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Book of Abstracts

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List of talks

Abstracts

On large eigenvalues of Jacobi matrices of Jaynes-Cummings type

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The Jaynes-Cummings model describes the interaction between a single two-level atomic system and a quantized radiation mode. The mathematical version of this model is given by a Jacobi matrix with discrete spectrum. We determine the large n asymptotics of its nth eigenvalue in a form that allows us to recover the parameters of the model. (Joint work with Lech Zielinski.)

Uniqueness for an inverse problem in electromagnetism with partial data

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A uniqueness result for the recovery of the electric and magnetic coefficients in the time-harmonic Maxwell equations from local boundary measurements is showed. No special geometrical condition are imposed on the inaccessible part of the boundary of the domain, apart from that that the boundary of the domain is $C^{1,1}$. The coefficients are assumed to coincide on a neighbourhood of the boundary: a natural property in many applications.

Boundary triples, Krein formula, and resolvent estimates for one-dimensional high-contrast periodic problems

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I will discuss operator-norm resolvent convergence estimates for one-dimensional periodic differential operators with rapidly oscillating coefficients in the non-uniformly elliptic high-contrast setting, which has been out of reach of the existing homogenisation techniques. Our analysis is based on a special representation of the resolvent of the operator in terms of the M-matrix of an associated boundary triple, due to M. G. Krein. The resulting asymptotic behaviour is shown to be described, up to a unitary equivalent transformation, by a non-standard version of the Kronig-Penney model on R. This is joint work with Alexander V. Kiselev.

Polynomial orthogonality: entropy, complexity and entanglement

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Polynomial orthogonality of Shohat-Favard type (also called classical or hypergeometric in some contexts) has not only played a key role in the development of the theory of special functions, it has been instrumental in numerous scientific problems. In particular, it has allowed analytically calculate the exact solutions of quantum mechanical equation of non-relativistic motion (i.e., Schrödinger equation) of a reduced set of realistic physical systems , including hydrogen. This has recently enabled the determination of the theoretic-informational measures of such systems in terms of polynomial functionals of entropy and complexity kind. The meaning and mathematical calculation of the entropy and complexity measures of the hypergeometric polynomials will be addressed in this talk. The usefulness of these quantities will be illustrated in some non-relativistic quantum systems. The need for other types of polynomial orthogonality (e.g., matricial, multivariate) will be discussed to explain the relativistic and entanglement properties in quantum physics.

Asymptotic expansions for singular Schrödinger operators

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The subject of this talk are Schrödinger operators with an attractive singular 'potential' supported by a manifold of a lower dimensionality; formally one can write them as $-\Delta-\alpha\delta(x-\Gamma)$ with $\alpha > 0$, where Γ is a curve in $\mathbb{R}^d$, $d = 2, 3$, or a surface in $\mathbb{R}^3$. It is known that spectral properties of these operators depend on the geometry of Γ, in particular, that a non-trivially shaped interaction support may give rise to a discrete spectrum. In this talk we discuss asymptotic properties of such eigenvalues with respect the parameters involved, both with respect to the coupling strength α and to the geometry of Γ.

Two interacting particles on the half-line

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In this talk we discuss a system consisting of two particles moving on $\mathbb{R}_+$ and whose Hamiltonian is (formally) given by

$$H = -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + v(x,y)\,[\delta(x)+\delta(y)] \ ,$$

$v(x,y) = v(y,x)$ being some symmetric interaction potential. The considered two-particle interactions are of singular type, namely, the particles are interacting only whenever at least one particle is situated at the origin. Although this model originated from the theory of many-particle quantum chaos, it may also be of interest to other areas such as many-body quantum mechanics or applied superconductivity.

From a mathematical point of view, our system is translated into a boundary value problem for the two-dimensional Laplacian on $\mathbb{R}^2_+$ subject to Robin boundary conditions with variable coefficient. We will discuss spectral properties of H, i.e., we will describe the essential part of the spectrum and derive conditions on $v(x,y)$ which ensure the existence of at least one eigenstate below $\inf \sigma_{ess}(H)$. We will also estimate the number of such eigenvalues and provide an estimation of the ground state energy. Finally, we will prove exponential decay of the ground state in the case of $v(x,y)$ having compact support.

(This talk is based on joint work with T. Mühlenbruch (Hagen)).

Solutions of the system of pde's for Appell hypergeometric F_2, a tribute to Per O.M. Olsson

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Appell introduced four different two-variable analogues of the Gauss hypergeometric series. Each of these is a solution of a system of two pde's. In particular, the system of pde's associated with Appell's hypergeometric $F_2(a; b_1, b_2; c_1, c_2; x, y)$ admits a rich set of explicit solutions, which was most comprehensively studied by Per O.M. Olsson (Dept. of Theoretical Physics, Royal Institute of Technology, Stockholm) in a paper [2] which should have deserved more attention. The lecture will survey these solutions. Next double integral representations for some of these solutions, some possibly new, will be produced and proved in a unified way, using the concept of transmutation. This extends work recently done by the speaker [1] in the one-variable case. The research presented here is inspired by work in progress by Enno Diekema, and will include some of his results.

References

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Multiplicative non-Hermitian perturbations of Jacobi matrices

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We fully classify possible eigenvalue configurations of rank-one multiplicative non-Hermitian perturbations of Jacobi matrices. We provide a Hermite-Biehler type theorem for the characteristic polynomial and an application in random matrix theory. Joint work with M. Tyaglov.

On factorisation of a class of Schrödinger operators

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The aim of this talk is to prove sharp inequalities for 3/2-moments for the negative eigenvalues of Schrödinger operators on half-line that have a "Hardy term" in addition to a potential function.

New spectral estimations for a class of integral-difference operators and generalisation to higher dimensions

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Quadratic form approach allows for new results in the analysis of a class of integral-difference operators in finite domains: non-negativity, spectral estimations, a new property of Legendre polynomials; and establishing links with weighted mean-square deviation functionals and with infinite Jacobi matrices with not-bounded coefficients. Generalisation of integral-difference operators to higher dimensions is provided and application to matter relaxation in a field is considered. A new class of special functions naturally appears.

Abstract graph-like space and vector-valued metric graphs

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In this note we present some abstract ideas how one can construct spaces from building blocks according to a graph. The coupling is expressed via boundary pairs, and can be applied to very different spaces such as discrete graphs, quantum graphs or graph-like manifolds. We show a spectral analysis of graph-like spaces, and consider as a special case vector-valued quantum graphs. Moreover, we provide a prototype of a convergence theorem for shrinking graph-like spaces with Dirichlet boundary conditions.

Symmetry of spectra of nuclear operators

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It was proved in a paper [1] by M. I. Zelikin: The spectrum of a nuclear operator A acting on a separable Hilbert space is central-symmetric iff $\mathit{trace}\, A^{2n-1} = 0$, $n \in \mathbf{N}$.

Recall that the spectrum of A is central-symmetric, if together with any eigenvalue $\lambda \neq 0$ it has the eigenvalue $-\lambda$ of the same multiplicity.

The part "only if" is evident if one applies the Lidskiĭ trace formula for trace class operators in Hilbert spaces [2]. An analogue of Lidskiĭ theorem for operators in Banach spaces is the famous Grothendieck trace formula for the 2/3-nuclear operators [3].

Our aim is to present the following generalization (Theorem 1) of both Grothendieck and Lidskiĭ theorems and *to apply this generalization* for getting an analogue of Zelikin's theorem for subspaces of quotients of L_p-spaces (Theorem 2). In a sense, these theorems can be considered as "interpolation results" in the scale "L_1 through L_2 to L_∞".

We need a definition: An operator T in a Banach space Y is s-nuclear, $0 < s \leq 1$, if T admits a representation

$$T = \sum_i \lambda_i y_i' \otimes y_i,$$

with $(\lambda_i) \in l_s$, (y_i') and (y_i) are bounded in Y^* and Y, respectively.

Theorem 1 [4]. Let Y be a subspace of a quotient (or a quotient of a subspace) of an L_p-space, $1 \leq p \leq \infty$. If $T \in N_s(Y,Y)$ (s-nuclear), where $1/s = 1 + |1/2 - 1/p|$, then

1. the (nuclear) trace of T is well defined,
2. $\sum_{n=1}^{\infty} |\lambda_n(T)| < \infty$, where $\{\lambda_n(T)\}$ is the system of all eigenvalues of the operator T (written in according to their algebraic multiplicities)

and

$$\text{trace}\; T = \sum_{n=1}^{\infty} \lambda_n(T).$$

Theorem 2. *Let Y be a subspace of a quotient (or a quotient of a subspace) of an L_p-space, $1 \leq p \leq \infty$, and $T \in N_s(Y,Y)$ (s-nuclear), where $1/s = 1 + |1/2 - 1/p|$. The spectrum of T is central-symmetric iff $\mathit{trace}\, T^{2n-1} = 0, n = 1, 2, \ldots$.*

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Harmonic oscillator and its inverse

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Abstract We study the inverse of the harmonic oscillator. In particular, we consider the Weyl symbol to this inverse. We establish precise expressions and convenient estimates. We also explain how such estimates can be used to ensure that this operator image analytical and super analytical functions to functions of the same class. We also show how our results lead to estimates of certain special functions which, to our knowledge, seems to be unknown until now. Finally we explain some results on mapping properties of radial symmetric elements under the Bargmann transform, deduced during the investigations of the harmonic oscillator.

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Topological resonances on quantum graphs

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We consider metric graphs which consist of a finite graph with some leads attached to some vertices. To this graph is associated a Laplacian using the Kirchoff conditions. We describe some asymptotic properties of the resonances close to the real axis. This is a joint work with Y. Colin de Verdière.

On Kotani's program

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We were inspired by a program of Shinichi Kotani presented in his several papers and talks (e.g. a mini course of lectures at CRM, 2011). He was focused on the idea to construct KdV flow on a space of continuum Schrödinger operators with reflectionless potentials, which form the so-called Marchenko class, by applying Sato's Grassmanian approach. Particularly his aim was to clarify which of these potentials are almost periodic. Jointly with A. Volberg and D. Damanik we found a quite comprehensive theory for such operators with absolutely continuous spectrum (in a generic position). It is based on and actually develops the Widom theory of Hardy spaces in multi-connected domains. The main part of the talk deals with a presentation of this theory. Besides that, we are going to discuss some related results in the function theory in multi-connected domains and some open problems.

Stone theorem and the three-Hilbert-space representation of quantum systems

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In the context of the recent boom of formalism in which the observables (typically, of the bound-state energy levels) of a closed and unitarily evolving quantum system are represented, paradoxically, by non-selfadjoint operators with real spectra it will be pointed out that in the language of mathematics, one encounters here merely a number of painful terminological misunderstandings. Conceptual remedy of some of the misunderstandings will be outlined leading, typically, to several possible resolutions of the Big Bounce paradox in quantum cosmology, etc.
