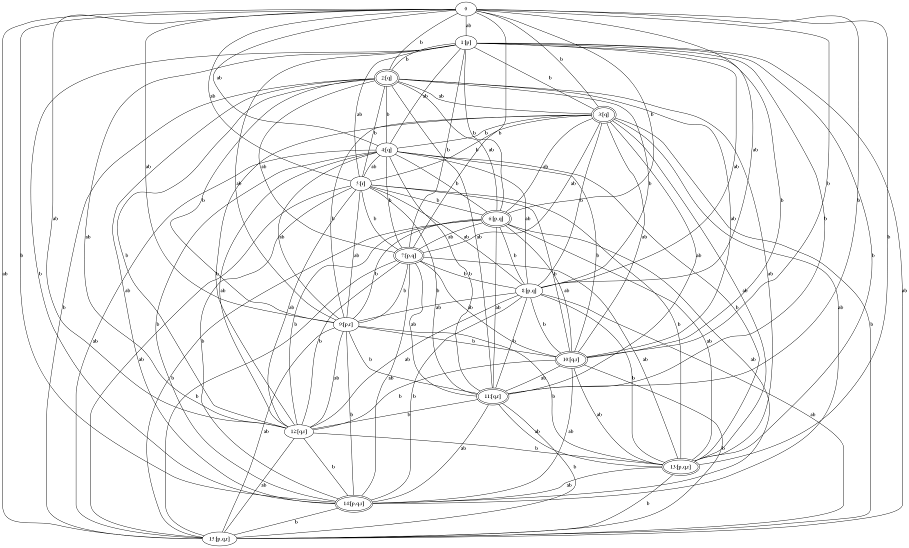

Belief Calculator(ver. 0.6.4) *
File Edit View Run Help
Kripke Model Editor
Calculator
Submit task: verify frame properti
Thread : pool-1-thread-1(27)
<---begin: frame property verificat
Reflexivity(T) ---> 1
Symmetricity(B) ---> 0
Transitivity(4) ---> 1
Seriality(D) ---> 1
Euclidianness(5)---> 0
- Valid formulas: T4D
<---end: frame property verificati
Done[58] verify frame properties
Truths
Frame Property
Visualize
- Given formula is valid on the model.
- Given formula is satisfiable on the model.
- The model is NOT updated.
Clear
Copy
Help
Ready.


Belief Calculator(ver. 0.6.4) *
File Edit View Run Help
$4 \quad \Box p \to \Box\Box p$
Calculator
Agents(G) : Cardinality of agents(#G)
Truths
Frame Property
Visualize
resultant truth set =
w_1 | 1.0000
w_2 | 1.0000
w_3 | 1.0000
w_4 | 1.0000
w_5 | 1.0000
- Given formula is valid on the model.
- Given formula is satisfiable on the model.
- The model is NOT updated.
---- end stmt calculation ----
Done[52] calculate truth set
Valuations(V) : Cardinality of propositions(#Prop)
Control
Progress
Clear
Copy
Help
Ready.

Spaghetti of Accessibility



Spaghetti of Accessibility



Interim Summary

Our approach can cover the following topics:

- Matrix representation of graph of a Kripke model
- Computation of the truth value of a formula
- The validity and the satisfiability of a formula
- Frame properties (reflexivity, symmetricity, transitivity, seriality and Euclideanness)

Outline

Multi-agent Communication

For Multi-agent system, we propose:

- Logic of belief with communication channels and its dynamic operators
- A linear algebraic reformulation for proposed operators
- Dynamic change of belief by matrix calculation

Digression: what is communication?

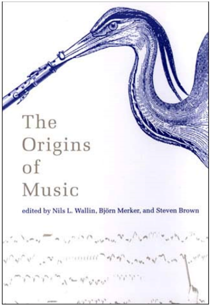
If you are alone in the universe ...

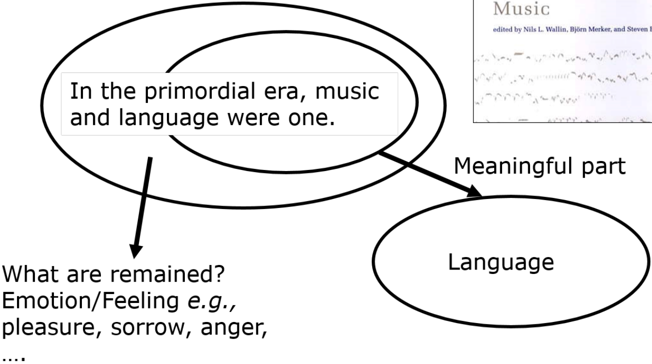
11

- Have you possessed language?
- Some claim we need language to think.

Further Digression: what is language?

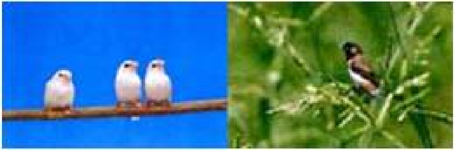
Is there any meaning in music?

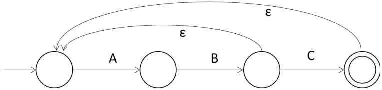




CFG and RG

- Language of domestic finch: $((ab)^+c)^+$





- Human language
  - CFG - NPDA
    - We came to see a movie to Shibuya.
    - We came to Shibuya to see a movie.
    - (*) We came to see to Shibuya a movie.
  - Dutch crossing
    - He said that A saw B help C feed the dogs.
    - Hij zegt dat A B C de honden zag helpen voeren.

Logical Studies for Multi-agent communication

Historical development:

- **DEL** for public announcements (Plaza 1989 etc.)
- Integration of communication channels into DEL
  - Two-dimensional approach of Facebook logic (Seligman et al. 2011, Sano & Tojo 2013).
  - Linear algebraic approach of DEL (Tojo 2013).

It is unknown whether resulting logics of two-dimensional approach is decidable.

We extend our linear algebraic approach for Dynamic Logic of Relation Changers (DLRC) to handle communication channels.

Syntax

- $\mathrm{PROP} = \{ p, q, \ldots \}$ is a finite set of propositional variables.
- $\mathrm{G} = \{ a, b, \ldots \}$ is a finite set of agents.

$$A ::= p \mid \mathsf{c}_{ab} \mid \neg A \mid A \vee B \mid \mathsf{B}_a A$$

- c_{ab}: "there is a channel from agent a to agent b."
- $\mathsf{B}_a A$: "agent a believes A."

Kripke Semantics

Let us extend our Kripke semantics by a channel relation.

- $\mathfrak{M} = (W, (R_a)_{a \in \mathsf{G}}, (C_{ab})_{a,b \in \mathsf{G}}, V)$ is a Kripke model.
- $C_{ab} \subseteq W$ is a channel relation s.t. $C_{aa} = W$.

Given any $\mathfrak{M} = (W, (R_a)_{a \in \mathsf{G}}, (C_{ab})_{a,b \in \mathsf{G}}, V)$ and any $w \in W$,

$$\begin{array}{lll} \mathfrak{M}, w \models \mathsf{c}_{ab} & \text{iff} & w \in C_{ab} \\ \mathfrak{M}, w \models \mathsf{B}_a A & \text{iff} & \text{for all } v \in W : wR_a v \text{ implies } \mathfrak{M}, v \models A. \end{array}$$

The truth set $[\![A]\!]_{\mathfrak{M}}$ is defined by:

$$[\![A]\!]_{\mathfrak{M}} = \{\, w \in W \mid \mathfrak{M}, w \models A \,\}.$$

Hilbert-style Axiomatization H$\mathbf{K}_c$

(**Taut**)	A, A is a tautology
($\mathbf{K}_{[B]}$)	$B_a(A \to B) \to (B_a A \to B_a B) \quad (a \in G)$
(**Selfchn**)	$c_{aa} \quad (a \in G)$
(**MP**)	From A and $A \to B$, infer B
($\mathbf{Nec}_{[B]}$)	From A, infer $B_a A \quad (a \in G)$

Theorem

This axiomatization is decidable, sound and complete for the previous Kripke semantics.

Conditional Private Announcement $[A{\downarrow}_b^a]$

$[A{\downarrow}_b^a]$: "Agent a sends a message A to agent b via a channel."

When the communication succeeds?
Our assumptions:

- There should be a channel from a to b.
- Agent a believes the content of the message, to avoid Moore sentences.

Semantics of $[A{\downarrow}^a_b]$

$$\mathfrak{M}, w \models [A{\downarrow}^a_b]B \quad \text{iff} \quad \mathfrak{M}^{A\downarrow^a_b}, w \models B$$

where $\mathfrak{M}^{A\downarrow^a_b} = (W, (R'_a)_{a\in \mathsf{G}}, (C_{ab})_{a,b\in \mathsf{G}}, V)$ and $(R'_c)_{c\in \mathsf{G}}$ is defined as:

- If $c = b$, for all $x \in W$,

$$R'_b(x) := \begin{cases} R_b(x) \cap [\![A]\!]_{\mathfrak{M}} & \text{if } \mathfrak{M}, x \models \mathsf{c}_{ab} \wedge \mathsf{B}_a A \\ R_b(x) & \text{otherwise.} \end{cases}$$

- Otherwise, $R'_c := R_c$.

Agent b restricts his/her belief by $[\![A]\!]_{\mathfrak{M}}$ if

- there is a channel from a to b,
- agent a believes the content of the message.

Other agents than b do not change beliefs.

Hilbert-style Axiomatization $\mathrm{H}\mathbf{K}_{\mathsf{c}[\cdot\downarrow^a_b]}$

In addition to all the axioms and rules of $\mathrm{H}\mathbf{K}_{\mathsf{c}}$, we add:

$$
\begin{array}{lcl}
[A\downarrow^a_b]p & \leftrightarrow & p, \\
[A\downarrow^a_b]\mathsf{c}_{cd} & \leftrightarrow & \mathsf{c}_{cd}, \\
[A\downarrow^a_b]\neg B & \leftrightarrow & \neg [A\downarrow^a_b]B, \\
[A\downarrow^a_b](B \vee C) & \leftrightarrow & [A\downarrow^a_b]B \vee [A\downarrow^a_b]C, \\
[A\downarrow^a_b]\,\mathsf{B}_c\, B & \leftrightarrow & \mathsf{B}_c\,[A\downarrow^a_b]B \quad (c \neq b) \\
[A\downarrow^a_b]\,\mathsf{B}_b\, B & \leftrightarrow & ((\mathsf{c}_{ab} \wedge \mathsf{B}_a\, A) \to \mathsf{B}_b(A \to [A\downarrow^a_b]B))\wedge \\
 & & (\neg(\mathsf{c}_{ab} \wedge \mathsf{B}_a\, A) \to \mathsf{B}_b\,[A\downarrow^a_b]B)
\end{array}
$$

($\mathbf{Nec}_{[A\downarrow^a_b]}$) From B, infer $[A\downarrow^a_b]B$

Theorem

This is a decidable, sound and complete axiomatization for the previous Kripke semantics.

PDL-extension of Our Syntax

- PROP $= \{ p, q, \dots \}$ is a finite set of propositional variables.
- $\mathsf{G} = \{ a, b, \dots \}$ is a finite set of atomic programs.
- We regard each agent's belief as an atomic program.

$$\alpha ::= a \mid (\alpha \cup \alpha) \mid (\alpha ; \alpha) \mid ?A$$
$$A ::= p \mid \mathsf{c}_{ab} \mid \neg A \mid A \vee A \mid [\alpha]A$$

- $[a]$ corresponds to the accessibility of agent a, that is R_a.

The Relation Changer $[A{\downarrow}^a_b]$

$[A{\downarrow}^a_b]$: If $\mathsf{c}_{ab} \wedge \mathsf{B}_a\, A$ then restrict R_b to A else keep R_b.

$$\text{If } X \text{ then } \alpha \text{ else } \beta \quad \overset{\text{def}}{\Leftrightarrow} \quad (?X; \alpha) \cup (?\neg X; \beta).$$

$$\begin{aligned} \alpha_b \quad &:= \quad \text{if } \mathsf{c}_{ab} \wedge \mathsf{B}_a\, A \text{ then } b; ?A \text{ else } b \\ &:= \quad (?(\mathsf{c}_{ab} \wedge \mathsf{B}_a\, A); b; ?A) \cup (?\neg(\mathsf{c}_{ab} \wedge \mathsf{B}_a\, A); b) \end{aligned}$$

$$R_b'^M = \|?(\mathsf{c}_{ab} \wedge \mathsf{B}_a\, A)\| R_b^M \|?A\| + \|?\neg(\mathsf{c}_{ab} \wedge \mathsf{B}_a\, A)\| R_b^M$$

PDL semantics

$$
\begin{array}{lcl}
[\![\mathsf{R}_a]\!]_{\mathfrak{M}} & := & R_a \\
[\![\pi \cup \pi']\!]_{\mathfrak{M}} & := & [\![\pi]\!]_{\mathfrak{M}} \cup [\![\pi']\!]_{\mathfrak{M}} \\
[\![\pi ; \pi']\!]_{\mathfrak{M}} & := & [\![\pi]\!]_{\mathfrak{M}} \circ [\![\pi']\!]_{\mathfrak{M}} \\
[\![?\varphi]\!]_{\mathfrak{M}} & := & \{(w, w) \in W^2 \mid w \in [\![\varphi]\!]_{\mathfrak{M}}\} \\
[\![p]\!]_{\mathfrak{M}} & := & V(p) \\
[\![\mathsf{c}_{ab}]\!]_{\mathfrak{M}} & := & C_{ab} \\
[\![\neg\varphi]\!]_{\mathfrak{M}} & := & W \setminus [\![\varphi]\!]_{\mathfrak{M}} \\
[\![\varphi \vee \psi]\!]_{\mathfrak{M}} & := & [\![\varphi]\!]_{\mathfrak{M}} \cup [\![\psi]\!]_{\mathfrak{M}} \\
[\![[\pi]\varphi]\!]_{\mathfrak{M}} & := & \{w \in W \mid [\![\pi]\!]_{\mathfrak{M}}(w) \subseteq [\![\varphi]\!]_{\mathfrak{M}}\}
\end{array}
$$

Matrix Representation of Channel and Programs

A column vector $\|A\|_{\mathfrak{M}}$ is defined by:

$$
\begin{array}{lll}
\|p\|_{\mathfrak{M}} & := & (V(p))^{M}, \\
\|\mathsf{c}_{ab}\|_{\mathfrak{M}} & := & C^{M}_{ab}, \\
\|\neg A\|_{\mathfrak{M}} & := & \overline{\|A\|_{\mathfrak{M}}}, \\
\|A \vee A\|_{\mathfrak{M}} & := & \|A\|_{\mathfrak{M}} + \|A\|_{\mathfrak{M}}, \\
\|[\alpha]A\|_{\mathfrak{M}} & := & \overline{R^{M}_{\alpha}\overline{\|A\|_{\mathfrak{M}}}}, \\
\|a\|_{\mathfrak{M}} & := & R^{M}_{a}, \\
\|\alpha \cup \beta\|_{\mathfrak{M}} & := & \|\alpha\|_{\mathfrak{M}} + \|\beta\|_{\mathfrak{M}}, \\
\|\alpha ; \beta\|_{\mathfrak{M}} & := & \|\alpha\|_{\mathfrak{M}}\|\beta\|_{\mathfrak{M}}, \\
\|?A\|_{\mathfrak{M}} & := & \begin{cases} 1 & \text{if } i = j \text{ and } \|A\|_{\mathfrak{M}}(i) = 1, \\ 0 & \text{otherwise.} \end{cases}
\end{array}
$$

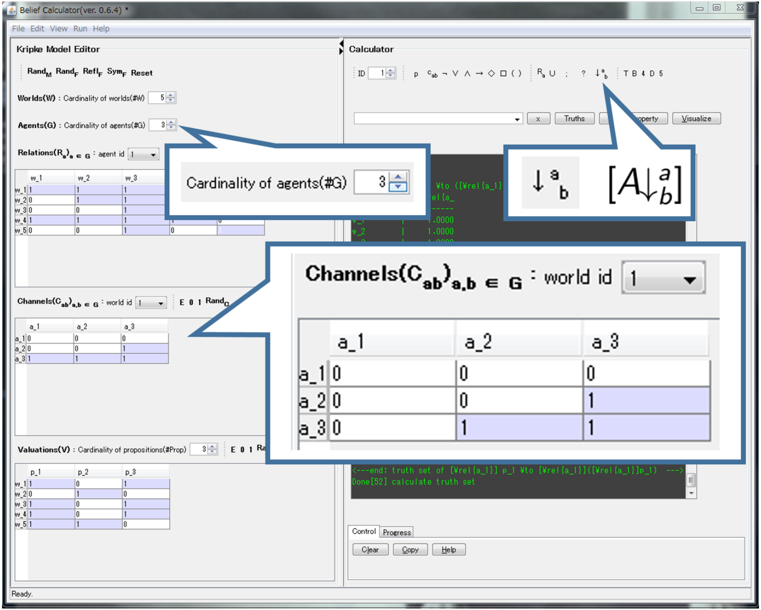
Example

Suppose that there are channels between agent a and b in every world, and agent a believes p at w_2.

$$[\![?(\mathsf{c}_{ab} \wedge \mathsf{B}_a\, p)]\!]^M R_b^M [\![?p]\!]^M = [\![?\mathsf{c}_{ab}]\!]^M [\![?\,\mathsf{B}_a\, p]\!]^M R_b^M [\![?p]\!]^M$$
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

After we calculate also the remaining part of R_b', *i.e.*, $[\![?\neg(\mathsf{c}_{ab} \wedge \mathsf{B}_a\, p)]\!]^M R_b^M$, we combine both results to obtain updated relation R_b' of agent b as:

$$R_b' = [\![?(\mathsf{c}_{ab} \wedge \mathsf{B}_a\, p)]\!]^M R_b^M [\![?p]\!]^M + [\![?\neg(\mathsf{c}_{ab} \wedge \mathsf{B}_a\, p)]\!]^M R_b^M$$
$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$


Belief Calculator(ver. 0.6.4) *
File Edit View Run Help
Kripke Model Editor
Worlds(W) : Cardinality of worlds(#W)
Agents(G) : Cardinality of agents(#G)
Cardinality of agents(#G) 3
Calculator
Truths
Visualize
Channels(C_{ab})$_{a,b \in G}$: world id 1
Valuations(V) : Cardinality of propositions(#Prop)
Control
Progress
Clear
Copy
Help
Ready.

Collective Belief Revision

We do not specify the recipients in advance. We may expand our static syntax $\mathcal{L}$ with a dynamic operator $[\varphi\downarrow^{H}]$ ($H \subseteq G$) whose reading is 'after a group H of agents sends information φ via communication channels'. Given a Kripke model $\mathfrak{M} = (W, (R_a)_{a \in G}, (C_{ab})_{a,b \in G}, V)$ and a world $w \in W$, we define the semantics of $[\varphi\downarrow^{H}]\psi$ by:

$$\mathfrak{M}, w \models [\varphi\downarrow^{H}]\psi \quad \text{iff} \quad \mathfrak{M}^{\varphi\downarrow^{H}}, w \models \psi,$$

where $\mathfrak{M}^{\varphi\downarrow^{H}} = (W, (R'_a)_{a \in G}, (C_{ab})_{a,b \in G}, V)$ and R'_a is defined as follows: for all $w \in W$, if there is some $b \in H$ such that $w \in C_{ba}$ and $\mathfrak{M}, w \models \mathsf{B}_b\,\varphi$, we put

$$R'_a(w) := R_a(w) \cap [\![\varphi]\!]_{\mathfrak{M}}.$$

Otherwise, we put $R'_a(w) := R_a(w)$.

Hilbert Style Axiomatization

In addition to all the axioms and rules of $\mathbf{K}_{\mathsf{c}}$, we add:

$$
\begin{array}{lcl}
[\varphi{\downarrow}^H]p & \leftrightarrow & p, \\
[\varphi{\downarrow}^H]\mathsf{c}_{ab} & \leftrightarrow & \mathsf{c}_{ab}, \\
[\varphi{\downarrow}^H]\neg\psi & \leftrightarrow & \neg[\varphi{\downarrow}^H]\psi, \\
[\varphi{\downarrow}^H](\psi \vee \chi) & \leftrightarrow & [\varphi{\downarrow}^H]\psi \vee [\varphi{\downarrow}^H]\chi, \\
[\varphi{\downarrow}^H]\,\mathsf{B}_a\,\psi & \leftrightarrow & (\bigvee_{b\in H}(\mathsf{c}_{ba} \wedge \mathsf{B}_b\,\varphi) \to \mathsf{B}_a(\varphi \to [\varphi{\downarrow}^H]\psi)) \\
 & & \wedge(\neg(\bigvee_{b\in H}(\mathsf{c}_{ba} \wedge \mathsf{B}_b\,\varphi)) \to \mathsf{B}_a[\varphi{\downarrow}^H]\psi)
\end{array}
$$

($\mathbf{Nec}_{[\varphi{\downarrow}^H]}$) From ψ, infer $[\varphi{\downarrow}^H]\psi$

Outline

Conclusions

- What we have done
  - Matrix representation of accessibility in Kripke semantics
  - Matrix representation of relation changer: a sequence of program (transitivity of relation) is represented by a product of matrices
- What we have not done
  - Rumor: a transitive closure of collective belief revision
  - Reliability: each agent may choose which to believe
  - So many indices; can we control the order of matrix/vector calculation by covariant/contra-variant tensors?

References

- Ryo Hatano, Katsuhiko Sano, and Satoshi Tojo. Linear algebraic semantics for multi-agent communication. In Proc. of the 7th International Conference on Agents and Artificial Intelligence, volume 1, pages 172-181, 2015.
- Ryo Hatano, Katsuhiko Sano, and Satoshi Tojo. Teaching modal logic from linear algebraic viewpoints. In Proc. of the 4th International Conference on Tools for Teaching Logic, pages 55-64, 2015. (Also, extended version is in: Journal of Logics and their Applications vol.4, no.1, 2017)