Proposal for a Constructive Tarski-Grothendieck Set Theory

Erik Palmgren

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It is known since the work of Aczel (1986) that a good constructive counterpart to regular cardinal is the notion of regular set. In CZF a regular set is a set $A$ such that $A$ is inhabited and transitive, and that moreover, for every set $a \in A$, and every set $R \subseteq a \times A$, with the property $(\forall x \in a)(\exists y \in A)(x, y) \in R$, there exists a set $c \in A$ such that $(\forall x \in a)(\exists y \in c)(x, y) \in R$, and $(\forall y \in c)(\exists x \in a)(x, y) \in R$. In particular, if $R$ is a function $a \rightarrow A$, then the image of $a$ under $R$ is included in a set $c \in A$.

In Rathjen et al. (1998) a constructive theory of inaccessible sets is presented and interpreted in constructive set theory (following Aczel 1986). In CZF a set $I$ is set-inaccessible if $I$ is regular, such that $(\forall x \in I)(\exists y \in I)(x \subseteq y \land y$ is regular), and moreover that $(I, \in_{|I \times I})$ is a first-order model of CZF.

An obvious suggestion for a Tarski-Grothendieck axiom for CZF is then

$(\forall x)(\exists I)(x \in I \land I$ is set-inaccessible).

References


Peter Aczel and Michael Rathjen. Constructive set theory. Book draft 2010
