SLIDES FOR OTTAWA OCT 2015 TALK
MM

Part 1

Intro

Ehresmann sketch

Christian Lair 1981:

sketchable = accessible

Bob Paré – MM 1985:

accessible categories are closed under small pseudo-limits.

$\rightsquigarrow$ category of sketches

Example: finite-limit sketches

S_{cat} : finite-limit-sketch for "category"

$T_{cat} = \langle S_{cat} \rangle$: finite-limit theory [category] for "category"

Lex : category of small fin-lim cat's

Lex Sketch : category of small fin-lim sketches

$$\boxed{\text{Lex} \underset{\text{full subcat}}{\subseteq} \text{Lex Sketch}}$$

General: $\mathbb{A}$: locally presentable category

$$\Sigma \text{ small } \subseteq \mathrm{Arr}(\mathbb{A})$$

$A \in \mathrm{Ob}(\mathbb{A})$:

$$\boxed{A \text{ is } \Sigma\text{-}\underline{\text{injective}}} \equiv \underbrace{A \models \Sigma}_{\text{notation}}$$

DEFINITION

[1] $\boxed{A \text{ is } \Sigma\text{-}\underline{\text{injective}}}$ if for all $f : U \to V$ in Σ and all $u : U \to A$ there is $v : V \to A$ such that $vf = u$

$$\begin{tikzcd}
 & A & \\
U \arrow[ur, "\forall u"] \arrow[rr, "f"'] \arrow[rr, phantom, shift left=4, "="] & & V \arrow[ul, "\exists v"']
\end{tikzcd}$$

DEFINITION

[2] $X \subseteq \mathrm{Arr}(S)$ is closed under pushout

If
$$\begin{tikzcd} C \arrow[r, "c"] & D \\ A \arrow[u] \arrow[r, "a"'] \arrow[ur, phantom, "\lrcorner"] & B \arrow[u] \end{tikzcd}$$
is a pushout

then $a \in X \Rightarrow c \in X$

DEFINITION

[3] ($X \subseteq Arr(S)$ is closed under transfinite composition)

Let α be an ordinal, and

$$\underset{\substack{\| \\ \{\beta : \beta \le \alpha\} \\ \text{as a poset}}}{A : \alpha+1} \longrightarrow \mathbb{A} \quad \text{be a}$$

diagram. A is continuous if for all limit ordinals $\beta \le \alpha$, $\underbrace{A \restriction (\beta+1)}_{\substack{\text{restriction} \\ \text{to ordinals } \gamma \le \beta}} : \beta+1 \longrightarrow \mathbb{A}$

is a colimit diagram, with colimit-object $A(\beta)$ and coprojections $A(\gamma < \beta) : A(\gamma) \to A(\beta)$.

$\boxed{X \text{ is closed under transfinite composition}}$ if : for all continuous $A : \alpha+1 \longrightarrow \mathbb{A}$ such that every link $A(\beta) \xrightarrow[A(\beta<\beta+1)]{} A(\beta+1)$ $(\beta < \alpha)$ is in X

$A(\beta < \beta+1) \in X$, we have that $A(0,\alpha) : A(0) \to A(\alpha)$ is in X.

A class of arrows is

$\boxed{\text{cellular}}$

if it is closed under

pushout

and

transfinite composition

$\mathbb{A}$ category, $\Sigma \subseteq \mathrm{Arr}(\mathbb{A})$

$a : A \to B$ in $\mathbb{A}$:

DEF

$a : A \longrightarrow B$ is a

$\boxed{\Sigma\text{-completion of } A}$

if a is Σ-cellular

and B is Σ-injective

$\boxed{\text{Proposition}}$ ($\mathbb{A}$ locally presentable, $\Sigma \subseteq \mathrm{Arr}(\mathbb{A})$ small)

For any $A \in \mathbb{A}$, 'the' Σ-completion

$$\gamma_A : A \longrightarrow A\langle\Sigma\rangle$$

exists.

If f and g in

$$\begin{tikzcd}
 & B \arrow[d, dashed, shift right, "h"'] \\
A \arrow[ur, "f"] \arrow[dr, "g"'] & \\
 & C \arrow[u, dashed, shift right, "k"']
\end{tikzcd}
\qquad
\begin{aligned} hf &= g \\ kg &= f \end{aligned}$$

are Σ-completions, h & k as shown exist. 'Equivalence ...'

In fact: $\gamma_A : A \longrightarrow A\langle\Sigma\rangle$

is weakly initial among all

$\varphi : A \longrightarrow B$ where $B \models \Sigma$:

$$\begin{tikzcd}
& A\langle\Sigma\rangle \arrow[dd, dashed, "\exists"] \\
A \arrow[ur, "\gamma_A"] \arrow[dr, "\varphi"'] \arrow[r, phantom, "="] & \\
& B
\end{tikzcd}$$

(non-unique $\exists$)

<u>Example</u>:

$\mathbb{A}$ = LexSketch :

category of small finite-limit sketches

presheaf category

$\left(\begin{array}{c} \text{Lex} \subseteq \text{Lex Sketch} \\ \text{full subcategory} \end{array} \right)$

Have: $\underbrace{\Sigma_{finlim}}_{\text{finite set of finite morph's}} \subset \text{Arr}(\text{Lex Sketch})$

For any $S \in$ Lex Sketch, the Σ_{finlim} completion

$$\gamma_S : S \longrightarrow S\langle \Sigma_{finlim} \rangle = T$$

is the finite-limit category completion.

In particular

$$\text{Mod}(S) \simeq \text{Lex}(T, \text{Set})$$

$\boxed{\text{Part 2: Linton sketches}}$

$\mathbb{G}$: fixed category

(main example :
$\mathbb{G}$ = Glob = ω Graph)

DEF'N $\boxed{\underline{\text{Linton sketch}}}$ S over $\mathbb{G}$:

has : (i) underlying graph $|S|$
(always ordinary graph!)

(ii) subgraph P ($= P[S]$) of $|S|$
'scaffolding'
$Ob(P) = Ob(S)$

(iii) sketch-for-categories structure on $|S|$

(iv) <u>required</u>: P subgraph of $\mathbb{G}$.

$\boxed{\text{Morphism}}$ of (Linton) sketches:

$$\frac{\varphi : S \longrightarrow T}{\varphi : |S| \longrightarrow |T|}$$

such that

(i) φ acts as the identity on objects and arrows of $P[S]$ — hence, $P[S]$ is a subgraph of $P[T]$

(ii) φ is a morphism of sketches for categories ('functor').

— · —

$\boxed{\text{Linton}[\mathbb{G}]}$: category of small sketches

Examples of sketches and maps between them:

$\mathbb{G} = \omega Graph = GlobularSet$

$$\begin{tikzcd}
S_1 \arrow[r, "\checkmark"] & S_2 \arrow[r, "!"] \arrow[d, "\checkmark"'] & S_3 \\
& S_4 \arrow[r, "!"'] & S_5
\end{tikzcd}$$

as follows:

$\boxed{S_1}$:

$P_1 = P[S_1]$:

(the scaffolding of S_1)

$$\boxed{\begin{tikzcd}
\langle 0 \rangle \arrow[loop below, "id"] \arrow[r, shift left, "p_0"] \arrow[r, shift right, "p_1"'] & \langle 1 \rangle \arrow[loop below, "id"] \arrow[r, shift left, "p_0"] \arrow[r, shift right, "p_1"'] & \langle 2 \rangle \arrow[loop below, "id"] \arrow[r, shift left, "p_0"] \arrow[r, shift right, "p_1"'] & \langle 3 \rangle
\end{tikzcd}}$$

notation used:

$\langle n \rangle$: the graph

$$0 \xrightarrow{01} 1 \xrightarrow{12} \cdots \xrightarrow{(n-1,n)} n$$

for $m \le n$: all morphisms

$$\langle m \rangle \longrightarrow \langle n \rangle$$

are of the form

$$p_i = p_i^{m,n} : \langle m \rangle \longrightarrow \langle n \rangle \quad (i = 0, \dots, n-m)$$

$$p_i((k, k+1)) = (k+i, k+1+i)$$

$$p_0^{n,n} = id_{\langle n \rangle}$$

Proper operations (arrows not in P):

$$\boxed{\langle 0 \rangle \xleftarrow{i} \langle 1 \rangle \xrightarrow{m} \langle 2 \rangle}$$

end of def'n of S_1

$\boxed{S_2}$: S_1 plus:

$$\boxed{\begin{tikzcd} \langle 1 \rangle & \langle 2 \rangle \arrow[l, shift right, "i_0"'] \arrow[l, shift left, "i_1"] \arrow[r, shift left, "m_0"] \arrow[r, shift right, "m_1"'] & \langle 3 \rangle \end{tikzcd}}$$

'defined' (see later!) by the commutative diagrams:

$$\begin{tikzcd} \langle 1 \rangle \arrow[r, "p_0"] \arrow[d, "i"'] & \langle 2 \rangle \arrow[d, "i_0"'] & \\ & & \langle 1 \rangle \arrow[ul, "p_1"'] \arrow[dl, "id"] \\ \langle 0 \rangle \arrow[r, "p_0"'] & \langle 1 \rangle & \end{tikzcd} \qquad \begin{tikzcd} \langle 1 \rangle \arrow[r, "p_1"] \arrow[d, "i"'] & \langle 2 \rangle \arrow[d, "i_1"'] & \\ & & \langle 1 \rangle \arrow[ul, "p_0"'] \arrow[dl, "id"] \\ \langle 0 \rangle \arrow[r, "p_1"'] & \langle 1 \rangle & \end{tikzcd}$$

$$\begin{tikzcd} \langle 1 \rangle \arrow[r, "p_1"] \arrow[d, "p_1"'] & \langle 2 \rangle \arrow[d, "m_0"] & \langle 1 \rangle \arrow[l, "p_0"'] \arrow[d, "m"] \\ \langle 2 \rangle \arrow[r, "p_1"'] & \langle 3 \rangle & \langle 2 \rangle \arrow[l, "p_0"] \end{tikzcd} \qquad \begin{tikzcd} \langle 1 \rangle \arrow[r, "p_1"] \arrow[d, "m"'] & \langle 2 \rangle \arrow[d, "m_1"] & \langle 1 \rangle \arrow[l, "p_0"'] \arrow[d, "p_0"] \\ \langle 2 \rangle \arrow[r, "p_1"'] & \langle 3 \rangle & \langle 2 \rangle \arrow[l, "p_0"] \end{tikzcd}$$

end of def'n of S_2

Remark: the inclusion $S_1 \longrightarrow S_2$ is 'tautological' in a sense explained later

$\boxed{S_3} := S_{cat}$: S_2 plus:

domain / codomain laws: commutative diagrams:

$$
\begin{tikzcd}
\langle 0 \rangle \arrow[r, "p_0"] \arrow[dr, "id"'] & \langle 1 \rangle \arrow[d, "i"] & \langle 0 \rangle \arrow[l, "p_1"'] \arrow[dl, "id"] \\
& \langle 0 \rangle &
\end{tikzcd}
$$

$$
\begin{tikzcd}
\langle 0 \rangle \arrow[r, "p_1"] \arrow[d, "p_1"'] & \langle 1 \rangle \arrow[d, "m"] & \langle 0 \rangle \arrow[l, "p_0"'] \arrow[d, "p_0"] \\
\langle 1 \rangle \arrow[r, "p_1"'] & \langle 2 \rangle & \langle 1 \rangle \arrow[l, "p_0"]
\end{tikzcd}
$$

unit laws:

$$
\begin{tikzcd}
\langle 1 \rangle & \langle 1 \rangle \arrow[l, "id"'] \arrow[r, "id"] \arrow[d, "m"] & \langle 1 \rangle \\
& \langle 2 \rangle \arrow[ul, "i_0"] \arrow[ur, "i_1"'] &
\end{tikzcd}
$$

associative law:

$$\begin{tikzcd} \langle 1 \rangle \arrow[r, "m"] \arrow[d, "m"'] & \langle 2 \rangle \arrow[d, "m_0"] \\ \langle 2 \rangle \arrow[r, "m_1"'] & \langle 3 \rangle \end{tikzcd}$$

end of def'n of S_3

$S_4 := S_2$ plus

$$\begin{tikzcd} \langle 0 \rangle \arrow[r, shift left, "p_0"] \arrow[r, shift right, "p_1"'] & \langle 1 \rangle \arrow[r, shift left, "m_0 m"] \arrow[r, shift right, "m_1 m"'] & \langle 3 \rangle \end{tikzcd}$$

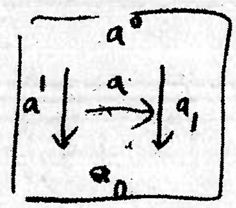
$$m_0 m p_0 = m_1 m p_0$$

$$m_0 m p_1 = m_1 m p_1$$

end of def'n of S_4

$\boxed{S_5}$: S_4 plus :

new object ② $= \underline{g}_2 =$ the 2-globe



new P-arrows: (coprojections):

$$\boxed{\langle 1\rangle \overset{d}{\underset{c}{\rightrightarrows}} ②}$$

$$\begin{tikzcd} 0 \arrow[d, "01"'] & \longmapsto_{d} & a^1 \\ 1 & \longmapsto_{c} & a_1 \end{tikzcd}$$

new proper operation ('contraction': Tom Leinster)

$$② \xrightarrow{\alpha} \langle 3\rangle$$

with two commutativities

$$\begin{tikzcd} & \langle 3 \rangle & \\ ② \arrow[ur, "\alpha"] \arrow[r, phantom, "\Theta"] & {} & ② \arrow[ul, "\alpha"'] \arrow[l, phantom, "\Theta"] \\ & \langle 1 \rangle \arrow[ul, "d"] \arrow[ur, "c"'] \arrow[uu, bend left, "m_0 m"] \arrow[uu, bend right, "m_1 m"'] & \end{tikzcd}$$

(end of def'n of S_5)

S : sketch over $\mathbb{G}$

$\boxed{\text{Definition}}$ Model M of S:

(i) object $|M|$ of $\mathbb{G}$

(underlying "graph" of M)

(ii) $M : S^{op} \longrightarrow Set$

(a model of the category-sketch S

<u>such that</u>:

$$
\begin{tikzcd}
P^{op} \arrow[r, "\text{inclusion}"] \arrow[d, "\text{inclusion}"'] \arrow[dr, phantom, "="] & S^{op} \arrow[d, "M"] \\
\mathbb{G}^{op} \arrow[r, "\hom_{\mathbb{G}}(-, |M|)"'] & Set
\end{tikzcd}
$$

<u>commutes</u>

end of def'n of model

<u>Notation</u>: For $U \in Ob(S)$ ($\subseteq ob(\mathbb{G})$!);

$$M^U \underset{def}{=} hom_{\mathbb{G}}(U, |M|)$$

M^U is a <u>set</u>, an object of *Set*; the <u>set</u> "of U-tuples of elements of M"

(a "U-tuple": $U \longrightarrow |M|$ an arrow in $\mathbb{G}$)

For $f: U \to V$ in S:

$$M(f) : M^V \longrightarrow M^U.$$

The condition in (ii) says:

<u>if</u> $(f=)p: U \to V$ is in P, hence also in $\mathbb{G}$,

$$M(p) = hom_{\mathbb{G}}(p, |M|) = (-)\circ p$$

and

$$\left(U \xrightarrow{p} V \xrightarrow{v} |M|\right) \longmapsto \underset{M(p)(v)}{\underset{\shortparallel}{\left(U \xrightarrow{vp} |M|\right)}}$$

We can extend the notation to all $f : U \to V$ in $\mathbb{S}$: we then have

$$\underbrace{U \xrightarrow{f} V}_{\text{in } \mathbb{S}} \quad \underbrace{\xrightarrow{g} W}_{\text{in } \mathbb{S}} \quad \underbrace{\xrightarrow{w} |M|}_{\text{in } \mathbb{G}}$$

gives

$$V \underbrace{\xrightarrow{\quad vg \quad}}_{\text{in } \mathbb{G}} |M|$$

$$U \xrightarrow[v(gf) \,=\, (vg)f \,=\, vgf]{} |M|$$

(so the notation is good)

Example:

diagram from p. 13:

$$\begin{tikzcd}
\langle 0 \rangle \arrow[r, "p_1"] \arrow[d, "p_1"'] \arrow[dr, phantom, "="] & \langle 1 \rangle \arrow[d, "m"] \arrow[dr, phantom, "="] & \langle 0 \rangle \arrow[l, "p_0"'] \arrow[d, "p_0"] \\
\langle 1 \rangle \arrow[r, "p_1"'] & \langle 2 \rangle & \langle 1 \rangle \arrow[l, "p_0"]
\end{tikzcd} \qquad (*)$$

M: model; meaning of $M \models (*)$:

Let $(f,g) = a \xrightarrow{f} b \xrightarrow{g} c \in M\langle 2 \rangle$

Chase

$$\begin{tikzcd}
\hat{c} \arrow[d, phantom, "\parallel"] \arrow[d, phantom, "\text{asserted}"'] & \hat{a} \xrightarrow{gf} \hat{c} \arrow[l, mapsto, "M(p_1)"'] \arrow[r, mapsto, "M(p_0)"] & \hat{a} \arrow[d, phantom, "\parallel"] \arrow[d, phantom, "\text{asserted}"] \\
c & & a \\
\begin{pmatrix} b \\ \downarrow g \\ c \end{pmatrix} \arrow[u, mapsto, "M(p_1)"] & \begin{pmatrix} a \\ \downarrow f \\ b \\ \downarrow g \\ c \end{pmatrix} \arrow[uu, mapsto] \arrow[l, mapsto, "M(p_1)"] \arrow[r, mapsto, "M(p_0)"] & \begin{pmatrix} a \\ \downarrow f \\ b \end{pmatrix} \arrow[u, mapsto, "M(p_0)"']
\end{tikzcd}$$

$\boxed{\text{Morphism}}$ of algebras of Linton sketch S:

$$h : M \longrightarrow N$$

$$h : |M| \longrightarrow |N|$$

Such that; [8]

for all $f : U \to V$ in S,

$$\begin{tikzcd} \hom_{\mathcal{C}}(V,|M|) = M^V \arrow[r, "M(f)"] \arrow[d, "h\circ(-)"'] \arrow[dr, phantom, "="] & M^U \arrow[d, "h\circ(-)"] \\ N^V \arrow[r, "N(f)"'] & N^U \end{tikzcd}$$

commutes.

Note: when $f = p : U \to V$ is in P, then this is automatic! And, since it is inherited by composites in S, it is enough to require it for generating proper operation-arrows.

$\mathrm{Alg}(S)$: category of algebras of S

$$\mathcal{U} = \mathcal{U}_S : \mathrm{Alg}(S) \longrightarrow \mathbb{G}$$

faithful
iso reflecting
creates limits

$\boxed{\text{Proposition}}$ ($\mathbb{G}$: locally presentable
S : small

Then: (i) $\mathrm{Alg}(S)$ locally presentable

(ii) $\mathcal{U} : \mathrm{Alg}(S) \longrightarrow \mathbb{G}$ has a left adjoint and

(iii) $\mathcal{U}$ is monadic

$$\mathrm{Alg} : \mathrm{Linton}[\mathbb{G}]^{op} \longrightarrow \mathrm{CAT}$$

For $\varphi : S \to T$ in $\mathrm{Linton}[\mathbb{G}]$:

$$\mathrm{Alg}(\varphi) = (-)\restriction\varphi : \mathrm{Alg}(T) \longrightarrow \mathrm{Alg}(S)$$

'<u>reduct</u>' : $\quad M \longmapsto M\restriction\varphi$

$$|M| = |M\restriction\varphi|$$

etc

Moreover:

$$\begin{tikzcd}
\mathrm{Alg}(T) \arrow[rr, "(-)\restriction\varphi"] \arrow[dr, "U_S"'] \arrow[rr, phantom, shift right=6, "="] & & \mathrm{Alg}(S) \arrow[dl, "U_T"] \\
& \mathbb{G} &
\end{tikzcd}$$

— · —

$\varphi : S \to T$ is <u>tautological</u> if

$$(-)\restriction\varphi : \mathrm{Alg}(T) \xrightarrow[\text{iso}]{\cong} \mathrm{Alg}(S)$$

Fact:

The class Taut of tautological morphisms in Linton $[\mathbb{G}]$ is cellular (closed under pushout and transfinite composite)

– also, under retracts.

There is a specific set

$$\Sigma_{taut}[X]$$

of tautological morphisms – with X a fixed small subgraph of $\mathbb{G}$, with effect:

for $T \in \mathrm{Linton}[\mathbb{G}]$

$T \models \Sigma_{taut}[X] \;\; \big(\underset{def}{\Leftrightarrow} \;\; T$ is injective

w.r. to each $\in \Sigma_{taut}[X] \big)$

if and only if:

(i) $P[T]$ contains X (at least);

and (ii) T is a category;

(iii) for every diagram in $P[T]$

that is a colimit in $\mathbb{G}$ is a

colimit diagram in T.

($\begin{tikzcd} & B \arrow[dr] & \\ A \arrow[ur] \arrow[dr] & & D \\ & C \arrow[ur] & \end{tikzcd}$ in $P[T]$:

it is a pushout in $\mathbb{G}$ $\Rightarrow$ it is a pushout

in T

$\boxed{\text{Part 3 \quad Batanin sketches}}$

$\mathbb{G} = \mathrm{Glob}$

$\mathrm{Glob} :=: \mathrm{Set}^{(\underline{gl})^{op}}$ (symbol)

where $\underline{gl}$ has objects (k) for $k = (0), (1), (2), \ldots,$ arrows

$$(k) \overset{d}{\underset{c}{\rightrightarrows}} (k+1)$$

such that in

$$(k-1) \overset{d}{\underset{c}{\rightrightarrows}} (k) \overset{d}{\underset{c}{\rightrightarrows}} (k+1)$$

we have $dd = cd$, $dc = cc$.

$\underline{g}_k = \hom(-, (k)) : \underline{gl}^{op} \to \mathrm{Set}$;

$\underline{g}_k$ 'k-globe' $\in \mathrm{Glob}$

$$\begin{tikzcd} \underline{g}_{k+1} \arrow[r, shift left, "d"] \arrow[r, shift right, "c"'] & \underline{g}_k \end{tikzcd}$$

domain

codomain

$(d = \hom(-, d), \ldots)$

— · —

$H \in \mathrm{Glob}$: a <u>composite</u> of H (if any) is any <u>non-identity</u> element x of $\langle H \rangle =$

$=$ the free, $\boxed{\text{(strict)}}$ ω-category generated by H

such that "x uses all of H":

for <u>all</u> sub-omega-graphs K of H,

with $\langle K \rangle \xrightarrow[\text{canonical inclusion}]{} \langle H \rangle$,

$$x \in \langle K \rangle \implies K = H$$

An ω-graph B is a <u>Batanin-cell</u> (B-cell) if it has a unique composite, denoted

$$\mu_B \in \langle B \rangle$$

$$\dim B \underset{\text{def}}{=} \dim \mu_B = \text{maximal dimension of elements of } B$$

Let $\mathbb{B}$: full subcategory of Glob with objects (iso class representatives of) Batanin cells.

<u>Batanin</u> <u>-sketches</u> (B - sketch) : $\boxed{\text{Linton } [\mathbb{B}]}$

(however : models will have arbitrary underlying ω-graphs)

Specification of two particular $\mathbb{B}$-sketches

$$S_{magma}\ ,\quad S_{strict}\ :$$

P of both $:\underset{def}{=}:\ \mathbb{B}$

S_{magma} : primitive operations for (strict) ω-category, with domain / codomain laws

S_{strict} : S_{magma} plus laws for strict ω-cat's.

$$Alg(S_{strict}) = \omega\,Cat$$

= category of strict ω-cat's

$$T_{strict} \underset{def}{=} \Sigma_{taut}[\mathbb{B}]\text{-completion of } S_{strict}$$

$\boxed{\text{Proposition}}$ (fact about T_{strict}):

Let B : B-cell;

$$k \geq \dim(B) > 0$$

Suppose that in the category T_{strict} we have the diagram

$$\underbrace{\underline{g}_{k-1} \overset{d}{\underset{c}{\rightrightarrows}} \underline{g}_k}_{\text{(in the scaffolding)}} \underbrace{\overset{\delta}{\underset{\gamma}{\rightrightarrows}} B}_{\text{arbitrary}}$$

such that

$$\delta d = \gamma d$$

$$\delta c = \gamma c$$

<u>Then</u>: $\boxed{\delta = \gamma}$

"All generic diagrams that can commute will commute"

Let B be any B-cell,

$k > \dim(B)$.

I define sketch arrow

$$\Phi[B,k] : S_1[B,k] \longrightarrow S_2[B,k]$$

temporarily abbreviated

$$\Phi : S_1 \longrightarrow S_2$$

$\Phi[B,k]$ is a

"<u>contraction</u> axiom"

$S_1 :=: S_1[\mathbb{B}, k]$:

$$\begin{tikzcd} \underline{g}_{k-1} \arrow[r, shift left, "d"] \arrow[r, shift right, "c"'] & \underline{g}_k \arrow[r, shift left, "\underline{\delta}"] \arrow[r, shift right, "\underline{\gamma}"'] & \mathbb{B} \end{tikzcd}$$

$$\underline{\delta} d = \underline{\gamma} d$$

$$\underline{\partial} c = \underline{\gamma} c$$

$P[S_1]$ is the part $\begin{tikzcd} \underline{g}_{k-1} \arrow[r, shift left, "d"] \arrow[r, shift right, "c"'] & \underline{g}_k \end{tikzcd}$, $\mathbb{B}$

objects and arrows of the category $\mathbb{B}$

$\underline{\delta}$, $\underline{\gamma}$ are mere symbols (proper operations)

$S_2 :=: S_1$ plus : $\begin{tikzcd} \underline{g}_k \arrow[r, shift left, "d"] \arrow[r, shift right, "c"'] & g_{k+1} \arrow[r, "\underline{\alpha}"] & \mathbb{B} \end{tikzcd}$

with $\alpha d = \delta$

$\alpha c = \gamma$:

$$\begin{tikzcd} \underline{g}_k \arrow[r, shift left, "d"] \arrow[r, shift right, "c"'] \arrow[rr, bend left=50, "\delta"] \arrow[rr, bend right=50, "\gamma"'] & \underline{g}_{k+1} \arrow[r, "\underline{\alpha}"] & \mathbb{B} \end{tikzcd}$$

$$\Sigma_{\text{contraction}} \overset{\text{def}}{:=:}$$

set of all $\overline{\Phi}[B,k]$,

$B \in \mathbb{B}$, $k > \dim(B)$

$\overline{T}_B$ = the Linton theory of Batanin-weak omega categories

$\overline{\overline{\text{def}}}$ the $(\underbrace{\Sigma_{\text{taut}}}_{\text{see above!}} \cup \Sigma_{\text{contraction}})$-

-completion of S_{magma}

Proposition (i) There is a

<u>unique</u> morphism of sketches

$$T_B \longrightarrow T_{strict}$$

that extends the identity on S_{magma}

$$\begin{tikzcd}
 & T_B \arrow[dd, dashed] \\
S_{magma} \arrow[ur] \arrow[dr] \arrow[r, phantom, "="] & \\
 & T_{strict}
\end{tikzcd}$$

(ii) $S_{magma} \longrightarrow T_B$ is weakly initial

among all $\varphi : S_{magma} \longrightarrow S$

where $S \models \Sigma_{taut} \cup \Sigma_{contraction}$