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A strictly predicative version of Hyland's effective topos

The tripos-to-topos construction by Hyland-Johnstone-Pitts [2] provides a powerful machinery in categorical logic to build examples of elementary toposes suitable for modelling various set theories. The most famous and seminal example is Hyland's effective topos [1] which hosts a model of IZF [3].

A predicative generalization of Hyland's effective topos had been studied by Moerdijk and van den Berg in [8] by referring to CZF as the set theory to be modelled.

Here we generalize the notion of tripos-to-topos to a strictly predicative version (à la Feferman) to build a categorical universe for the (extensional level of the) Minimalist Foundation (for short MF) conceived by Maietti-Sambin in 2005 [6] and completed in 2009 in [4].

We base our construction on a categorical presentation of the model for the intensional level of MF in [7] that we call *predicative effective tripos.* The adjective *predicative* refers to the fact that the tripos is formalized in Feferman's theory of inductive definitions $\widehat{ID_1}$ and the adjective *effective* to the fact that the model validates the extended Church thesis.

Then we perform the elementary quotient completion in [5] on such a tripos and the resulting category can be considered as a strictly predicative version of Hyland's effective topos.

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