Matematikcentrum
Köpenhamns universitet
Lunds universitet

ÖRESUNDSSEMINARIUM ÖVER MATEMATISK ANALYS
Lund, 23 november 2007
Matematikhuset, sal C

Detta är ett i en rad seminarier som vi arrangerar med en frekvens av någon till ett par gånger per termin, och som alternerar mellan Lund och Köpenhamn. Föredragen hålles på en relativt allmän nivå och vänder sig inte bara till etablerade forskare utan även till studenter, såväl doktorander som studenter på C- och D-nivå.

- 13.30-14.20 Bernhard Krötz (Max-Planck-Institut für Mathematik, Bonn, Germany, presently at Copenhagen)
  Holomorphic extension of representations

- 14.25-15.15 Annemarie Luger (Lund University, Sweden)
  Hydrogen atom: An operator theoretic interpretation of the generalized Titchmarsh-Weyl coefficient

  15.15-15.45 Coffee break

- 15.45-16.35 Hans Plesner Jakobsen (Copenhagen University, Denmark)
  Covariant Differential Operators and Indecomposable Representations

- 16.40-17.30 Anders Olofsson (Lund University, Sweden)
  Characterization of Bergman space Toeplitz operators with harmonic symbols

  18.15- Dinner at restaurant "Gräddhyllan"

Arrangörer: Gerd Grubb, Per-Anders Ivert, Pavel Kurasov, Jan Philip Solovej

Holomorphic extension of representations

Bernhard Krötz

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We report on the paper [1] whose introduction we attach below. We make a special emphasis for the group $G = \mathrm{Sl}(2, \mathbb{R})$ and refer to the overview article [2].

Let us consider a unitary irreducible representation $(\pi, \mathcal{H})$ of a simple, non-compact and connected Lie group G. Let us denote by K a maximal compact subgroup of G. According to Harish-Chandra, the Lie algebra submodule $\mathcal{H}_K$ of K-finite vectors of π consists of analytic vectors of the representation. We determine, and in full generality, their natural domain of definition as holomorphic functions:

Theorem 1. *Let $(\pi, \mathcal{H})$ be a unitary irreducible representation of G. Let $v \in \mathcal{H}$ be a non-zero K-finite vector and*

$$f_v : G \to \mathcal{H}, \quad g \mapsto \pi(g)v$$

the corresponding orbit map. Then there exists a maximal $G \times K_{\mathbb{C}}$-invariant domain $D_\pi \subseteq G_{\mathbb{C}}$, independent of v, to which f_v extends holomorphically. Explicitely:

1. *$D_\pi = G_{\mathbb{C}}$ if π is the trivial representation.*

2. *$D_\pi = \Xi^+ K_{\mathbb{C}}$ if G is Hermitian and π is a non-trivial highest weight representation.*

3. *$D_\pi = \Xi^- K_{\mathbb{C}}$ if G is Hermitian and π is a non-trivial lowest weight representation.*

4. *$D_\pi = \Xi K_{\mathbb{C}}$ in all other cases.*

Let us explain the objects Ξ, Ξ^+ and Ξ^- in the statement. We form $X = G/K$, the associated Riemann symmetric space, and view X as a totally real submanifold of its affine complexification $X_{\mathbb{C}} = G_{\mathbb{C}}/K_{\mathbb{C}}$. The natural G-invariant complexification of X, the crown domain, is denoted by $\Xi \ (\subseteq X_{\mathbb{C}})$. For a domain $D \subseteq X_{\mathbb{C}}$ we denote by $D K_{\mathbb{C}}$ its preimage in $G_{\mathbb{C}}$.

In [3] we observed that a $G \times K_{\mathbb{C}}$-invariant domain of definition of f_v, say $D_v \subseteq G_{\mathbb{C}}$, must be such that G acts properly on $D_v/K_{\mathbb{C}} \subseteq X_{\mathbb{C}}$. By our work with Robert J. Stanton we know that we can choose D_v such that $D_v \supseteq \Xi K_{\mathbb{C}}$ (see [4], [5]). Therefore it is useful to classify all G-domains $\Xi \subseteq D \subseteq X_{\mathbb{C}}$ with proper action. As it turns out, they allow a simple description. We extract from theorems below:

Theorem 2. *Let $\Xi \subseteq D \subseteq X_{\mathbb{C}}$ be a G-invariant domain on which G acts properly. Then:*

1. *If G is not of Hermitian type, then $D = \Xi$.*

2. *If G is of Hermitian type, then either $D \subseteq \Xi^+$ or $D \subseteq \Xi^-$ with Ξ^+ and Ξ^- two explicite maximal domains for proper G-action.*

Finally, let us emphasize that proofs in this paper are modelled after $G = \mathrm{Sl}(2,\mathbb{R})$ which was dealt with earlier in [3].

References

[1] B. Krötz, *Domains of holomorphy for irreducible unitary representations of simple Lie groups*, arXiv:math/0608284, submitted

[2] —, *Crown theory for the upper halfplane*, arXiv:0705.1243, to appear

[3] B. Krötz and E.M. Opdam, *Analysis on the crown domain*, math.RT/0606213, to appear in GAFA

[4] B. Krötz and R. Stanton, *Holomorphic extensions of representations: (I) automorphic functions*, Ann. Math. **159** (2004), 641–724.

[5] —, *Holomorphic extensions of representations: (II) geometry and harmonic analysis*, GAFA **15** (2005), 190–245.

Hydrogen atom: An operator theoretic interpretation of the generalized Titchmarsh-Weyl coefficient

Annemarie Luger
Lund University, SWEDEN

The singular differential expression

$$\ell(y) := -y'' + \frac{q_0 + q_1 x}{x^2} y, \qquad x \in (0,\infty)$$

is considered in an extended Hilbert space, which also includes locally not square integrable functions.

In this space – in a very similar way as in the regular situation – self-adjoint model operators are obtained by imposing certain boundary conditions and, in particular, the generalized Titchmarsh-Weyl-coefficient turns out to play an analogous role. Finally we use these model operators in order to deduce a new expansion result.

Covariant Differential Operators and Indecomposable Representations

Hans Plesner Jakobsen
Copenhagen University, DENMARK

The set of holomorphically induced representations associated with hermitian symmetric spaces is the natural setting for many of the covariant differential operators of physics. The representations may be viewed as extensions of representations induced from finite dimensional representations of a maximal parabolic subalgebra P - representations which moreover are trivial on the nilpotent part of P. In a special case, P is the Poincare Group together with scale transformations, and the representations one induces from in this case are trivial on the translation subgroup. The talk adresses the more general case of induction from indecomposable representations of P but will also review some of the background mentioned above.

Characterizations of Bergman space Toeplitz operators with harmonic symbols

Anders Olofsson
Lund University, SWEDEN

Joint work with Issam Louhichi. It is well-known that Toeplitz operators on the Hardy space of the unit disc are characterized by the equality

$$S_1^* T S_1 = T,$$

where S_1 is the Hardy shift operator. In this talk we present a generalized equality of this type which characterizes Toeplitz operators with harmonic symbols in a class of standard weighted Bergman spaces of the unit disc containing the Hardy space and the unweighted Bergman space. The operators satisfying this equality are also naturally described using a slightly extended form of the Sz.-Nagy-Foias functional calculus for contractions. This leads us to consider Toeplitz operators as integrals of naturally associated positive operator measures in order to take properties of balayage into account.