The authors were supported in part by a grant from the Spanish Ministry of Education.

1. Introduction

Holomorphy uniqueness theorem and support for real analytic Radon transforms

Theorems for real analytic Radon transforms
A. Let $d$ and $H$ be as in Corollary 2. Let $f$ be continuous on $H \setminus \partial d$. Then $f = 0$ if and only if $f$ is constant.

**Proof.** The proof is analogous to that of Proposition 1. The assumption in Proposition 1 allows us to consider the proposition in one in $[1]$, whereas the assumption in Corollary 2 allows us to consider the proposition in one in $[2]$.

**Corollary 3.** If $f$ is a continuous function on $H \setminus \partial d$, then $f = 0$ if and only if $f$ is constant.

**Proof.** The proof is analogous to that of Proposition 1.

Assumptions in the following corollaries do not depend on this choice.

**Corollary 4.** If $f$ is an analytic function on $H \setminus \partial d$, then the maximum principle in $H \setminus \partial d$ is equivalent to the maximum principle in $H$.

**Proof.** The proof is analogous to that of Proposition 1.

**Corollary 5.** If $f$ is an analytic function on $H \setminus \partial d$, then the minimum principle in $H \setminus \partial d$ is equivalent to the minimum principle in $H$.

**Proof.** The proof is analogous to that of Proposition 1.

**Remark.** It is sufficient to assume that $f$ is analytic on $H \setminus \partial d$ in order to obtain the same results for $f$ analytic on $H$.
3. The microlocal regularity theorem. Our Radon transform can be written

\[ R_\nu (H) = \int K(x, H, \eta) F(x) dx, \]

where the distribution \( K(x, H, \eta) \) on \( \mathbb{R}^n \) is a measure supported on the hypersurface \( Z = \{ \nu = \mu \} \) in \( T^*(\mathbb{R}^n) \) and \( \eta \) is a normal direction to \( Z \) at \( x \). The Radon transform is defined as

\[ \mathcal{F}(f) = \int_{\mathbb{R}^n} f(x) e^{-2\pi i x \cdot \xi} d\xi, \]

where \( f \) is a function on \( \mathbb{R}^n \) and \( \xi \) is a point in the dual space \( \mathbb{R}^n \).

Remark: The function \( f \) in the theorem and the corollaries may be any distribution on \( \mathbb{R}^n \) provided that the smooth condition needed here is replaced by an appropriate smooth condition valid for distributions (cf. Proposition 2).

4. Vanishing theorems for microanalytic distributions. Assume for a moment that we know the function \( f \) of \( \mathcal{F}(f), N, S, \) and \( \gamma = 0 \). For some closed analytic surface \( S \), one can show immediately from the definition of the integral of \( f \) over a family of concentric spheres \( S_t \) with radius \( t \) that the same would be true with \( f \) replaced by \( f \) for an arbitrary real analytic function on \( \mathbb{R}^n \). We assume that \( f = 0 \) in a neighborhood of \( \{ \gamma = 0 \} \).
and whose graphs are disjoint from $f$.}

(1) $\forall t \in I$ there exists $z \in \mathcal{H}$, $\rho \in \mathcal{R}$, $\gamma \in \mathcal{G}$, $\delta \in \mathcal{D}$ such that $f \ni (x, y, z, \rho, \gamma, \delta)$.

(2) $\forall t \in I$ there exists $z \in \mathcal{H}$, $\rho \in \mathcal{R}$, $\gamma \in \mathcal{G}$, $\delta \in \mathcal{D}$ such that $f \ni (x, y, z, \rho, \gamma, \delta)$.
\[
\mathcal{H} \neq x \quad \Leftrightarrow \quad \mathcal{I}(\mathcal{H} - x) \mathcal{B} \mathcal{F}(x) = (x)J
\]

Suppose \( \mathcal{H} \) is such that \( \mathcal{H} \neq x \). Then, \( \mathcal{H} \) \( \mathcal{H} \) is a homogenous differential equation in \( \mathcal{H} \) which is a linear homogeneous equation. We can therefore solve the equation \( \mathcal{H} \) in terms of \( \mathcal{H} \) and \( \mathcal{H} \) in the region where \( \mathcal{H} \) is not identified. Thus, we have \( \mathcal{H} = (x)J \).

Moreover, if \( \mathcal{H} \) is such that \( \mathcal{H} \neq x \), then \( \mathcal{H} \) is a homogenous differential equation in \( \mathcal{H} \) which is a linear homogeneous equation. We can therefore solve the equation \( \mathcal{H} \) in terms of \( \mathcal{H} \) and \( \mathcal{H} \) in the region where \( \mathcal{H} \) is not identified. Thus, we have \( \mathcal{H} = (x)J \).

6. Complementary To show that the differential equation \( \mathcal{H} \) is solved by \( x \) in the region where \( \mathcal{H} \) is not identified, we need to find a solution \( x \) of the differential equation \( \mathcal{H} \) and \( \mathcal{H} \) in the region where \( \mathcal{H} \) is not identified.

Assume now that \( x \not\in \mathcal{H} \). Suppose \( x \) is such that \( x \neq \mathcal{H} \). Then, \( x \) is a homogenous differential equation in \( \mathcal{H} \) which is a linear homogeneous equation. We can therefore solve the equation \( \mathcal{H} \) in terms of \( \mathcal{H} \) and \( \mathcal{H} \) in the region where \( \mathcal{H} \) is not identified. Thus, we have \( \mathcal{H} \neq x \).