Higher-Order Situation Theory in Artificial Intelligence

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8th International Conference on Agents and Artificial Intelligence
ICAART 2016

Rome, Italy, 24–26 February, 2016
1 Brief Intro to Situation Theory
   • Origins and Present
   • Atomic and basic objects

2 More Complex Objects
   • Infons
   • Propositions
   • Complex Relations
   • Complex Types and Parameters
   • Restricted Parameters

3 Linguistic Contexts and Agents

4 Situation-Semantics Representations of Human Language

5 Applications

6 Some References
Barwise [1] (1981) is the most influential and debated, early work on SitT
Devlin [5, 6] (2008) an intuitive introduction to informal SitT
Seligman and Moss [14] (2011): a model theory of SitT
Loukanova [11] (2014) is an intro to the mathematics of set-theoretical (non-well founded) foundations of SitT
  - information in context, w.r.t. agents

This work: new type-theoretic approach to formal syntax and semantics of typed information
  - pure variables (for abstractions) and recursion variables (for memory)
  - theory of recursion on relations and functions
  - recursion terms
  - generalized variables with constraints
Sets of basic situation theoretical objects

- **Primitive individuals**: $\mathcal{A}_{\text{IND}} = \{a, b, c, \ldots\}$
- **Space-time locations**: $\mathcal{A}_{\text{LOC}} = \{l, l_0, l_1, \ldots\}$
  associated with some space and time relations, e.g.:

  \[
  \begin{align*}
  l_i &\prec l_j & \text{(time precedence)} \\
  l_i &\circ l_j & \text{(time overlapping)} \\
  l_i &\diamond l_j & \text{(space overlapping)} \\
  l_i &\subseteq_t l_j & \text{(time inclusion)} \\
  l_i &\subseteq_s l_j & \text{(space inclusion)} \\
  l_i &\subseteq l_j & \text{(space-time inclusion)}
  \end{align*}
  \]

- **Primitive relations**: $\mathcal{A}_{\text{REL}} = \{r_0, r_1, \ldots\}$
Primitive (basic) types

\[ B_{\text{TYPE}} = \{ \text{IND, REL, ARG_R, LOC, POL,} \]
\[ \quad \text{INFON, SIT, PROP, PAR, TYPE, } \models \} \] (2a)

- **IND**: primitive and complex individuals;
- **REL**: primitive and complex relations;
- **ARG_R**: primitive and complex argument roles;
- **LOC**: space-time locations;
- **POL**: polarities 0 and 1;
- **INFON**: basic or complex information units;
- **SIT**: situations;
- **PROP**: basic or complex propositions;
- **PAR**: primitive and complex parameters;
- **TYPE**: basic and complex types;
$\models$ is a special type called “supports” ("holds") used in propositions that a situation $s$ and an infon $\sigma$ are of the type “supports”, i.e., “$s$ supports $\sigma$”:

\[
(s \models \sigma) \quad \text{(a proposition)} \\
\models s \models \sigma \quad \text{(a verified proposition)}
\]

- **Primitive and complex types $T_{\text{TYPE}}$**

\[
B_{\text{TYPE}} \subseteq T_{\text{TYPE}} \quad \text{(4)}
\]
Basic argument roles with appropriateness constraints

- basic argument roles: $\mathcal{BA}_{\text{ARGR}}$, e.g.,
  $\mathcal{BA}_{\text{ARGR}} = \{ \rho_1, \ldots, \rho_m \}$;

- basic and complex argument roles: $\mathcal{BA}_{\text{ARGR}} \subseteq \mathcal{A}_{\text{ARGR}}$

- A set of argument roles is assigned to the primitive relations and types by a function $\text{ArgR}$. I.e.:

  - for every $\gamma \in \mathcal{A}_{\text{REL}} \cup \mathcal{B}_{\text{TYPE}}$

    $$\text{ArgR}(\gamma) = \{ \langle \text{arg}_1, T_1 \rangle, \ldots, \langle \text{arg}_n, T_n \rangle \}$$

    $$\equiv \{ T_1 : \text{arg}_1, \ldots, T_n : \text{arg}_n \} \quad (n \geq 0)$$

  where $\text{arg}_1, \ldots, \text{arg}_n \in \mathcal{A}_{\text{ARGR}},$
  $T_1, \ldots, T_n \in \mathcal{P}(\mathcal{T}_{\text{TYPE}})$ are sets of types (basic or complex).

- The objects $\text{arg}_1, \ldots, \text{arg}_n$ are called the argument roles or argument slots of $\gamma$.

- $T_1, \ldots, T_n$ are specific for $\gamma$ and are called the appropriateness constraints of the argument roles of $\gamma$. 
Relations and Types with Argument Roles

- Each relation is associated with a set $\text{ArgR}$ of argument roles

$$\text{ArgR}(\text{smile}) = \{ T_a : \text{smiler} \} \quad (7a)$$

$$\text{ArgR}(\text{read}) = \{ T_{a1} : \text{reader}, T_m : \text{read-ed}, T_{a2} : \text{readee} \} \quad (7b)$$

$$\text{ArgR}(\text{read}_1) = \{ T_a : \text{reader}, T_o : \text{read-ed} \} \quad (7c)$$

$$\text{ArgR}(\text{give}) = \{ T_a : \text{giver}, T_r : \text{receiver}, T_g : \text{given} \} \quad (7d)$$

- Each type is associated with a set $\text{ArgR}$ of argument roles, e.g., for the “supports” type $\models$ of situations and infons:

$$\text{ArgR}(\models) = \{ \text{SIT} : \text{arg}_{\text{SIT}}, \text{INFON} : \text{arg}_{\text{INFON}} \}. \quad (8)$$
Primitive parameters

- Typed primitive parameters (sometimes called indeterminates):

\[ P_{\text{IND}} = \{ \dot{a}, \dot{b}, \dot{c}, \ldots \}, \]  
\[ P_{\text{LOC}} = \{ l_0, l_1, \ldots \}, \]  
\[ P_{\text{REL}} = \{ r_0, r_1, \ldots \}, \]  
\[ P_{\text{POL}} = \{ i_0, i_1, \ldots \}, \]  
\[ P_{\text{SIT}} = \{ s_0, s_1, \ldots \}. \]
We will define complex objects recursively

- Infons
- states
- events
- situations
- propositions
- situated propositions
- complex relations
- complex types
- restricted parameters
A **basic infon** is every tuple \( \langle \gamma, \theta, \tau, i \rangle \), where

- \( \gamma \in \mathcal{R}_{\text{REL}} \) is a relation (primitive or complex)
  
  \[ \text{ArgR}(\gamma) = \{ \langle \text{arg}_1, T_1 \rangle, \ldots, \langle \text{arg}_n, T_n \rangle \} \quad (n \geq 0) \quad (10a) \]
  \[ T_1, \ldots, T_n \in \mathcal{P} (\mathcal{T}_{\text{TYPE}}) \quad (10b) \]

- \( \theta \) is an argument filling for \( \gamma \), i.e.:
  
  \[ \theta = \{ \langle \text{arg}_1, \xi_1 \rangle, \ldots, \langle \text{arg}_n, \xi_n \rangle \} \], \quad (11) \]

  for \( \xi_1, \ldots, \xi_n \) that satisfy the type constraints over \( \gamma \):

  \[ T_1 : \xi_1, \ldots, T_n : \xi_n \quad (12) \]

- \( \text{LOC} : \tau \) (basic or complex), \( \text{POL} : i \), \( i \in \{0, 1\} \),
Definition (Infons)

The class $\mathcal{I}_{INF}$ of infons has basic and complex infons:

$$\mathcal{B} \mathcal{I}_{INF} \subset \mathcal{I}_{INF}$$

- Complex infons (for representation of conjunctive, disjunctive, and negated information), e.g.:

For any infons $\sigma_1, \sigma_2, \sigma \in \mathcal{I}_{INF}$,

$$\langle \land, \sigma_1, \sigma_2 \rangle \in \mathcal{I}_{INF} \quad (13a)$$
$$\langle \lor, \sigma_1, \sigma_2 \rangle \in \mathcal{I}_{INF} \quad (13b)$$
$$\langle \neg, \sigma \rangle \in \mathcal{I}_{INF} \quad (13c)$$
basic infons in linear notations:

\[
\ll \gamma, T_1 : \text{arg}_1 : \xi_1, \ldots, \\
T_n : \text{arg}_n : \xi_n, \\
\text{LOC} : \text{Loc} : \tau, \text{POL} : \text{Pol} : i \gg 
\]

(14)

\[
\ll \gamma, \text{arg}_1 : \xi_1, \ldots, \text{arg}_n : \xi_n, \text{Loc} : \tau; \text{Pol} : i \gg 
\]

(15)

\[
\ll \gamma, \xi_1, \ldots, \xi_n, \tau; i \gg 
\]

(16)
An infon can be specific or parametric, e.g.

- *a* reads *b* to *c* at the space-time location *l* (specific objects)

\[
\ll \text{read, } T_{a1} : \text{reader} : a, \\
T_m : \text{read-ed} : b, \\
T_{a2} : \text{readee} : c, \\
\text{LOC} : \text{Loc} : l; \text{POL} : \text{Pol} : 1 \gg
\] (17)

- *a* reads *b* to the unknown \(\dot{c}\) at the unknown location \(\dot{l}\)

\[
\ll \text{read, } T_{a1} : \text{reader} : a, \\
T_m : \text{read-ed} : b, \\
T_{a2} : \text{readee} : \dot{c}, \dot{l}; : 1 \gg
\] (specific)

\[
\ll \text{read, } T_{a1} : \text{reader} : a, \\
T_m : \text{read-ed} : b, \\
T_{a2} : \text{readee} : \dot{c}, \dot{l}; : 1 \gg
\] (parametric)
Example (infons in linear notations)

Other parametric infons, e.g.

- **a** reads
  
  (the unknown $\dot{b}$ to the unknown $\dot{c}$ at the unknown location $\dot{l}$)

  \[
  \ll read, \ T_{a_1} : reader : a, \ T_m : read-ed : \dot{b}, \ T_{a_2} : readee : \dot{c}, \ \dot{l}; \ 1 \gg \]

  (specific)

  (parametric)

- the info that **a** either reads or does not — unknown polarity $\dot{p}$

  \[
  \ll read, \ T_{a_1} : reader : a, \ T_m : read-ed : \dot{b}, \ T_{a_2} : readee : \dot{c}, \ \dot{l}; \ \dot{p} \gg \]

  (specific)

  (parametric)
Definition (Propositions)

Proposition is any tuple \( \langle \text{PROP}, T, \theta \rangle \), where

- \( T \in \mathcal{T}_{\text{TYPE}} \) is a type with a set of argument roles

\[
\text{ArgR}(T) = \{ \langle \text{arg}_1, T_1 \rangle, \ldots, \langle \text{arg}_n, T_n \rangle \}, \quad n \geq 0 \tag{21}
\]

- \( \theta \) is an argument filling for \( T \), i.e.:

\[
\theta = \{ \langle \text{arg}_1, \xi_1 \rangle, \ldots, \langle \text{arg}_n, \xi_n \rangle \}, \quad (22)
\]

for some objects \( \xi_1, \ldots, \xi_n \) that satisfy the appropriateness type constraints of the type \( T \), i.e.:

\[
T_1 : \xi_1, \ldots, T_n : \xi_n \tag{23}
\]
### Notation

\[ \langle T, \theta \rangle \equiv (T : \theta) \quad (24a) \]
\[ \equiv (\theta : T) \quad (24b) \]
\[ \equiv \langle \text{PROP}, T, \theta \rangle \quad (24c) \]

- The variant notations (24a) and (24b) are used depending on context.
- The notation (24a) resemble the application operation.
Definition (Situated propositions)

- The type $\models$ (“supports”):

$$\text{ArgR}(\models) = \{\text{SIT} : \text{arg}_{\text{SIT}}, \text{INFON} : \text{arg}_{\text{INFON}}\}$$  \hspace{1cm} (25)

- Situated proposition:

$$\langle \text{PROP}, \models, s, \sigma \rangle, \text{ where } s \in \mathcal{P}_{\text{SIT}} \text{ and } \sigma \in \mathcal{I}_{\text{INFON}}$$  \hspace{1cm} (26)

Notation

$$\langle \models, s, \sigma \rangle \equiv (s \models \sigma)$$  \hspace{1cm} (27a)

$$\equiv \langle \text{PROP}, \models, s, \sigma \rangle$$  \hspace{1cm} (27b)
Example (The proposition that $s$ supports a positive infon)

$$(s \models \ll book, \text{IND} : \text{arg} : b, \text{LOC} : \text{Loc} : l; \text{POL} : \text{Pol} : 1 \gg)$$ (28a)

$$(s \models \ll book, \text{IND} : \text{arg} : b, \text{LOC} : \text{Loc} : l; \text{POL} : \text{Pol} : 1 \gg)$$ (28b)

Example (The proposition that $s$ supports a negative infon)

$$(s \models \ll book, \text{IND} : \text{arg} : b, \text{LOC} : \text{Loc} : l; \text{POL} : \text{Pol} : 0 \gg)$$ (29a)

$$(s \models \ll book, \text{IND} : \text{arg} : b, \text{LOC} : \text{Loc} : l; \text{POL} : \text{Pol} : 0 \gg)$$ (29b)
Example (The situation $s$ does not support a positive infon)

$$(s \not\models \ll book, \text{IND} : \text{arg} : b, \text{LOC} : \text{Loc} : l; \text{POL} : \text{Pol} : 1 \gg)$$ (30a)

$$(s \not\models \ll book, \text{IND} : \text{arg} : b, \text{LOC} : \text{Loc} : l; \text{POL} : \text{Pol} : 1 \gg)$$ (30b)

Example (The situation $s$ does not support a negative infon)

$$(s \not\models \ll book, \text{IND} : \text{arg} : b, \text{LOC} : \text{Loc} : l; \text{POL} : \text{Pol} : 0 \gg)$$ (31a)

$$(s \not\models \ll book, \text{IND} : \text{arg} : b, \text{LOC} : \text{Loc} : l; \text{POL} : \text{Pol} : 0 \gg)$$ (31b)
Example (actual vs. fallible situations)

\[
\begin{align*}
(s_1 \models \langle \text{book, b, } l; 1 \rangle) \quad (32a) \\
(s_2 \models \langle \text{book, b, } l; 0 \rangle) \quad (32b)
\end{align*}
\]

- In case that both propositions (32a), (32b) are true, (without being part of perspective environments) at least one of the situations \(s_1, s_2\) is not actual, because of the shared location \(l\).
- It may be that
  - \(s_1\) is actual situation, corresponding to a part of the reality
  - \(s_2\) is erroneous, i.e., “carries” wrong information
    E.g., \(s_2\) can be a state of an informational entity.
Example (actual vs. fallible situations)

\[(s_1 \models \ll \text{book}, b, l; 1 \gg)\]  \hspace{1cm} (32a)
\[(s_2 \models \ll \text{book}, b, l; 0 \gg)\]  \hspace{1cm} (32b)

- In case that both propositions (32a), (32b) are true, (without being part of perspective environments) at least one of the situations \(s_1, s_2\) is not actual, because of the shared location \(l\).
- It may be that
  - \(s_1\) is actual situation, corresponding to a part of the reality
  - \(s_2\) is erroneous, i.e., “carries” wrong information
  E.g., \(s_2\) can be a state of an informational entity.
Example (actual vs. fallible situations)

\[(s_1 \models \langle \text{book, b, l;} 1 \rangle)\]  \hspace{1cm} (32a)
\[(s_2 \models \langle \text{book, b, l;} 0 \rangle)\]  \hspace{1cm} (32b)

- In case that both propositions (32a), (32b) are true, (without being part of perspective environments) at least one of the situations \(s_1\), \(s_2\) is not actual, because of the shared location \(l\).
- It may be that
  - \(s_1\) is actual situation, corresponding to a part of the reality
  - \(s_2\) is erroneous, i.e., “carries” wrong information
  E.g., \(s_2\) can be a state of an informational entity.
Example (A situation $s$ can “carry” partial information)

$(s \not\models \langle \text{book, } b, \ l; \ 1 \rangle)$ \hspace{1cm} (33a)

$(s \not\models \langle \text{book, } b, \ l; \ 0 \rangle)$ \hspace{1cm} (33b)

Both propositions (33a) and (33b) can be true.
Example (conjunctive information)

- a conjunctive infon in a proposition

\[(s \models \ll smiles, \text{IND: arg: } a, \text{LOC: } Loc: l; 1 \gg) \land \ll animate, \text{IND: arg: } a, l_1; 1 \gg \land (l \circ l_1)\]  

- a conjunctive proposition

\[(s \models \ll smiles, \text{IND: arg: } a, l; 1 \gg) \land (s \models \ll animate, \text{IND: arg: } a, l_1; 1 \gg) \land (l \circ l_1)\]  

- There is another way to present the information (34b) and (35b). More on this later.
Example (conjunctive information)

- a conjunctive infon in a proposition

\[(s \models \langle \text{smiles}, \text{IND} : \text{arg} : a, \text{LOC} : \text{Loc} : l; 1 \rangle) \land \langle \text{animate}, \text{IND} : \text{arg} : a, l_1; 1 \rangle \land (l \circ l_1) \]  

- a conjunctive proposition

\[(s \models \langle \text{smiles}, \text{IND} : \text{arg} : a, l; 1 \rangle) \land (s \models \langle \text{animate}, \text{IND} : \text{arg} : a, l_1; 1 \rangle) \land (l \circ l_1) \]  

- There is another way to present the information (34b) and (35b). More on this later.
Example (conjunctive information)

- a conjunctive infon in a proposition

\[(s \models \langle \text{smiles}, \ \text{IND} : \text{arg} : a, \ \text{LOC} : \text{Loc} : l; 1 \rangle) \quad (34a)\]
\[\land \langle \text{animate}, \ \text{IND} : \text{arg} : a, \ l_1; 1 \rangle \quad (34b)\]
\[\land (l \circ l_1) \quad (34c)\]

- a conjunctive proposition

\[(s \models \langle \text{smiles}, \ \text{IND} : \text{arg} : a, \ l; 1 \rangle) \quad (35a)\]
\[\land (s \models \langle \text{animate}, \ \text{IND} : \text{arg} : a, \ l_1; 1 \rangle) \quad (35b)\]
\[\land (l \circ l_1) \quad (35c)\]

- There is another way to present the information (34b) and (35b). More on this later.
The propositional content of the sentence (36) might be expressed by the proposition (37a)–(37c), with some (great) approximation.

The book \( b \) is read

\[
(s \models \ll read, \ reader : \dot{x}, \ readed : b, \ readee : \dot{y}, \ Loc : l; 1 \gg)
\]

\[
\land \ll book, \ arg : b, \ Loc : l_1; 1 \gg) \quad (37b)
\]

\[
\land (l \subset l_1) \quad (37c)
\]

(37b) and (37c) are presented as parts of the propositional content of (36). There are other ways to include this information (later).
Definition (Complex relations and appropriateness constraints)

- Let $\sigma$ be a given infon, and $
\{\xi_1, \ldots, \xi_n\}$ a set of parameters that occur in $\sigma$.
- Let, for each $i \in \{1, \ldots, n\}$,
  $T_i$ be the union of the constraints over the argument roles filled up by $\xi_i$.
- Then $\lambda\{\xi_1, \ldots, \xi_n\}\sigma$ is a complex relation,
  with abstract argument roles denoted by $[\xi_1], \ldots, [\xi_n]$ and having $T_1$, $\ldots$, $T_n$ as appropriateness type constraints, respectively, i.e.:

$$\text{ArgR}(\lambda\{\xi_1, \ldots, \xi_n\}\sigma) = \{\langle[\xi_1], T_1\rangle, \ldots, \langle[\xi_n], T_n\rangle\}$$ (38)
Example (A complex infon)

\[
\ll book, b, l_1; 0 \rr \quad (39a)
\]
\[
\land \ll writes, a, b, l_2; 1 \rr \quad (39b)
\]
\[
\land \ll book, b, l_3; 1 \rr \quad (39c)
\]
\[
\land l_1 < l_2 \land l_2 < l_3 \quad (39d)
\]

Example (A complex relation between \(\dot{x}, \dot{y}\), and locations \(\dot{l}_1, \dot{l}_2, \dot{l}_3\))

\[
\lambda\{\dot{x}, \dot{y}, \dot{l}_1, \dot{l}_2, \dot{l}_3\} \ll book, \dot{y}, \dot{l}_1; 0 \rr \quad (40a)
\]
\[
\land \ll writes, \dot{x}, \dot{y}, \dot{l}_2; 1 \rr \quad (40b)
\]
\[
\land \ll book, \dot{y}, \dot{l}_3; 1 \rr \quad (40c)
\]
\[
\land \dot{l}_1 < \dot{l}_2 \land \dot{l}_2 < \dot{l}_3 \quad (40d)
\]
Example (A complex infon)

\[
\begin{align*}
&\ll \text{book}, \ b, \ l_1; \ 0 \gg \\
\wedge &\ll \text{writes}, \ a, \ b, \ l_2; \ 1 \gg \\
\wedge &\ll \text{book}, \ b, \ l_3; \ 1 \gg \\
\wedge &l_1 < l_2 \wedge l_2 < l_3
\end{align*}
\]

(39a) \quad (39b) \quad (39c) \quad (39d)

Example (A complex relation between \(\dot{x}, \dot{y}\), and locations \(\dot{l}_1, \dot{l}_2, \dot{l}_3\))

\[
\lambda\{\dot{x}, \dot{y}, \dot{l}_1, \dot{l}_2, \dot{l}_3\}[ \ll \text{book}, \ \dot{y}, \ \dot{l}_1; \ 0 \gg \\
\wedge &\ll \text{writes}, \ \dot{x}, \ \dot{y}, \ \dot{l}_2; \ 1 \gg \\
\wedge &\ll \text{book}, \ \dot{y}, \ \dot{l}_3; \ 1 \gg \\
\wedge &\dot{l}_1 < \dot{l}_2 \wedge \dot{l}_2 < \dot{l}_3]
\]

(40a) \quad (40b) \quad (40c) \quad (40d)
Definition (Complex types and appropriateness constraints)

- Let $\Theta$ be a given proposition, and $
\{\xi_1, \ldots, \xi_n\}$ a set of parameters that occur in $\Theta$.
- Let, for each $i \in \{1, \ldots, n\}$, $T_i$ be the union of the constraints over the argument roles filled up by $\xi_i$.
- Then $\lambda\{\xi_1, \ldots, \xi_n\}\Theta$ is a complex type, with abstract argument roles denoted by $[\xi_1], \ldots, [\xi_n]$ and having $T_1, \ldots, T_n$ as appropriateness type constraints, respectively, i.e.:

$$\text{ArgR}(\lambda\{\xi_1, \ldots, \xi_n\}\Theta) = \{\langle[\xi_1], T_1\rangle, \ldots, \langle[\xi_n], T_n\rangle\}$$ (41)
Alternative classic notations for the complex types (corresponding to the set-theoretical comprehension):

\[ \lambda\{\xi_1, \ldots, \xi_n\} \Theta \equiv \left[ T_1 : [\xi_1], \ldots, T_n : [\xi_n] \mid \Theta \right] \] (42a)

\[ \lambda\{\xi_1, \ldots, \xi_n\} \Theta \equiv \left[ [\xi_1], \ldots, [\xi_n] \mid \Theta \right] \] (42b)

If \( \Theta \) is a proposition, we sometimes use the following notation for types:

\[ \{ \xi_1, \ldots, \xi_n \mid \Theta \} \equiv \text{TYPE} : \lambda\{\xi_1, \ldots, \xi_n\} \Theta \] (43)
Example (A proposition)

\[(s_1 \not| \ll book, b, l_1; 0 \gg)\]  
\[\land (s_2 |= \ll writes, a, b, l_2; 1 \gg)\]  
\[\land (s_3 |= \ll book, b, l_3; 1 \gg)\]  
\[\land (l_1 \prec l_2 \prec l_3)\]

Example (Complex type of objects \(\dot{x}, \dot{y}\), and locations \(\dot{l}_1, \dot{l}_2, \dot{l}_3\))

\[\lambda\{\dot{x}, \dot{y}, \dot{l}_1, \dot{l}_2, \dot{l}_3\}[(s_1 \not| \ll book, \dot{y}, \dot{l}_1; 0 \gg)\]
\[\land (s_2 |= \ll writes, \dot{x}, \dot{y}, \dot{l}_2; 1 \gg)\]  
\[\land (s_3 |= \ll book, \dot{y}, \dot{l}_3; 1 \gg)\]  
\[\land (\dot{l}_1 \prec \dot{l}_2 \prec \dot{l}_3)\]
Example (A proposition)

\[
(s_1 \not\models \ll book, b, l_1; 0 \gg)
\] (44a)
\[
\land (s_2 \models \ll writes, a, b, l_2; 1 \gg)
\] (44b)
\[
\land (s_3 \models \ll book, b, l_3; 1 \gg)
\] (44c)
\[
\land (l_1 \prec l_2 \prec l_3)
\] (44d)

Example (Complex type of objects \(\dot{x}, \dot{y}\), and locations \(\dot{l}_1, \dot{l}_2, \dot{l}_3\))

\[
\lambda\{\dot{x}, \dot{y}, \dot{l}_1, \dot{l}_2, \dot{l}_3\}[(s_1 \not\models \ll book, \dot{y}, \dot{l}_1; 0 \gg)
\]
\[
\land (s_2 \models \ll writes, \dot{x}, \dot{y}, \dot{l}_2; 1 \gg)
\] (45b)
\[
\land (s_3 \models \ll book, \dot{y}, \dot{l}_3; 1 \gg)
\] (45c)
\[
\land (\dot{l}_1 \prec \dot{l}_2 \prec \dot{l}_3)
\] (45d)
Definition (Complex propositions)

- Let $\text{TYPE} : \lambda\{\xi_1, \ldots, \xi_n\}\Theta$, and
  
  $$\text{Arg}_R(\lambda\{\xi_1, \ldots, \xi_n\}\Theta) = \{\langle[\xi_1], T_1\rangle, \ldots, \langle[\xi_n], T_n\rangle\} \quad (46)$$

- Let $a_i$ be an object of all the types in $T_i$, i.e.,
  
  $$T_i = \bigcup\{T_{i,1}, \ldots, T_{i,k_i}\} \quad (47a)$$
  
  $$T_{i,1} : a_i, \ldots, T_{i,k_i} : a_i, \quad \text{for } i = 1, \ldots, n \quad (47b)$$

- Then we can form the proposition
  
  $$\left(\lambda\{\xi_1, \ldots, \xi_n\}\Theta, \theta\right) \quad (48)$$

  where $\theta = \{\langle[\xi_1], a_1\rangle, \ldots, \langle[\xi_n], a_n\rangle\}$. 
### Notation

\[(\lambda\{\xi_1, \ldots, \xi_n\} \Theta, \theta)\]  
\[(\lambda\{\xi_1, \ldots, \xi_n\} \Theta, \{T_1 : [\xi_1] : a_1, \ldots T_n : [\xi_n] : a_n\})\]  
\[(\{T_1 : [\xi_1] : a_1, \ldots T_n : [\xi_n] : a_n\} : \lambda\{\xi_1, \ldots, \xi_n\} \Theta)\]

### Linear Notations

By assuming an order over the argument roles

\[(\lambda\{\xi_1, \ldots, \xi_n\} \Theta, \theta)\]  
\[(a_1, \ldots, a_n : \lambda\{\xi_1, \ldots, \xi_n\} \Theta)\]  
\[(\lambda\{\xi_1, \ldots, \xi_n\} \Theta \{a_1, \ldots, a_n\})\]  
\[(\lambda\{\xi_1, \ldots, \xi_n\} \Theta : a_1, \ldots, a_n)\]

(reminds application)
Example (Complex proposition)

\[
\left( \lambda\{\dot{x}, \dot{y}, \dot{l}_1, \dot{l}_2, \dot{l}_3\} \left[ (s_1 \not= \ll book, \dot{y}, \dot{l}_1; 0 \gg) \right. \right.
\left. \left. \land (s_2 \models \ll writes, \dot{x}, \dot{y}, \dot{l}_2; 1 \gg) \right. \right.
\left. \left. \land (s_3 \models \ll book, \dot{y}, \dot{l}_3; 1 \gg) \right. \right.
\left. \left. \land (\dot{l}_1 \prec \dot{l}_2 \prec \dot{l}_3) \right] \right)
\]

: \ a, b, l_1, l_2, l_3 \)
Definition (Complex restricted parameters)

Given that

- \( \xi \) is a parameter and \( \Theta(\xi) \) is a proposition
- \( T \) is the set of the types that are constraints over the argument roles in \( \Theta(\xi) \) that are filled up by \( \xi \)
- \( x \) is a parameter of type \( \tau \), i.e., \( \tau : x \), and \( \tau \) is compatible with the types (constraints) \( T \),
- then \( x^{\lambda\xi}\Theta(\xi) \) is a complex parameter of type \( \tau \), which is called a parameter restricted by the type \( \lambda\xi\Theta(\xi) \).
- An object \( a \) can be anchored to the parameter \( x^{\lambda\xi}\Theta(\xi) \)
  \[ \iff \]
  \( a \) is of type \( \tau \), i.e., \( \tau : a \),
  \( T_i : a \), for each type \( T_i \in T \),
  and \( \lambda\xi\Theta(\xi) : a \), i.e., the proposition \( \Theta(a) \) is true.
Definition (States of Affairs, Events, Situations)

- A set of infons that have the same location is called a state of affairs (soa).
- A set of infons with multiple locations is called an event (also, a course of affairs/events — coa).
- A situation is a collection of infons (a set or a proper class that is a non-well founded set).
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- A situation is a collection of infons (a set or a proper class that is a non-well founded set).
• Need of further refinement of the definitions of situations, states of affairs and courses of events, by considering:
  • Sets of infons may include inconsistency, e.g., by modelling contradictory or circular information.
  • The purpose is not to exclude the inconsistency, but to detect it (where possible).
  • There are definitions of (in)consistent situations.
  • How to distinguish between states and events based on kinds of relations that are components of infons (there are classifications of verbs reflecting such differentiations)
  • models of processes?
  • space-time locations; models of space-time?
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- models of processes?
- space-time locations; models of space-time?
Example (A Situated Proposition)

\[(s \models \langle \text{read}, \text{reader} : x, \text{readed} : b, \text{Loc} : l_1; 1 \rangle \land \langle \text{book}, \text{arg} : b, \text{Loc} : l_2; 1 \rangle \land (l_1 \circ l_2))\]  

- The proposition (52a)-(52c) is true iff
  - \(x\) reads \(b\) in the location \(l_1\), in the situation \(s\):
    \[(s \models \langle \text{read}, \text{reader} : x, \text{readed} : b, \text{Loc} : l_1; 1 \rangle)\]  
  - \(b\) is having the property \(\text{book}\) in \(l_2\), in the situation \(s\):
    \[(s \models \langle \text{book}, \text{arg} : b, \text{Loc} : l_2; 1 \rangle)\]  
  - and
    \[(l_1 \circ l_2)\]
Example (A Situated Proposition)

\[(s \models \ll read, reader : x, readed : b, Loc : l_1; 1 \gg \land \ll book, arg : b, Loc : l_2; 1 \gg \land l_1 \circ l_2) \quad (52a)\]

The proposition (52a)-(52c) is true iff

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\[s \models \ll read, reader : x, readed : b, Loc : l_1; 1 \gg \quad (53)\]

- \(b\) is having the property \textit{book} in \(l_2\), in the situation \(s\):

\[s \models \ll book, arg : b, Loc : l_2; 1 \gg \quad (54)\]

- and

\[l_1 \circ l_2 \quad (55)\]
Example (A Situated Proposition)

\[(s \models \iff \text{\textit{read}, reader} : x, \text{\textit{readed}} : b, \text{\textit{Loc}} : l_1; 1 \quad \wedge \quad (52a)\]
\[\iff \text{\textit{book}, arg} : b, \text{\textit{Loc}} : l_2; 1 \quad \wedge \quad (52b)\]
\[l_1 \circ l_2) \quad (52c)\]

- The proposition (52a)-(52c) is true iff
  - \(x\) reads \(b\) in the location \(l_1\), in the situation \(s\):
    \[s \models \iff \text{\textit{read}, reader} : x, \text{\textit{readed}} : b, \text{\textit{Loc}} : l_1; 1 \quad (53)\]
  - \(b\) is having the property \(\text{\textit{book}}\) in \(l_2\), in the situation \(s\):
    \[s \models \iff \text{\textit{book}, arg} : b, \text{\textit{Loc}} : l_2; 1 \quad (54)\]
  - and
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Example (A Situated Proposition)

\[(s \models \ll read, \text{reader} : x, \text{readed} : b, \text{Loc} : l_1; 1 \gg) \land (52a)\]
\[\ll book, \text{arg} : b, \text{Loc} : l_2; 1 \gg \land (52b)\]
\[l_1 \circ l_2) (52c)\]

- The proposition (52a)-(52c) is true iff
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    \[s \models \ll read, \text{reader} : x, \text{readed} : b, \text{Loc} : l_1; 1 \gg (53)\]
  - \(b\) is having the property \(book\) in \(l_2\), in the situation \(s\):
    \[s \models \ll book, \text{arg} : b, \text{Loc} : l_2; 1 \gg (54)\]
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  - and
    \[ l_1 \circ l_2 \] (55)
Quantificational scheme in Situation Semantics

Semantic quantifiers as relations between types of situated objects:

\[
(s \models \llcorner\textit{every}\lrcorner,\ [x/(s_i \models \llcorner\textit{student}, x, l_i; 1 \gg)],[y/(s_j \models \llcorner\textit{walk}, y, l_j; 1 \gg)], \ l; 1 \gg)
\]

(56a)

\[
(s \models \llcorner\textit{some}\lrcorner,\ [x/(s_i \models \llcorner\textit{student}, x, l_i; 1 \gg)],[y/(s_j \models \llcorner\textit{walk}, y, l_j; 1 \gg)], \ l; 1 \gg)
\]

(56b)

\[
(s \models \llcorner\textit{two}\lrcorner,\ [x/(s_i \models \llcorner\textit{student}, x, l_i; 1 \gg)],[y/(s_j \models \llcorner\textit{walk}, y, l_j; 1 \gg)], \ l; 1 \gg)
\]

(56c)
The proposition $pu(u, l, x, y, \alpha)$, where

$$pu(u, l, x, y, \alpha) \equiv (u \models \ll tells\_to, x, y, \alpha, l; 1 \gg)$$  \hspace{1cm} (57)

$pu(u, l, x, y, \alpha)$ states that the situation $u$ is an utterance situation.

The proposition $pu(u, l, x, y, \alpha)$ is true iff $u$ supports the uttering act:

$$u \models \ll tells\_to, x, y, \alpha, l; 1 \gg$$  \hspace{1cm} (58)

i.e., iff

- $x$ is the speaker agent in $u$
- $y$ is the listener agent in $u$
- $l$ is the space-time location of the act of $x$ uttering $\alpha$
- $\alpha$ is the expression uttered in $u$ by the speaker agent $x$

The type of an utterance situation is

$$ru(l, x, y, \alpha) \equiv [u \mid pu(u, l, x, y, \alpha)]$$  \hspace{1cm} (59)
The proposition $pu(u, l, x, y, \alpha)$, where

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The proposition \( pu(u, l, x, y, \alpha) \), where

\[
pu(u, l, x, y, \alpha) \equiv (u \models \ll tells_to, x, y, \alpha, l; 1 \gg) \tag{57}
\]

\( pu(u, l, x, y, \alpha) \) states that the situation \( u \) is an utterance situation.

The proposition \( pu(u, l, x, y, \alpha) \) is true iff \( u \) supports the uttering act:

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u \models \ll tells_to, x, y, \alpha, l; 1 \gg \tag{58}
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$$pu(u, l, x, y, \alpha) \equiv (u \models \ll tells_to, x, y, \alpha, l; 1 \gg) \quad (60)$$

- the type of a speaker agent in $u$ is:

$$rsp(u, l, y, \alpha) \equiv [x \mid pu(u, l, x, y, \alpha)] \quad (61)$$

- the type of a listener agent in $u$ is:

$$rlst(u, l, x, \alpha) \equiv [y \mid pu(u, l, x, y, \alpha)] \quad (62)$$

- the type of the utterance space-time location is

$$rdl(u, x, y, \alpha) \equiv [l \mid pu(u, l, x, y, \alpha)] \quad (63)$$

- in $u$, $x$ is the speaker agent and $y$ is the listener agent iff $u$ supports an uttering act:

$$u \models \ll tells_to, x, y, \alpha, l; 1 \gg \quad (64)$$
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pu(u, l, x, y, \alpha) \equiv (u \models \ll tells_to, x, y, \alpha, l; 1 \gg) \quad (60)
\]

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  \[
  rsp(u, l, y, \alpha) \equiv [x \mid pu(u, l, x, y, \alpha)] \quad (61)
  \]

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  \[
  rlst(u, l, x, \alpha) \equiv [y \mid pu(u, l, x, y, \alpha)] \quad (62)
  \]

- the type of the utterance space-time location is
  \[
  rdl(u, x, y, \alpha) \equiv [l \mid pu(u, l, x, y, \alpha)] \quad (63)
  \]

- in \( u \), \( x \) is the speaker agent and \( y \) is the listener agent iff \( u \) supports an uttering act:
  \[
  u \models \ll tells_to, x, y, \alpha, l; 1 \gg \quad (64)
  \]
Speaker’s References: referent agents

- **the type of the speaker’s referent agent** of the expression $\alpha$

$$r_\alpha(u, l, x, y) = [z \mid q(u, l, x, y, z, \alpha)]$$  \hspace{1cm} (65)

where $q(u, l, x, y, z, \alpha)$ is a proposition such as (66a)

$$q(u, l, x, y, z, \alpha) \equiv$$  \hspace{1cm} (66a)

$$(u^{ru(l, x, y, \alpha)} \models \ll refers-to, x^{rsp(u, l, y, \alpha)}, z, \alpha, l^{rdl(u, x, y, \alpha)}; 1 \gg)$$  \hspace{1cm} (66c)

The proposition $q(u, l, x, y, z, \alpha)$ in (66a) states that

- in the utterance $u^{ru(l, x, y, \alpha)}$, the speaker $x^{rsp(u, l, y, \alpha)}$ refers to the referent agent $z$ of the expression $\alpha$.  


Speaker's References: referent agents

- the type of the speaker’s referent agent of the expression $\alpha$

$$r_\alpha(u, l, x, y) = [z \mid q(u, l, x, y, z, \alpha)] \quad (65)$$

where $q(u, l, x, y, z, \alpha)$ is a proposition such as (66a)

$$q(u, l, x, y, z, \alpha) \equiv$$

$$(u^{ru(l, x, y, \alpha)} \models$$

$\ll ref\text{-}to, x^{rsp(u, l, y, \alpha)}, z, \alpha, l^{rdl(u, x, y, \alpha)}; 1 \gg) \quad (66c)$$

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- in the utterance $u^{ru(l, x, y, \alpha)}$, the speaker $x^{rsp(u, l, y, \alpha)}$ refers to the referent agent $z$ of the expression $\alpha$
Speaker’s References: referent agents

- **the type of the speaker’s referent agent** of the expression \( \alpha \)

\[
r_\alpha(u, l, x, y) = [z | q(u, l, x, y, z, \alpha)]
\] (65)

where \( q(u, l, x, y, z, \alpha) \) is a proposition such as (66a)

\[
q(u, l, x, y, z, \alpha) \equiv \\
(u^{ru(l,x,y,\alpha)} \models \\
\ll \text{refers-to}, x^{rsp(u,l,y,\alpha)}, z, \alpha, l^{rdl(u,x,y,\alpha)}; 1 \gg)
\] (66c)

The proposition \( q(u, l, x, y, z, \alpha) \) in (66a) states that

- in the utterance \( u^{ru(l,x,y,\alpha)} \), the speaker \( x^{rsp(u,l,y,\alpha)} \) refers to the referent agent \( z \) of the expression \( \alpha \)
Speaker’s denotations of referent agents

Denotation of a proper name, e.g., MARIA, as a referent agent

- a referent agent $z^r$: determined by a reference restriction $r$,
- in an utterance situation (context) $u$,
- by a speaker agent $x^{rsp(u,l,y,\alpha)}$

where the type restriction $r$ may be

- general, sincere reference
  
  \[
  r = [z \mid (u \vDash \ll \text{refers to by}, x^{rsp(u,l,y,\alpha)}, z, \text{MARIA}, l^{rdl}; 1 \gg) \land \\
  (u \vDash \ll \text{named}, \text{MARIA}, z; 1 \gg)]
  \]

- belief reference
  
  \[
  r = [z \mid (u \vDash \ll \text{refers to by}, x^{rsp(u,l,y,\alpha)}, z, \text{MARIA}, l^{rdl}; 1 \gg) \land \\
  (u \vDash \ll \text{believes}, x^{rsp(u,l,y,\alpha)}, \\
  (s_{res} \vDash \ll \text{named}, \text{MARIA}, z; 1 \gg), \\
  l^{rdl}; 1 \gg)]
  \]
Speaker’s denotations of referent agents

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$$r = [z | (u \models \llrefers{to}{by}{x^{rsp(u,l,y,\alpha)}, z, MARIA, l^{rdl}; 1 \gg}) \land (u \models \llnamed{MARIA, z; 1 \gg})]$$

- belief reference

$$r = [z | (u \models \llrefers{to}{by}{x^{rsp(u,l,y,\alpha)}, z, MARIA, l^{rdl}; 1 \gg}) \land (u \models \llbelieves{x^{rsp(u,l,y,\alpha)}, (s_{res} \models \llnamed{MARIA, z; 1 \gg}), l^{rdl}; 1 \gg})]$$
Speaker’s denotations of referent agents

Denotation of a proper name, e.g., MARIA, as a referent agent

- a referent agent \( z^r \): determined by a reference restriction \( r \),
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  \[
  r = [z \mid (u \models \ll \text{refers to by}, x^{rsp(u,l,y,\alpha)}, z, \text{MARIA, } l^{rdl}; 1 \gg) \land \\
  (u \models \ll \text{named}, \text{MARIA, } z; 1 \gg)]
  \]

- belief reference
  \[
  r = [z \mid (u \models \ll \text{refers to by}, x^{rsp(u,l,y,\alpha)}, z, \text{MARIA, } l^{rdl}; 1 \gg) \land \\
  (u \models \ll \text{believes}, x^{rsp(u,l,y,\alpha)}, \\
  (s_{res} \models \ll \text{named}, \text{MARIA, } z; 1 \gg), \ll l^{rdl}; 1 \gg)]
  \]
Speaker’s denotations of referent agents

Denotation of a proper name, e.g., MARIA, as a referent agent

- a referent agent $z^r$: determined by a reference restriction $r$,
- in an utterance situation (context) $u$,
- by a speaker agent $x^{rsp}(u,l,y,\alpha)$

where the type restriction $r$ may be

- general, sincere reference
  $$r = [z | (u \models \llrefers\to\by, x^{rsp}(u,l,y,\alpha), z, \text{MARIA}, l^{rdl}; 1 \gg) \land (u \models \llnamed, \text{MARIA}, z; 1 \gg)]$$

- belief reference
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Speaker’s denotations of referent agents

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Linguistic meaning vs. interpretations with respect to different agents

- A restricted (constrained) utterance situation $u[pu(u,l,x,z,\alpha)]$ is restricted by the proposition:

$$pu(u, l, x, y, \alpha) = (u \models \ll tells\_to, x, y, \alpha, l; 1 \gg) \quad (67)$$

which introduces:

- pure linguistic meaning of $\alpha$
- interpretation of the utterance of $\alpha$ with respect to various agents:
  - the speaker
  - various listeners
  - actual vs. intended or (mis)understood agents
Linguistic meaning vs. interpretations with respect to different agents

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Assume that

- $\sigma(x, l)$ is a parametric infon, with $x$ and $l$ among its parameters; $l$ is a parameter for a space-time location, i.e.:

$$\sigma(x, l) : \text{INFON}$$

such that $x, l : \text{PAR}; l : \text{LOC}; x, l$ - free in $\sigma$ (68b)

- $\sigma(x, l)$ is the linguistic information contributed by $\varphi$.
- $T_s$ is the type of the situation described by the expression $\varphi$, in (“ignoring”) abstraction of an utterance $u$ of $\varphi$:

$$T_s = \lambda s[x \mid (s \models \sigma(x, l))]$$

$$T_{s,l} = \lambda s, l[x \mid (s \models \sigma(x, l))]$$

where $s$ is a parameter for a situation described by a potential utterance $u$ of the sentence $\varphi$. 
Verbs and Common Nouns, as components of sentences, designate types

$s, l$ are parameters for the resource situation and location of the components “book” / “read”:

\[ t_{book} = \lambda s, l[ x \mid (s \models \ll book, x, l; 1 \gg) ] \]  
\[ t_{read} = \lambda s, l\lambda x[ y \mid (s \models \ll read, y, x, l[l\circ l^{rd}] l; 1 \gg) ] \]
Given that $T$ is the type of a described situation and location

$$T = \lambda s, l[x \mid (s \models \sigma(x, l))] \quad (71)$$

- the speaker’s reference $c$ is a function over the parameters of $T$
- The extension $E(T, c)$ of the type $T$, with respect to $c$, is the set of all objects $c'(x)$ of type $T$ in the described situation $c(s)$ and location $c(l)$:

$$E(T, c) = \{ c'(x) \mid c(s) \models c'(\sigma(x))) \},$$
where $c'$ differs from $c$ only possibly for $x$. 
• $E(t_{book}, c)$ is the extension of the type of objects being a book in a resource situation $c(s_2), c(l_2)$:

$$E(t_{book}, c) = \{ b | c(s_2) \models \ll book, b, c(l_2); 1 \gg \}$$ (72)

• $E(t_{read}, c)$ is the extension of the type of objects $a$ being read by an individual $c(y^r) = m$:

$$c(y^r) = m$$ (73)

$$E(t_{read}, c) = \{ a | c(s_1) \models \ll read, c(y^r), a, c(l_1[|l|_{o^{rdl}}]); 1 \gg \}$$ (74)

where $c$ is the function of the speaker’s references in the utterance (context) $u$, e.g., $c(y^r)$ can be $c(y^r) = m$ to which the speaker refers to by the name MARIA

$$r = [z | (u \models \ll refers\_to\_by, x^{rsp}(u, l, y, \alpha), z, MARIA, l^{rdl}; 1 \gg ) \wedge (u \models \ll named, MARIA, , z; 1 \gg )]$$
The type $T$ of the situation $s$ described by an utterance of (75), wrt resource situations $s_1, s_2$ and space-time locations $l_1, l_2$ is:

$$T \equiv [s, s_1, s_2, l_1, l_2 |$$

$$(s \models \ll exist, [x \models (s_2 \models \ll book, x, l_2; 1 \gg)]),$$

$$[x \models (s_1 \models \ll read, y^r, x, l_1[l\circ l^{rd}]; 1 \gg)]; 1 \gg))$$

In a given utterance $u$, with speaker’s reference $c$, s.t.

- $c(s) = \dot{s}$ (the described situation)
- $c(s_1) = \dot{s}_1$, $c(s_2) = \dot{s}_2$ (the resource situations)
- $c(l_1) = \dot{l}_1$, $c(l_2) = \dot{l}_2$ (the resource locations)

the proposition $P$ expressed by the speaker is

$$P \equiv (\dot{s} \models \ll exist, [x \models (\dot{s}_2 \models \ll book, x, \dot{l}_2; 1 \gg)]),$$

$$[x \models (\dot{s}_1 \models \ll read, y^r, x, l_1[l\circ l^{rd}]; 1 \gg)); 1 \gg))$$
Applications that use the formal notions in this lecture

The notions introduced here are model-theoretic, i.e., they are per se semantic. They represent objects in mathematical structures of typed information, which has components including situations and space-time locations. The following collection of papers uses such objects for computational semantics of human language:

- **Generalized Quantification in Situation Semantics.**
  See Loukanova [9]

- **Quantification and Intensionality in Situation Semantics.**
  See Loukanova [10]

- **Russellian and Strawsonian Definite Descriptions in Situation Semantics.**
  See Loukanova [8]
Some other applications that use Situation Theory

Situation Semantics for computational analysis of human language.

- Head-driven Phrase Structure Grammar (HPSG)
  See Pollard and Sag [12, 13]
- Semantic analysis of questions
  See Ginzburg and Sag [7]
- Semantics of tense and aspect, in settings of logic programming, from cognitive perspective
  See Lambalgen and Hamm [15]
- Minimal Recursion Semantics (MRS), for handling scope ambiguities (see Copestake et al. [4])
Existing and potential applications

- Type-theoretic syntax-semantics interfaces
  - programming languages
  - algorithm specifications: higher-order type theory of algorithms
  - data basis
  - information representation systems, e.g., in
    - health and medical systems
    - medical sciences
    - legal systems

- Syntax-semantics interface in grammar systems

- Applications to:
  - Human language processing
  - AI
  - Neuroscience

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