Homological algebra and algebraic topology
Problem set 10

due: Tuesday Nov 26 in class.

Problem 1 (4pt)**.** Let X be the space obtained from S^2 by identifying antipodal points on the equator $S^1 \subset S^2$.

(1) Show that X admits the structure of a cell complex.
(2) Calculate the homology groups of X.
(3) Do the same thing for S^3 with antipodal points of the equator $S^2 \subset S^3$ identified.

Problem 2 (3pt)**.** Suppose we are given a cell complex homeomorphic to S^2 that is built from a finite collection of polygons by identifying edges in pairs, for example as in the following picture:

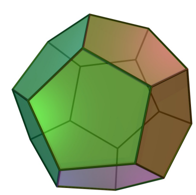
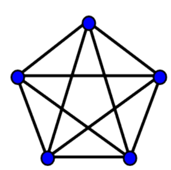
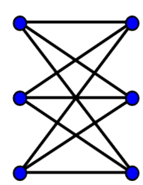


FIGURE 1. Cell complex with 20 vertices, 30 edges and 12 faces.

Show that the 1-skeleton of any such a cell complex cannot be either of the following two graphs.





Problem 3 (3pt)**.** Show that the space $\{0\} \cup \{1/n \mid n = 1, 2, \ldots\} \subset \mathbf{R}$ is not homotopy equivalent to a cell complex.